SPACE CHARGE EFFECT IN THE PRESENCE OF X-Y COUPLING IN J-PARC MR

K. Ohmi, Y. Satoh, KEK, 1-1 Oho, Tsukuba, 305-0801 Japan
S. Hatakeyama, JAEA, Tokai, Japan

Abstract

It is crucial issue to suppress beam loss due to space charge force in J-PARC MR. We focus x-y coupling as a source of the beam loss. x-y coupling is measured by turn-by-turn beam position monitors in J-PARC MR. Emittance growth and beam loss are evaluated by a space charge simulation (SCTR) under the measured x-y coupling. We discuss how x-y coupling affect the emittance growth and beam loss.

INTRODUCTION

x-y coupling is characterized by 4 parameters, r_1-r_4 in this paper. The 4x4 revolution matrix is diagonalized block-wise by a matrix consist of the coupling parameters,

$$M_4(s) = R(s)M_{2\times 2}(s)R(s)^{-1} \qquad (1)$$

where

$$R = \begin{pmatrix} r_0 & 0 & r_4 & -r_2 & 0 & 0 \\ 0 & r_0 & -r_3 & r_1 & 0 & 0 \\ -r_1 & -r_2 & r_0 & 0 & 0 & 0 \\ -r_3 & -r_4 & 0 & r_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2)$$

and

$$M_{2\times 2} = \begin{pmatrix} M_u & 0 \\ 0 & M_v \end{pmatrix} \qquad (3)$$

$$M_i = \begin{pmatrix} \cos\mu_i + \alpha_i \sin\mu_i & \beta \sin\mu_i \\ -\gamma_i \sin\mu_i & \cos\mu_i - \alpha_i \sin\mu_i \end{pmatrix}$$

Effect of x-y coupling is evaluated using several model lattices of J-PARC MR. x-y coupling also measured using turn-by-turn monitors.

X-Y COUPLING IN MODEL LATTICES

We use three model lattices:

1. Design
2. Measurement of magnet positions in 2010.
3. Measurement in 2011, after the earthquake on 11 Mar.

The third lattice does not really exist now, because positions of magnets are aligned. Closed orbit is corrected every models. Residual x-y coupling for each lattice is plotted in Figure 1. x-y coupling of design lattice is 1/5-1/10 compare to 2010, but is not zero, because of an injection bump. Figure 2 shows detailed behavior of the coupling parameters. The figure indicates that the coupling is induced by the rotation of the quadrupoles.

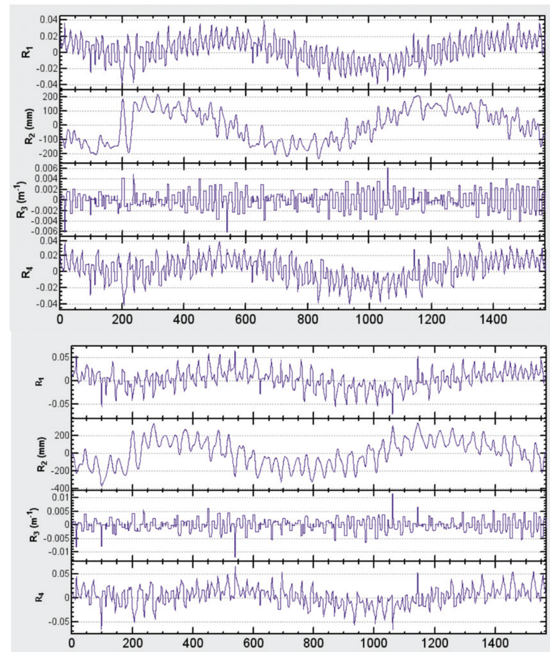


Figure 1: x-y coupling parameters along s for lattices with errors. Top and bottom are the lattices at 2010 and 2011, respectively.

