STRETCHED-WIRE MEASUREMENTS OF SMALL BORE MULTIPOLE MAGNETS

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Abstract

Stretched-wire (SW) measurements of magnetic multipoles have been performed at radii ranging from 0.5 mm to 4 mm, with an accuracy of 10^{-3} of the main multipole component. The theoretical aspects of SW measurements were investigated. The processing of the measured signals is based on a least square approach, instead of the Fourier transform widely used for rotating coil measurements. It allows the position errors of the SW to be corrected numerically and the design of SW trajectories which are not sensitive to the main multipole, as with "bucked" rotating coils. This SW measurement bench was developed for the characterization of ESRF magnets. It was first tested with large aperture multipole magnets. An accuracy of 10^{-4} was obtained for a measurement radius of 30 mm. A small permanent magnet quadrupole was built in order to test the bench at small measurement radii.

INTRODUCTION

A stretched-wire magnetic measurement bench has been developed at the ESRF. This bench has been used successfully for the fiducialization and the harmonic analysis of conventional accelerator magnets [1]. There is a demand in the magnetic measurement community for the measurement of small aperture multipole magnets, with a radius smaller than 5 mm: the use of classical "bucked" rotating coils is difficult in this context, but it is always possible to pass a wire through the bore.

Theoretical aspects of stretched-wire multipole measurements are summarized in the next section. A much more detailed analysis can be found in reference [1]. The experimental setup and the results are described in the last sections.

STRETCHED-WIRE THEORY

Least Square Multipole Estimation

The complex longitudinal integral of the magnetic field is $\boldsymbol{B}_{XY} = \boldsymbol{B}_Y + i\,\boldsymbol{B}_X$, where X and Y denote the transverse and the vertical field components. This field can be expressed in a frame attached to the wire:

$$\boldsymbol{B}_{\parallel\perp} = B_{\perp} + i\,B_{\parallel} = e^{i\theta}\boldsymbol{B}_{XY}\,, \tag{1}$$

where θ is the angle between the speed of the wire and the **x** axis. The term $B_\perp$ corresponds to the measured field component which is perpendicular to the wire motion; the wire is not sensitive to the field component $B_\parallel$ parallel to its motion. A set of M measurements can be written in matrix form:

$$\begin{pmatrix} \boldsymbol{B}^1_{\parallel\perp} \\ \vdots \\ \boldsymbol{B}^1_{\parallel\perp} \end{pmatrix} = \begin{pmatrix} e^{i\theta_1} & \cdots & e^{i\theta_1}\left(\dfrac{z_1}{\rho_0}\right)^{N-1} \\ \vdots & & \vdots \\ e^{i\theta_M} & \cdots & e^{i\theta_M}\left(\dfrac{z_M}{\rho_0}\right)^{N-1} \end{pmatrix} \begin{pmatrix} b_1 + i\,a_1 \\ \vdots \\ b_N + i\,a_N \end{pmatrix}, \tag{2}$$

where $z_k = x_k + i\,y_k$ is the position of the measurement point k, ρ_0 is the normalization radius, b_n and a_n are the normal and the skew multipole coefficients. Keeping only the real part of Eq. (2), rearranging the terms and introducing $\boldsymbol{Z}_{mn} = \exp(i\theta_m)(z_m/\rho_0)^{n-1}$ yield

$$\begin{pmatrix} B^1_\perp \\ \vdots \\ B^M_\perp \end{pmatrix} = \left(\operatorname{Re}\boldsymbol{Z}, -\operatorname{Im}\boldsymbol{Z}\right) \begin{pmatrix} \vdots \\ b_n \\ \vdots \\ a_n \\ \vdots \end{pmatrix}. \tag{3}$$

In the following, the vector on the left part of Eq. (3) will be denoted by **B**, the matrix build with the $\boldsymbol{Z}_{mn}$ coefficients will be denoted by **T** (for Trajectory) and the multipole coefficient vector will be denoted by **C**:

$$\mathbf{B} = \mathbf{TC}\,. \tag{4}$$

Writing the measured field in this form has the following applications:

- The measured field can be simulated for any trajectory. The sensitivity to position errors can be computed easily.
- The multipoles can be obtained from the pseudo-inverse of the matrix **T**. The least square solution is given by $\hat{\mathbf{C}} = \left(\mathbf{T}^{\mathsf{T}}\mathbf{T}\right)^{-1}\mathbf{T}^{\mathsf{T}}\mathbf{B}$.

With this method, multipoles can be obtained from an arbitrary trajectory. Figure 1 shows the matrix **T** for two different trajectories.

The above equations are valid for punctual measurements; it can be extended to finite wire motions by integrating the $\boldsymbol{Z}_{mn}$ coefficients over the wire displacement.

