# FREQUENCY MAP ANALYSIS FOR SUPERB

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### Abstract

The frequency map analysis is applied to the SuperB lattices including the Final Focus, in order to understand the dynamic aperture limitations and provide insight for a working point optimization. In this respect, frequency and diffusion maps are evaluated applying random magnet misalignments and tilts, before and after correction of orbit, dispersion and coupling using Low Emittance Tuning techniques. The same analysis is performed for on and off momentum particles. The lattice properties are further investigated using working point scans and the correction of non linear resonance driving terms and amplitude detuning.

#### **INTRODUCTION**

In this paper, a review of the work performed on the SuperB lattices using the Frequency Map Analysis (FMA) is presented. The SuperB B-factory is a  $e^+e^-$  asymmetric collider [1] (6.7 GeV  $e^+$  and 4.18 GeV  $e^-$ ) with a goal peak luminosity of  $10^{36}$  cm<sup>-2</sup>s<sup>-1</sup>. The design is based on very low emittances (2 nmrad horizontal and 5 pmrad in vertical) and very low vertical beta functions at the interaction point. Very accurate studies are henceforth required to guarantee these performances since strong non linearities arise from both the Final Focus (FF) and the arcs. In particular the FF contribution is due to the strong sextupoles used for local chromaticity compensation and to the presence of very strong quadrupoles in the final doublets. The Frequency Map Analysis (FMA) method is employed in order to gain more insight on the lattice and in particular its non linear properties. In particular this analysis is focused on the working point optimization and on the correction of observed resonances using the tools available in MADX-PTC [2] in order to increase the dynamic aperture of the lattices.

#### **FREQUENCY MAPS**

The FMA technique has been used in recent years [3] to explore the tunes and phase-space planes in order to optimize non linearities. In a frequency map, a direct relation is determined between the position of a particle in the space, its frequencies (tunes) of motion in all dimensions and their diffusion along time due to the presence of non linearities in the lattice. For every particle tracked over N turns the diffusion rate D is evaluated as follows:

$$e^{D} = \sqrt{\frac{(\nu_{x,1} - \nu_{x,2})^{2} + (\nu_{y,1} - \nu_{y,2})^{2}}{N/2}}$$

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where subscripts denote tunes evaluated for two consecutive time spans. This quantity allows to give a direct estimation of the dynamical behavior of the tune during these N turns. When D is large, the motion is chaotic and the particle will probably be lost, over the next few turns [4]. The tune however needs to be estimated by accurate techniques that allow higher resolutions compared to a standard FFT. In this study, the algorithm NAFF [4] is applied, originally used for this analysis due to its very high precision, growing like the fourth power of the number of turns.

The FMA technique will be used to study off momentum particle dynamics, effects of misalignments and correction, working point optimisation and correction of observed resonances.

#### SuperB Frequency Map

The tracking is performed in MADX-PTC (MADX-4.00), using the matrix-kick-matrix model and the EXACT Hamiltonian option. The points in the space are chosen to be equally spaced in the invariant emittance ( $\sigma^2 \sim \beta \epsilon$ ) and the region of the scan is determined to allow high resolution and a common range for the various conditions that will be studied. The frequency map obtained for the reference lattice of SuperB HER is shown in Figure 1, and should be used as a baseline for the comparison among all the following maps. The top figure shows the footprint in the tune space for 4900 tracked particles, while the bottom plot shows the same particles as distributed in configuration space at the beginning of the tracking, where black dots are lost particles. In both plots the color refers to the diffusion rates D (red is large, blue is small) evaluated for every tracked particle over N = 1024 turns. Resonances up to the  $5^{th}$  order are also plotted as dotted lines. The map evidences the presence of  $2Q_x - 2Q_y$  and  $5Q_x = 3$ resonances that will be analyzed later. In the same figure the frequency maps with a relative momentum deviation of  $\delta = \frac{\Delta p}{r} = \pm 0.01$  are presented, where the dynamic aperture is radically reduced, especially in the horizontal plane.

#### Misalignments and Low Emittance Tuning

The frequency map obtained by applying random misalignments and tilts to quadrupoles and sextupoles and correcting orbit, dispersion and coupling with the Low Emittance Technique (LET) [5] is presented in Figure 3. LET uses, in this case, only horizontal and vertical correctors to recover from the random misalignments' set. The lattice has not been retuned after correction, and the working point determined by the misalignments and the subsequent correction shows a net improvement in the vertical

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Figure 1: SuperB HER V16 Frequency Maps. Tune plane (top) and transverse configuration space (bottom): starting point for ideal SuperB lattice (center), with  $\frac{\Delta p}{p} = -0.01$  (left) and  $\frac{\Delta p}{p} = +0.01$  (right). The maps are computed by tracking 4900 particles for 1024 turns over a range of  $45\sigma_x$  in the horizontal plane and  $22.5\sigma_x/2$  in the vertical plane. Black dots are lost particles.

plane, giving strong indications that a tune scan is indeed needed. The frequency map also evidences the resonance node as the cause of the sharp cut observed in the horizontal dynamic aperture. The frequency map evaluated without correction is not shown here, since it consists of only few survived particles mainly distributed on the  $2Q_x - 2Q_y$ resonance.

