STUDY OF RESONANCE DRIVING TERM IN ELECTRON STORAGE RINGS*

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Abstract

Resonance driving terms including the information of nonlinear resonance can be used to analyse the nonlinear effect of the accelerator. In recent years, the driving terms have been successfully measured from the spectral decomposition of the turn-by-turn BPM data in proton storage rings and driving term measurements become powerful tool to calibrate nonlinear model of storage ring. In HLSII upgrade project, several TBT BPM systems will be installed in storage ring. The possibility of application driving term measurement techniques in HLSII storage ring is studies theoretically and numerically in this paper.

INTRODUCTION

Nonlinear dynamics have played an important role in most of modern advanced accelerators. The model of nonlinearity must be calibrated to make sure the system running at design parameters. Measuring resonance driving terms which including the information of nonlinear resonance can be one of the best methods to analyse the nonlinearity quantitatively. Thanks to the development of perturbation theory [1] and Normal Form techniques [2], a relation between resonance driving term and the spectral lines of the turn-by-turn BPM data has been build, which realized the measurement of resonance driving term from turn-by-turn BPM position data. The first experiments using this method were tried at the SPS and LEP in the years 1996 until 1999 [3], and from then on many experiments had been performed successfully in proton storage rings at some other laboratories. More recently, the effect of decoherence due to the tune shift with amplitude and chromaticity has been taken into account, and the relation between the spectral lines of the centroid motion and the resonance driving term has been found. The experiment had been performed at CERN and the result turned out very good [5].

But up to now this method is applied in electron storage ring rarely. After the undergoing upgrade project, the beam position monitor (BPM) system at Hefei Light Source (HLS) will have the capability to measure turn-by-turn beam position. To calibrate the model of nonlinearity, measuring resonance driving term with the same technique will be tried to understand the nonlinearity.

Compared with Hadron machine, the radiation damping is stronger in electron storage ring. In this report, how large the effect of radiation damping is will be calculated and simulated by tracking data. The relation between the spectral lines of the centroid motion and the resonance driving term will be introduced considering the effect of radiation damping and decoherence due to the tune shift with amplitude and the first order chromaticity.

NORMAL FORM

According to the theory of Normal Form [2], the relation between the spectral lines of the single particle motion and the generating function terms can be obtained in the form

$$\begin{aligned} h_x^-(N) &= x - ip_x = \sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{x0})} - 2i \\ &\times\sum_{jklm} jf_{jklm}(2I_x)^{(j+k-1)/2}(2I_y)^{(l+m)/2} \\ &\times e^{i[(1-j+k)(2\pi\nu_x N+\psi_{x0})+(m-l)(2\pi\nu_y N+\psi_{y0})]} \\ &= \sum_{jklm} HSL_{jklm} e^{2\pi i[(1-j+k)\nu_x+(m-l)\nu_y]N} \end{aligned} \tag{1}$$

Where $h_x^-(N)$ is linearly normalized horizontal variable, $I_{x,y}$ are the action variables, $\nu_{x,y}$ are the transverse tunes, $\psi_{x0,y0}$ are the initial phases, f_{jklm} are the generating function terms. The resonance driving terms are expressed in the Hamiltonian coefficients which can be calculated from the generating function coefficients by the well known expression:

$$f_{jklm} = \frac{h_{jklm}}{1-e^{i\{2\pi[(j-k)\nu_x+(l-m)\nu_y]\}}} \tag{2}$$

The information of resonance driving terms is included by the complex Fourier coefficient of the spectral lines HSL_{jklm} with amplitude $|HSL_{jklm}|$, which can be observed from turn-by-turn BPM data. But in a real machine, the BPM are used to record the turn-by-turn centroid position of the beam instead of the single particle. Because of the tune spread of individual particles in the beam due to transverse nonlinearity and the chromaticity [4], the motion of the beam will be decohere and the amplitude of the spectral lines transferred from the turn-by-turn centroid position will be different with that of single particle. So the effect of decoherence cannot be ignored.

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DECOHERENCE

It is assumed that the particles in a bunch are non-interacting, and their distribution is Gaussian. Then the centroid motion can be given by averaging over the beam bunch [5].