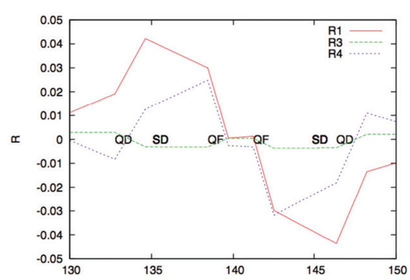
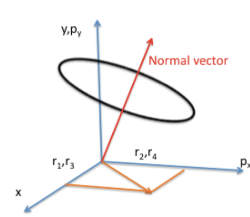


Figure 2: Detailed behavior of the coupling parameters, 130<s<150m.

x-y coupling is measured by a trajectory in 4 dimensional phase space at single mode excitation [1]. Figure 3 shows the measured coupling at Feb 2011 (before the earthquake). The values agree with Figure 1 in the order of magnitude. Note that the unit of r_2 in Figures 1 (mm) and 3 (m).



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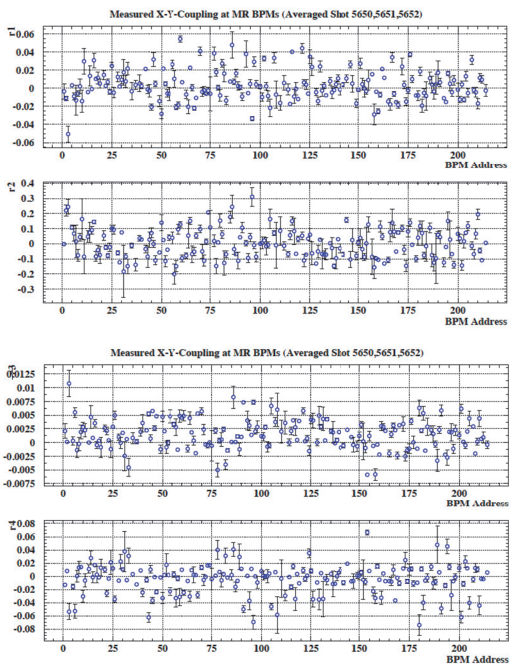

Figure 3: Measured x-y coupling. Units of r_2 and r_3 are m and m^{-1}, respectively.

SPACE CHARGE SIMULATION IN THE PRESENCE OF X-Y COUPLING

We study the effects of x-y coupling with a space charge simulation code, named SCTR [2]. In the space charge simulation, beam particles are kicked by self electro-magnetic field, which is evaluated every 1m traveling. Since minimum beta function is 4 m in MR, the integration between kicks is every betatron phase advance of 0.25 rad in maximum. One turn map $\mathcal{M}(s)$ is expressed by

$$\mathcal{M}(s) = \prod_{i=0}^{N-1} M(s_{i+1}, s_i) e^{-:\Phi(s_i):} \qquad (4)$$

where $M(s_{i+1},s_i)$ is transfer map from s_i to s_{i+1}, and $\Phi(s_i)$ is electric (magnetic) potential of the self field. Linear part of the transfer map is diagonalized blockwise using the coupling parameters,

$$M(s_{i+1}, s_i) = R(s_{i+1}) M_{2\times 2}(s_{i+1}, s_i) R(s_i)^{-1} \qquad (5)$$

Linear x-y coupling is removed when R matrix is multiplied before and after the space charge kicks.

$$\mathcal{M}(s) = \prod_{i=0}^{N-1} M_{2\times 2}(s_{i+1}, s_i) e^{-:\Phi(s_i):} \qquad (6)$$

In a scope of linear envelope theory, Eq.(6) corresponds to just uncoupled lattice (design). Comparing the simulation results with (4) and (6), an effect of x-y coupling is estimated.

Simulation is performed for J-PARC MR; the tune operating point is (ν_{x0}, ν_{y0})=(0.41, 0.76) and the space charge tune shift is around 0.07-0.1.