#### Working Point Scan

The tunes are then modified using the tune trombone in the straight section of the SuperB lattice. A change in tune using the quadrupoles of the arc cells is not performed here due to strong constraints on the phase advance between the arc sextupoles. The tune trombone allows however to scan an interval of  $\pm 0.05$  in tune. The figure of merit chosen to determine an alternative working point is the sum of the diffusion rates with a constant (empirically chosen) added for every lost particle [6].

$$WPS = 0.1N_{lost} + \sum e^D$$

This choice allows for taking into account at the same time: the increase in dynamic aperture (less lost particles) and the presence of less detrimental resonances. Figure 2 shows an interpolation of the results of this calculation simulating 100 particles for 512 turns for 441 different working points. A possible alternative working point is observed at  $Q_x = Q_x^0 + 0.045$  and  $Q_y = Q_y^0 + 0.015$ , and the ISBN 978-3-95450-115-1



Figure 2: Working Point Scan. The reference working point set is the center of the plot. The figure is an interpolation figure of  $11 \times 11$  simulations

frequency map evaluated at this point is shown in Figure 3. The effect of the  $2Q_x - 2Q_y$  resonance is not observed as the new working point is below it. However this alternative point may not be compatible with luminosity considerations, that are not presented here.



Figure 3: SuperB HER V16 Frequency Maps: corrected misalignments with LET (left), work point scan alternative point (center), resonant Driving Terms optimization (right). The maps are performed by tracking 4900 particles for 1024 turns over a range of  $45\sigma_x$  in the horizontal plane and  $22.5\sigma_x/2$  in the vertical plane. Black dots are lost particles.

#### Non Linear Optimization

Non linear resonances are evident in all the above figures. In particular a  $5^{th}$  order resonance  $5Q_x = 3$  and the difference resonance  $2Q_x - 2Q_y$  are emphasized by a higher diffusion rate. The MADX PTC-Normal routine allows to extract to any order the Hamiltonian resonance driving terms [7]. This may then be used as parameters to perform a matching routine aimed to minimize their effect by tuning the magnet strengths. The minimization of the term  $h_{2002}$  that drives the resonance  $2Q_x - 2Q_y$  is performed (starting from the reference lattice) using as knobs the strengths of 8 octupoles, 2 couples in arcs and 2 couples in the Final Focus (at constant chromaticity). The frequency map obtained is shown in figure 3. The dynamic aperture is improved by the correction of this resonance. In particular, the tune-shift with amplitude shrinks. More work is needed since same high diffusion rate spots are present in the configuration space at approximately  $22\sigma_x$ , evidently due to the same resonance.

## CONCLUSIONS

Frequency Map Analysis has been used for SuperB HER lattice to analyze the influence of various parameters on the lattice. Looking at the reference frequency map, two main resonances are observed, in particular a detrimental difference resonance. The effect of misalignments and correction on the map was shown and a tune scan was performed using the sum of diffusion rates as figure of merit. Using the informations retrieved from frequency maps it has been possible to use MADX to determine a tuning procedure for the lattice, focused on the cancellation of the specific resonances observed, with the result of an improvement in dynamic aperture in both planes. These studies are at an early stage for SuperB and will b extensively used in the near future. The same calculations have been performed for the Low Energy Ring, with and without crab waist sextupoles and with the influence of fringing filed in all magnets. A detailed study of all the possible resonances that may be removed using the resonance driving terms is also currently under study.

#### REFERENCES

- [1] M.E. Biagini, et al, Overview of Super B Factories, this conf.
- [2] MAD-X Home Page. http://mad.web.cern.ch/mad/.
- [3] C. Steier et al., Phys. Rev. E, 65, (056506) (2002)
- [4] J. Laskar, Frequency map analysis and particle accelerators, PAC03, Portland (2003)
- [5] S.M. Liuzzo, TUOAA03, this conference.
- [6] D. Robin et al., Phys. Rev. Let., 85, pp. 558-561 (2000).
- [7] J. Bengtsson, Tech.Rep. SLS 9/97.

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