The linear approximation of the tune with the tune shift due to amplitude and first order chromaticity is expressed by

$$\begin{aligned} \nu_x &= Q_x + \nu'_{xx} 2I_x + \nu'_{xy} 2I_y + \Delta\nu_x \\ \nu_y &= Q_y + \nu'_{yx} 2I_x + \nu'_{yy} 2I_y + \Delta\nu_y \end{aligned} \quad (3)$$

The additional tune oscillation due to off-momentum particles $\Delta\nu_{x,y}$ are given by

$$\Delta\nu_{x,y} = \frac{Q'_{x,y}\delta}{\pi Q_s N}\cos(\pi Q_s N + \psi_s)\sin(\pi Q_s N) \quad (4)$$

where $Q_{x,y}$ are the transverse nominal tunes, Q_s is the synchrotron tune, $Q'_{x,y}$ are the chromaticities, ν' are the amplitude detuning terms, N is the turn number.

The expression of the bunch's distribution is given by

$$\begin{aligned} \rho_x(I_x,\psi_x) &= \frac{1}{2\pi} e^{-(1/2)(2I_x + \bar{A}_x^2 - 2\bar{A}_x\sqrt{2I_x}\cos\psi_x)} \\ \rho_y(I_y,\psi_y) &= \frac{1}{2\pi} e^{-(1/2)(2I_y + \bar{A}_y^2 - 2\bar{A}_y\sqrt{2I_y}\cos\psi_y)} \\ \rho_s(\delta,\psi_s) &= \frac{\delta}{2\pi\sigma_s^2} e^{-\delta^2/(2\sigma_s^2)} \end{aligned} \quad (5)$$

where $\bar{A}_{x,y}$ are the kick amplitudes.

Therefore the relation between the spectral lines of centroid motion and the resonance driving terms can be derived as

$$\begin{aligned} \overline{h_x^-}(N) &= \int_0^\infty dI_x \int_0^\infty dI_y \int_0^\infty d\delta \int_0^{2\pi} d\psi_x \int_0^{2\pi} d\psi_y \\ &\times \int_0^{2\pi} d\psi_s \rho_x(I_x,\psi_x)\rho_y(I_y,\psi_y)\rho_s(\delta,\psi_s) h_x^-(N) \\ &= L_{1100}(N) - 2i\sum_{jklm} j f_{jklm} L_{(j+k-1)(1-j+k)(l+m)(m-l)}(N) \end{aligned} \quad (6)$$

where L_{nmlk} is expressed by

$$\begin{aligned} L_{nmlk} &= \int_0^\infty dI_x dI_y (2I_x)^{n/2}(2I_y)^{l/2} \\ &\times I_m(\bar{A}_x\sqrt{2I_x}) I_k(\bar{A}_y\sqrt{2I_y}) \\ &\times e^{-(1/2)(2I_x + \bar{A}_x^2 + 2I_y + \bar{A}_y^2)} \times e^{i2\pi(m\hat{\nu}_x + k\hat{\nu}_y)N - 2\gamma_{mk}^2 \sin^2(\pi Q_s N)} \end{aligned} \quad (7)$$

where $\gamma_{mk} = (mQ'_x + kQ'_y)\sigma_s / Q_s$, $\hat{\nu}_{x,y}$ contain only the amplitude detuning without the additional tune as expressed in Eq. (3).

RADIATION DAMPING

The Normal Form technique is developed from Hamiltonian mechanics, which provides a highly systematic framework for constructing the equations of motion. Therefore the map of the particles moving through an electromagnetic field must be symplectic. But in electron storage rings, the effect of radiation damping can conveniently be quantified in terms of deviations from symplecticity. How large the effect is must be calculated.

According to the theory [6] of radiation damping, the amplitude can be given by

$$A(N) = A_0 e^{-\alpha_{x,y} N T_0} \quad (8)$$

where the damping coefficients are

$$\alpha_x = (1 - \mathscr{D})\frac{U_0}{2E_0 T_0} \qquad \alpha_y = \frac{U_0}{2E_0 T_0} \quad (9)$$

and the damping partition $\mathscr{D}$ is given by

$$\mathscr{D} = 1 - \frac{2}{\theta_0}\tan\left(\frac{\theta_0}{2}\right) \quad (10)$$

Then the damping coefficients can be calculated with the parameter from Table 1:

Table 1: Beam Parameters at HLSII

Beam energy [MeV]	800
Energy lost per turn [MeV]	1.673
Angle of the bend [DEG]	45
Transverse tune	4.4141, 3.2234
Synchrotron tune	0.00677
Chromaticity	1.0017, 0.9921
Nominal rms bunch length [mm]	14.8

$$\alpha_x = 1.1036 \times 10^{-5} / T_0 \qquad \alpha_y = 1.0463 \times 10^{-5} / T_0$$

These show obviously that both of the amplitudes decay very slowly with the increasing turn. It is confirmed that the effect of the radiation damping is too slight to affect the use of the Normal Form technique when the turn number is small enough. But the expression of the relation between the spectral lines of centroid motion and the resonance driving terms given by Eq.(6) and (7) cannot be used in electron storage rings directly.

