

# PROPOSAL OF AN INVERSE LOGARITHM SCALING LAW FOR THE LUMINOSITY EVOLUTION

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## Abstract

A scaling law for the time-dependence of the dynamic aperture, i.e., the region of phase space where stable motion occurs, was proposed in previous papers, about ten years ago. It was showed that dynamic aperture has a logarithmic dependence on time, which would be suggested by some fundamental theorems of the theory of dynamical systems. Such a scaling law was recently extended also to the intensity evolution in a storage ring. In this paper, inspired by these results, and inverse logarithm scaling law for the luminosity in a circular collider is proposed. Such a time-dependence is then tested against the data from the LHC physics runs and also with some examples from other machines. The results are presented and discussed in detail.

## INTRODUCTION

The luminosity is the key figure-of-merit for colliders and is expressed as

$$L = \frac{\gamma_r f_{rev} N_b N_1 N_2}{4 \pi \epsilon^* \beta^*} F(\theta_c, \sigma_z, \sigma^*), \quad (1)$$

where  $\gamma_r$  is the relativistic  $\gamma$ -factor,  $f_{rev}$  the revolution frequency,  $N_b$  the number of colliding bunches,  $N_i$  the number of particles per bunch in each colliding beam,  $\epsilon^*$  is the rms normalised transverse emittance, and  $\beta^*$  is the value of the beta-function at the collision point. The factor  $F$  accounts for the reduction in volume overlap between the colliding bunches due to a crossing angle and is a function of the crossing angle  $\theta_c$ , the transverse ( $\sigma^*$ ) and longitudinal ( $\sigma_z$ ) rms dimensions. Eq. (1) is valid in the case of round beams ( $\epsilon_x^* = \epsilon_y^*$ ) and round optics ( $\beta_x^* = \beta_y^*$ ). Under normal conditions, i.e., excluding any levelling gymnastics or dynamic beta effects, only the emittances and the bunch intensities can change. Therefore, Eq. (1) is better interpreted as peak luminosity at the beginning of the fill, while in general  $L$  will be a function of time. When the burn off is the only relevant mechanism for a time-variation of the beam parameters, it is possible to estimate the time-evolution of the luminosity, which turns out to be

$$L(t) = \frac{L_{peak}}{(1 + at)^2}. \quad (2)$$

In reality, the situation is much more complex. In the case of a hadron collider, e.g., beam-beam and IBS affect the beam parameters in such a way that the model (2) is not valid anymore. Several approaches can be followed, for instance, in Refs. [1, 2] phenomenological fit models were proposed and applied with success to the characterisation

of luminosity evolution in the Tevatron machine. The functional form of the proposed models was suggested by considerations on scaling laws of key quantities, such as emittances, whenever IBS phenomena are considered.

Alternatively, in Ref. [3] the luminosity evolution is studied starting from numerical simulations taking into account the relevant physical processes. Also in this case the proposed approach was successful (in particular the one based on direct tracking) in reproducing the luminosity evolution in RHIC.

In this paper an alternative model to those used in Refs. [1, 2] is proposed. The basis for such a model is the evolution of the dynamic aperture (DA) with time in a hadron collider. Some years ago, the analysis of single-particle tracking results showed that the evolution of the DA follows a simple law [4, 5], whose justification is not entirely phenomenological. Recently, this approach was successfully applied to the analysis of intensity evolution in hadron machines [6]. So far, however, the results were obtained for single-particle simulations or for conditions in a running machine that were not including any collective effect. In fact, to extend the proposed scaling law to luminosity evolution, it is necessary to show that it is valid also in the presence of beam-beam effects. This seems to be case and the results of numerical simulations are discussed in a companion paper [7], which then opens the possibility to justify the approach proposed.

## LUMINOSITY EVOLUTION OVER TIME

The starting point is the dynamic aperture (defined as the radius of the region in phase space where stable motion over a given number of turns occurs) as a function of time. Assuming a polar grid in phase space (with co-ordinates expressed in units of beam  $\sigma$ )

$$x = r \cos \theta \quad y = r \sin \theta \quad \text{with} \quad 0 < \theta < \pi/2, \quad (3)$$

if  $r(\theta; t)$  stands for the last stable amplitude up to  $t$  turns in the direction  $\theta$ , then the dynamic aperture reads:

$$D(t) = \frac{2}{\pi} \int_0^{\pi/2} r(\theta; t) d\theta \equiv \langle r(\theta; t) \rangle. \quad (4)$$

According to the results reported in Refs. [4, 5], the following scaling law holds

$$D(t) = D_\infty + \frac{b}{[\log t]^\kappa}, \quad (5)$$

where  $D_\infty$  represents the asymptotic value of the amplitude of the stability domain, while  $b$  and  $\kappa$  are additional parameters. These three quantities can be obtained by fitting the results of numerical simulations. Under these assumptions  $D(t)$  is expressed in units of beam  $\sigma$ .