Figure 4 shows the emittance growth without the coupling correction, Eq (4). Figure 5 shows the emittance growth with linear x-y coupling correction in Eq.(6). The emittance growth due to the coupling is recovered clearly by the correction, especially in vertical. The coupling at nonlinear magnetic elements is not corrected in Eq.(6). This result means that coupling induced by the space charge force causes an emittance growth.

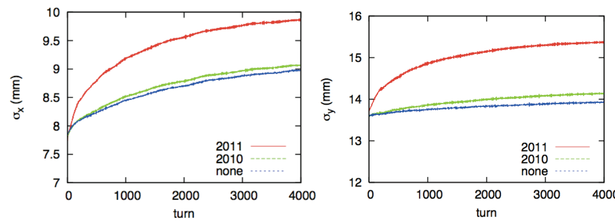

Figure 4: Evolutions of the horizontal and vertical beam for three lattices without linear coupling correction Eq.(4).

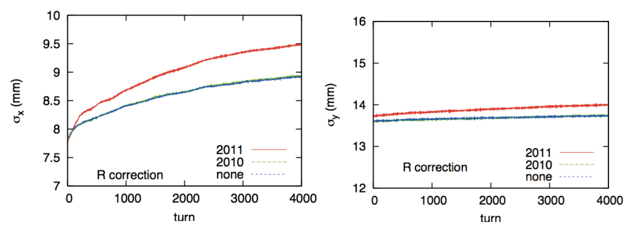

Figure 5: Evolutions of the horizontal and vertical beam for three lattices with linear coupling correction Eq.(6).

x-y coupling is assigned to the design lattice using

$$\mathcal{M}(s) = \prod_{i=0}^{N-1} R(s_{i+1}) M_0(s_{i+1}, s_i) R^{-1}(s_i) e^{-:\Phi(s_i):} \qquad (7)$$

where M_0 is the transfer map for the design lattice, and R is the coupling matrix for 2010 or 2011. In a scope of the envelope theory, Eq.(7) is equivalent to Eq.(4). The line with magenta in Figure 6 shows the emittance growth for the design lattice with linear coupling of 2011. Contribution to the horizontal emittance is a little, while that to the vertical is considerable.

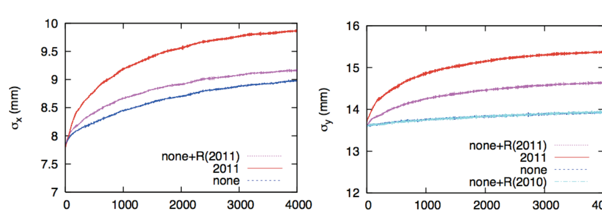

Figure 6: Evolutions of the horizontal and vertical beam for the design lattice with linear coupling 2011 and 2011 lattice, Eq.(7).

What can we get anything from the results

The space charge tune shift is similar for Eq.(6) and Eq.(7). The emittance growth is not caused by nonlinear transformations of magnets under the space charge tune spread, but is by nonlinear transformations of the space charge force.

Resonances $(m_x,m_y)=(2,-1)$ and $(1,-1)$ are a potential candidates, which cause emittance growth. Taylar map analysis gives the resonance strength defined by

$$H = H_{00} + \sum G_{m_x,m_y} \exp(m_x\phi_x + m_y\phi_y) \qquad (8)$$

The strength at the emittance amplitude $J_i=\varepsilon_i/2$ for the design and 2011 lattice are obtained as Eq.(9) and (10), respectively.

$$\begin{aligned} G_{1,-1}(\varepsilon_x/2,\varepsilon_y/2) &= -2.8\times10^{-10} - 5.6\times10^{-11}i \\ G_{2,-1}(\varepsilon_x/2,\varepsilon_y/2) &= -1.7\times10^{-10} - 7.4\times10^{-11}i \end{aligned} \qquad (9)$$

$$\begin{aligned} G_{1,-1}(\varepsilon_x/2,\varepsilon_y/2) &= 3.1\times10^{-8} + 5.0\times10^{-9}i \\ G_{2,-1}(\varepsilon_x/2,\varepsilon_y/2) &= 4.1\times10^{-10} + 1.4\times10^{-8}i \end{aligned} \qquad (10)$$