Multipole Compensation

Multipole magnets are usually measured with "bucked" rotating coils, which are not sensitive to the main harmonic and to the feed down terms [2, 3].

In the same way, some wire trajectories are not sensitive to the main harmonic: the basic idea is to move the magnet along the field lines of the main multipole component. Figure 2 shows an example of compensated trajectory. Measurements must be performed at two different radii in order to separate the dipole and the sextupole components; an alternative is to measure the field at the nominal radius and at the centre of the magnet.

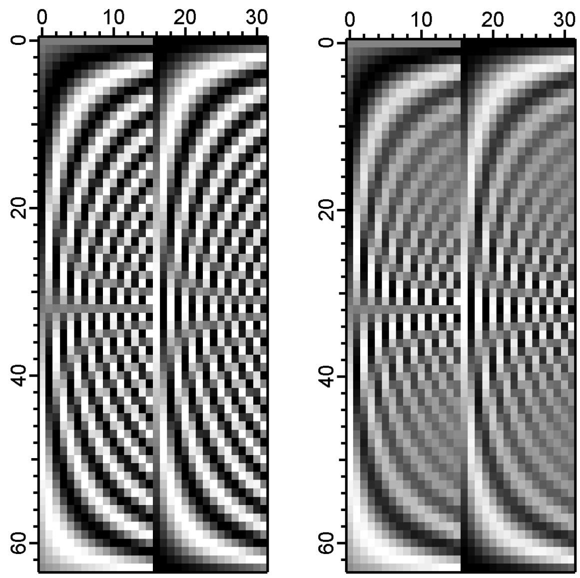

Figure 1: Trajectory matrix $\mathbf{T} = (\mathrm{Re}\,\mathbf{Z}, -\mathrm{Im}\,\mathbf{Z})$ for a circular trajectory (left) and an elliptic trajectory with a 0.5 eccentricity (right). The matrix are for 64 measurements, 16 normal and 16 skew multipole coefficients.

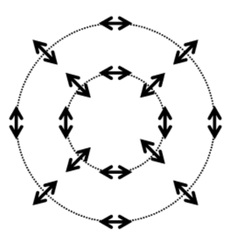

Figure 2: Quadrupole compensated trajectory.

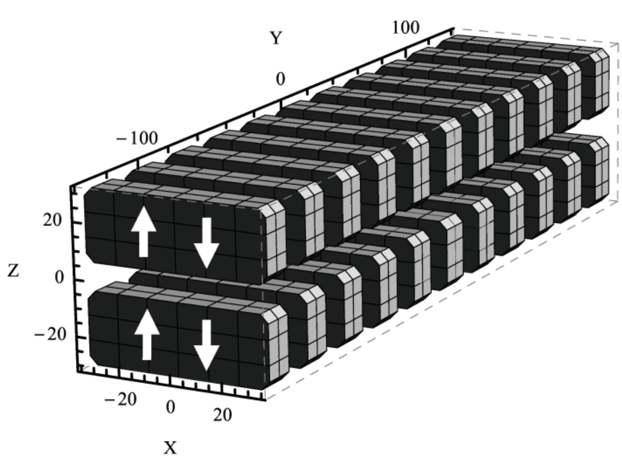

Figure 3: Permanent magnet planar quadrupole used for small radius field measurements.

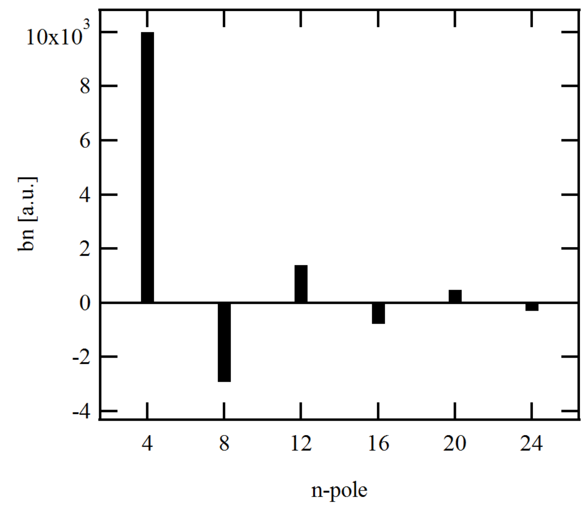

Figure 4: Normal multipoles of the small permanent magnet quadrupole (Radia simulation [4], normalized at 4 mm radius).