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The interesting point is that such a parametrisation is compatible with the hypothesis that the phase space can be partitioned into two regions: a central core, with  $r < D_\infty$ , where KAM [8] surfaces confine the motion, thus inducing a stable behaviour apart for a set of small measure where Arnold diffusion can take place; an outer part, with  $r > D_\infty$ , where chaotic motion occurs and the escape rate is given by a Nekhoroshev-like estimate [9, 10] such as

$$T(r) = N_0 \exp\left(\frac{r_*}{r}\right)^{1/\kappa} \quad (6)$$

where  $T(r)$  is the number of turns that are estimated to be stable for particles with initial amplitude smaller than  $r$ .

If the beam distribution is assumed to be Gaussian, then by integrating over  $x'$  and  $y'$  and after changing coordinates and a second integration over  $\theta$  one obtains the final expression for the beam distribution,  $\hat{\rho}(r) = r e^{-\frac{r^2}{2}}$ . By using the very definition of  $D(t)$ , it is clear that the evolution of the beam intensity  $N(t)$  can be found as

$$\frac{N(t)}{N_0} = 1 - \int_{D(t)}^{+\infty} \hat{\rho}(r) dr = 1 - e^{-\frac{D^2(t)}{2}}. \quad (7)$$

Starting from these considerations and assuming that Eq. (7) holds in a more general context than the original one and that, furthermore, the intensities of the two beams follow the same scaling law with the same (or similar) values of the parameters, then

$$L(t) = L_{peak} \left[ 1 - e^{-\frac{D^2(t)}{2}} \right]^2, \quad (8)$$

where  $L_{peak} = \gamma_r f_{rev} N_b N_{1,0} N_{2,0} / (4 \pi \epsilon^* \beta^*) F(\theta_c, \sigma_z, \sigma^*)$ . This would allow fitting the four parameters  $L_{peak}, D_\infty, b, \kappa$  from the measured luminosity evolution. The last three parameters could also be obtained by fitting the intensity evolution with time.

Even  $L_{peak}$  might depend on time because of emittance growth phenomena. However, in case total beam intensity measurements are available, these can be used to obtain the fit parameters of the inverse logarithm law. Then, one can define  $\varepsilon(t) = N_1(t)N_2(t)/L(t)$ , then the quantity  $\varepsilon(t)/\varepsilon(0) - 1$  can be easily derived from the luminosity measurement and analysed to determine its properties and hence study the emittance growth processes. Of course, the option of a blind fit of the measured luminosity using the model (8) is always possible. Nonetheless, the physical meaning of the fit parameters  $D_\infty, b, \kappa$  might be lost and the whole procedure would become rather a phenomenological approach.

## RESULTS OF DATA ANALYSIS

### LHC Case

The proposed model was applied to the LHC luminosity data for the 2011 physics run. Both proton and ion cases have been considered in this study, even if only the proton

case is reported here. The first step was to consider the intensity evolution. In Fig. 1 (upper) an example of the total intensity evolution for Beam 1 and 2 is showed, together with the fit (7) and the very good agreement is clearly observed. Next, the analysis of emittance growth effects was

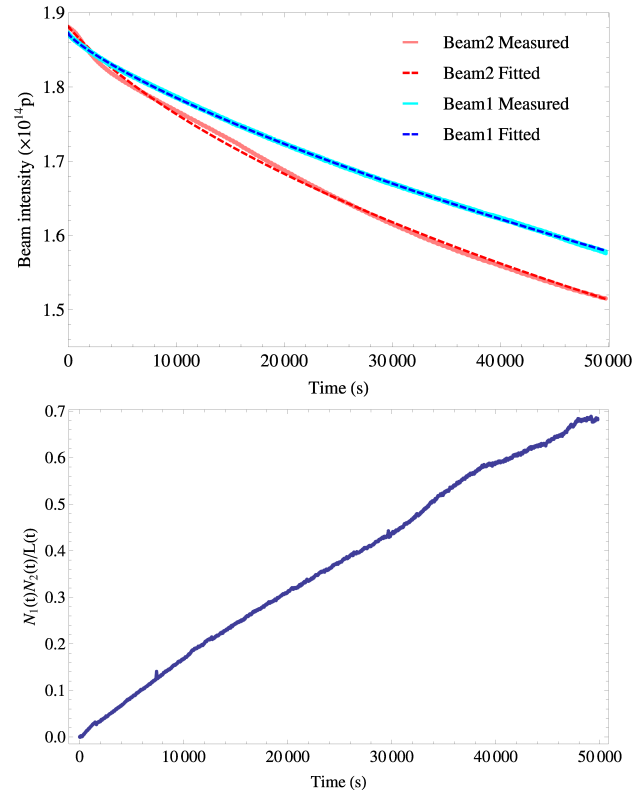


Figure 1: Measured intensity evolution for the LHC Beam 1 and 2 for proton fill 2208 (upper). The fits based on (7) are also reported, showing the good agreement, even if the two beams feature different behaviours. Relative variation of  $N_1(t)N_2(t)/L(t)$  for proton fill 2208 (lower). A non-negligible emittance growth is visible.

made to assess whether these effects play a role in the evolution of the luminosity. An example is plotted in Fig. 1 (lower).