Considering the effect of radiation damping, the action variables $I_{x,y}$ have to be modified including the damping terms as expressed in the Eq. (8), which can be expressed by

$$I_x(N) = I_x e^{-2a_x N T_0}$$
$$I_y(N) = I_y e^{-2a_y N T_0} \quad (11)$$

It seems impossible to integrate over $I_{x,y}$ in Eq. (7) with the replacement in Eq. (11) directly.

SIMULATIONS

In this section, the turn-by-turn BPM data of the centroid is tracked with the Elegant software using the model of the storage ring at HLSII. And the Fourier transform of the centroid is calculated with SUSSIX [7].

First of all, the difference of the Fourier spectral lines with the effect of radiation damping or not has been compared. The first 1024 turns centroid data have been tracked with the 0.01m kick amplitude.

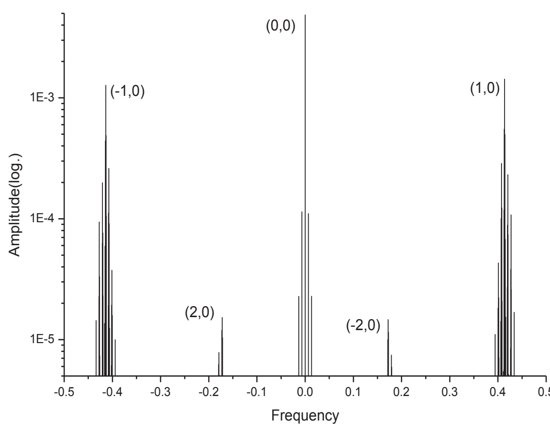

Figure 1: Spectrum of horizontal centroid motion of the beam for the HLS without the effect of radiation damping.

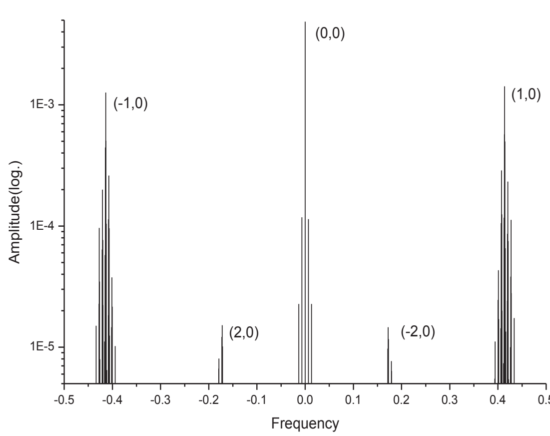

Figure 2: Spectrum of horizontal centroid motion of the beam for the HLS without the effect of radiation damping.

The amplitudes are compared in Table 2.

Table 2: Amplitude of the Spectral Lines (m)

Spectral lines	No radiation	Radiation
(-1,0)	1.27919e-3	1.26272e-3
(2,0)	1.53474e-5	1.52072e-5
(0,0)	4.88419e-3	4.86596e-3
(-2,0)	1.47348e-5	1.46026e-5
(1,0)	1.43889e-3	1.42104e-3

It shows that the effect of the radiation is very slight, and the theory of Normal Form can still be used in electron storage rings when the turn number isn't too large.

CONCLUSIONS

This paper tries to use the Normal Form theory in electron storage rings and founds a relation between the spectral lines of the turn-by-turn centroid motion and resonance driving term including the effect of radiation damping. The effect of radiation damping has been calculated in theory and simulated with tracking data, which turn out that the effect is too slight to affect the use of the Normal Form theory when the turn number is under several thousands. Therefore we can solve the expression of the relation between the spectrum and resonance driving term by ignoring the effect of radiation damping. We can also solve it precisely by using the numerical method which can be tried in future work.

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