INSTRUMENTS AND METHODS

Measurement Bench

A stretched-wire measurement bench has been built at the ESRF. This bench is based on two groups of linear stages (Newport M-ILS250CC) controlled by an up-to-date motion controller (Newport XPS). The stages and the magnets are supported by a granite table. A sensitive voltmeter (Keithley 2182A) is used for measuring the wire voltage. The length of the wire is 1.4 m.

Large Bore Magnets

Different magnets have been used for the commissioning of the bench:

- Permanent magnet steerers, with 14 mm vertical aperture.
- Quadrupole magnets with 66 mm bore diameter.
- Sextupole magnets with 82 mm bore diameter.

Accuracies of approximately 10^{-4} of the main harmonic have been obtained in each case.

Small Bore Magnets

A small vertical aperture quadrupole has been built from refurbished helical undulator magnets (Figure 3). Its magnetic gap is approximately 9.5 mm and its gradient is close to 150 T/m. The lack of symmetry of this magnet leads to large even harmonics (Figure 4). This harmonic content was seen as an advantage, as it allows measurements of very small radii.

Method

The small aperture magnet was placed at various positions, in order to estimate the sensitivity to the position of the linear stages. At each position, the following measurement steps were performed:

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- Alignment of the wire on the quadrupole axis. The centre of the magnet was determined from multipole measurements. The yaw and pitch angles were obtained by moving one extremity of the wire.
- The multipoles were measured with circular and compensated trajectories, at radii 4 mm, 2 mm, 1 mm and 0.5 mm. Measurement parameters are given in Table 1.

Table 1: Measurement Parameters

Circular trajectory		
Radius	0.5 ... 4	mm
Points	128 (R≥2 mm) or 64	
Averages	8	
Integration time	20	ms
Compensated trajectory		
Radius	0.5 ... 4	mm
Points	64 @ $R=R_{MAX}$ and 64 @ R = 0	
Averages	4	
Integration length	0.1	mm

The repeatability of the measurements has been estimated from several measurements performed without moving the magnet. The standard deviation was computed for each multipole; our figure of merit was the average of these values.

The accuracy was evaluated from the measurements performed at different positions. It was assumed that the position errors of the linear stages are independent if the measurements are performed on trajectories which do not intersect. The figure of merit was the same as for the repeatability. This accuracy study does not take into account the voltmeter errors.

RESULTS

The measured multipole content was in agreement with the simulations, with additional errors due to a transverse taper between the two magnet arrays.

The figures of merit for the repeatability and the accuracy of the measurements are shown in Figure 5. With a small measurement radius, the signal is weak and the repeatability is affected. The accuracy is close to 10^{-3} of the quadrupole for radii up to 2 mm and is three times larger for a radius of 4 mm. The large figure of merit at large radius may be due to the strong high order multipoles presents in this magnet: a one percent error on the octupole corresponds approximately to $3\ 10^{-3}$ of the quadrupole. We can reasonably assume that the accuracy would be better with a “better” magnet (*i.e.* a conventional electromagnet).

The accuracy can also be estimated by introducing position errors in the matrix **T** and by running several simulations. The order of magnitude of the accuracy obtained with this method is the same (Figure 5).

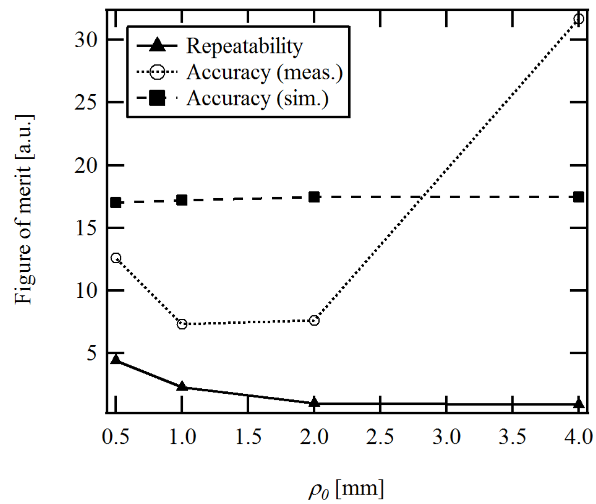

Figure 5: Repeatability and accuracy of the measurements, in 10^{-4} of the quadrupole. The figure of merit is defined above.

CONCLUSION

Multipole measurements have been performed on a small permanent magnet quadrupole. The repeatability is acceptable, even at a small radius. It can be improved by averaging the signals. The accuracy is close to 10 units (*i.e.* 10^{-3} of the quadrupole) for radii ranging from 0.5 mm up to 2 mm. With a larger radius, the measurements are affected by the strong even harmonics of the magnet. Better results are expected for conventional magnets with much lower harmonics. This bench seems to be an alternative to the rotating coils for small bore magnets.

REFERENCES

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