In the following, the direct application of the model (8) to  $L(t)$  is considered. In Fig. 2 an example is shown and the good agreement is clearly visible. A detailed analysis of the distribution of the fit parameters for the model (8) was performed. In particular, it was checked whether any correlation was found between the fill length and the fit parameters, but no relationship was found. The main result is shown in Fig. 3. It is worthwhile stressing that during the whole 2011 the LHC performance was increased, however, a clear trend is only observed for  $L_{peak}$ .

### Tevatron Case

To show the general character of the proposed scaling law, data from other colliders have been analysed. It is the case of luminosity evolution for the Tevatron published in Ref. [1]. The three different models used to analyse

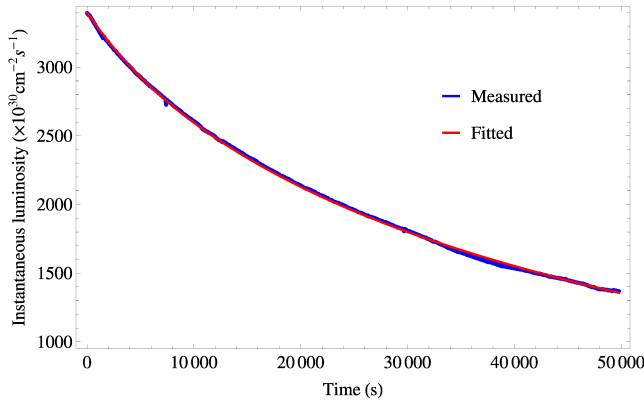


Figure 2: Measured  $L(t)$  by the ATLAS experiment for proton fill 2208 together with the fit (8).

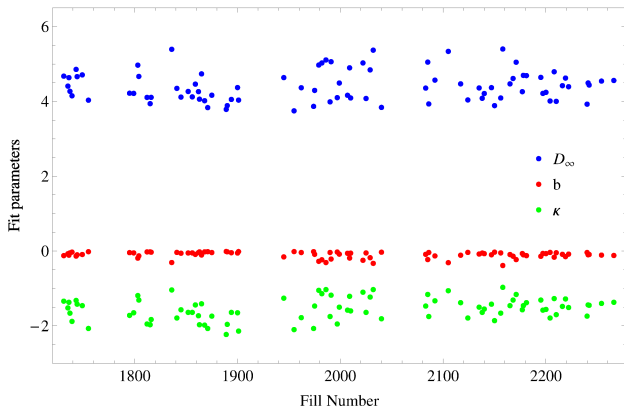


Figure 3: Evolution of the fit parameters  $D_\infty, b, \kappa$  for model (8) as a function of fill number. The fills considered are for the 2011 proton physics run.

$L(t)$  have been considered here together with the proposed model (8). The results of the fit are shown in Fig. 4, where the measured data is plotted together with the four fitting curves. If one excludes the pure exponential decay, the re-

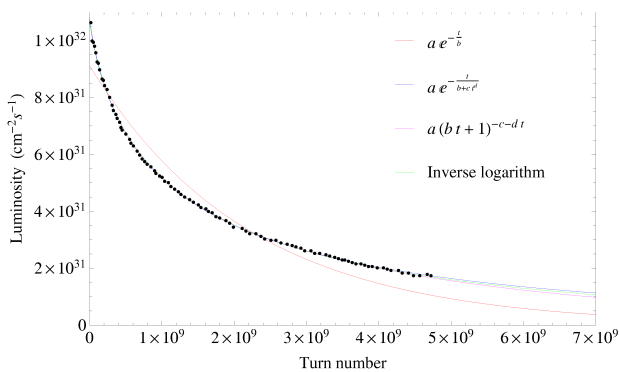


Figure 4: Measured luminosity evolution for the Tevatron (fill 4081 from Ref. [1]) together with the fit models [1] and (8). The latter is at least as good as the others, but with one fit parameter less.

maining three models are all very good approximations of

the real data. The one based on the inverse logarithm scaling law is intermediate between those from Ref. [1] and as good as the other two, in spite of having three instead of four fit parameters. Once more, this indicates that the proposed model is very accurate even as a pure phenomenological description of the data. A double-exponential fitting function, which has been successfully used in other studies [3], has not been tested here, but we believe that it would have provided a good agreement with the data.

## CONCLUSIONS

In this paper a model for describing the time-evolution of the luminosity in a hadron collider has been proposed. This was justified on a number of recent results concerning the scaling law of dynamic aperture and intensity variation with time for hadron machines. The proposed scaling law was successfully applied to LHC data, for which both proton and ion fills have been analysed and found to be well described by the model. Data from the Tevatron machine were analysed as well, showing the same good agreement with the scaling law (8). In the current studies a number of simplifying assumptions have been made, in particular, the burn off was not taken into account separately. Its effect will be considered and used to refine the proposed model.

We would like to stress that this paper reports on work still in progress and the next step is to find the appropriate physical meaning to the fit parameters, e.g., the interpretation of  $D_\infty$  whenever collimators are used. This is a crucial point to move from a pure phenomenological approach to using the proposed model as a predictive tool of the behaviour of luminosity in a hadron collider.

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