It is investigated how the coupling correction in Eq.(6) works using FMA analysis [3,4]. Particles are tracked in a frozen space charge potential for initial distribution, parabolic or realistic one injected from RCS. Figure 7 shows the diffusion index, $D=\log_{10}|\delta\nu|$, for the 2011 lattice and that corrected x-y coupling using Eq.(6), where the space charge tune shift is $\Delta\nu=0.07$. There is no clear difference at $\Delta\nu=0.07$. Figure 8 shows the diffusion index for the space charge tune shift $\Delta\nu=0.10$. (1,-1) coupling is reduced by the coupling correction at t $\Delta\nu$ =0.1.

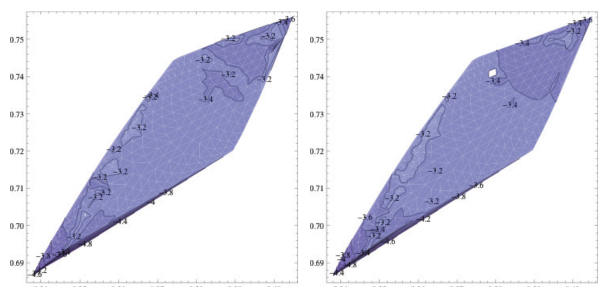

Figure 7: Diffusion index for the 2011 lattice (left) and that corrected linear x-y couplingusing Eq.(6) (right), where the space charge tune shift is $\Delta\nu=0.07$.

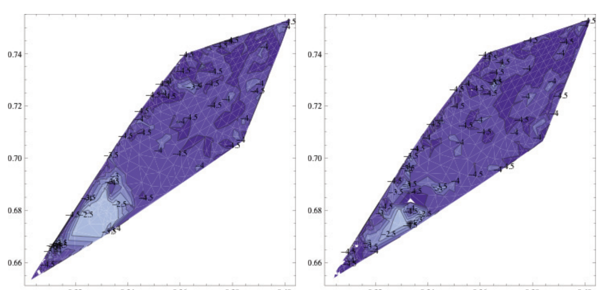

Figure 8: Diffusion index for the 2011 lattice (left) and that corrected linear x-y couplingusing Eq.(6) (right), where the space charge tune shift is $\Delta\nu=0.1$.

It is investigated how linear coupling assigned to the design lattice in Eq.(7) works using FMA analysis as shown in Figure 8. (1,-1) resonance is enhanced. In a scope of linear envelope theory, the lattice assigned linear coupling is equivalent to the 2011 lattice. The resonance strength (1,-1) seems weaker than that in left picture of Figure 8.

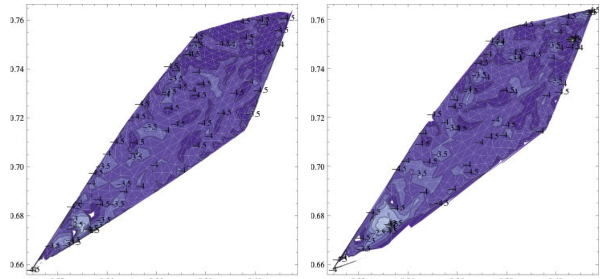

Figure 9: Diffusion index for the design lattice and that assigned linear x-y coupling of 2011 lattice using Eq.(7), where the space charge tune shift is $\Delta\nu=0.1$.

SUMMARY

We studied how x-y coupling causes emittance growth in a space charge dominant synchrotron, J-PARC MR. Space charge force under a coupled phase space distribution induces more coupling in (1,-1) resonance. Degradation of emittance growth is seen at space charge tune shift $\Delta\nu$ =0.07 in a simulation, but no clear sign for (2,-1) resonance in FMA analysis.

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