HIGH-FIDELITY 3D MODULATOR SIMULATIONS OF COHERENT ELECTRON COOLING SYSTEMS*

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Abstract

Next generation electron-hadron colliders will require effective cooling of high-energy, high-intensity hadron beams. Coherent electron cooling (CeC) can in principle cool relativistic hadron beams on orders-of-magnitude shorter time scales than other techniques [1]. The parallel Vorpal framework is used for 3D delta-f PIC simulations of anisotropic Debye shielding in a full longitudinal slice of the co-propagating electron beam, choosing parameters relevant to the proof-of-principle experiment under development at BNL. The transverse density conforms to an exponential Vlasov equilibrium for Gaussian velocities, with no longitudinal density variation. Comparison with 1D1V Vlasov/Poisson simulations shows good agreement in 1D. Parallel 3D simulations at NERSC show 3D effects for ions moving longitudinally and transversely. Simulation results are compared with the constant-density theory of Wang and Blaskiewicz [2].

COHERENT ELECTRON COOLING

Coherent electron cooling (CEC) is a novel technique for rapidly cooling high-energy hadron beams [1]. The proposed Brookhaven CEC consists of three sections: a *modulator*, where the ion imprints a density wake on the electron distribution, an *FEL*, where the density wake is amplified by an FEL, and a *kicker*, where the amplified wake interacts with the ion, resulting in dynamical friction for the ion.

In this paper we consider only the modulator section. We calculate the wake in the electron distribution due to the presence of a single ion, as the ion drifts with many co-propagating electrons. Although these particles are highly relativistic in the laboratory frame, particle velocities are non-relativistic in the beam frame drifting with the mean speed of the particles. In previous work [6] we modeled only a small portion of the beam, and the electron density was constant and uniform in space. Here we consider a more realistic beam where the electron density decreases to zero at the edge of the beam.

We assume the beam is close to an equilibrium solution with radial symmetry in x and y, and uniform in z (the direction of beam propagation). The formulation below uses a 2D transverse beam description, we can also simulate a 3D beam by assuming it is uniform in z. Simulation results are calculated using Vorpal [3] using delta-f PIC [4].

Wang and Blaskiewicz [2] found exact solutions to the Vlasov-Poisson equations in a uniform electron density assuming a special form of the electron velocity distribution, a kappa-2 distribution. Despite the different assumptions of this model, it is very easy to calculate its predictions, so this simple model provides a useful comparison to our numerical results.

DELTA-F FORMULATION

If $f(\vec{x},\vec{p},t)$ is the phase space electron density, f evolves according to the Vlasov equation, which specifies that the total time derivative of $f(\vec{x},\vec{p},t)$ is zero,

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{d\vec{x}}{dt}\cdot\nabla_x f + \frac{d\vec{p}}{dt}\cdot\nabla_p f = 0. \quad (1)$$

The particles accelerate due to the total electric field, which is composed of the field $-\nabla_x\phi(\vec{x},t)$ due to the perturbing ion and electron charge distribution plus an external field $\vec{E}_{ext}$,

$$\frac{d\vec{p}}{dt} = e(-\nabla_x\phi + \vec{E}_{ext}), \quad (2)$$

where $e<0$ is the electron charge. The potential ϕ satisfies a self-consistent Poisson equation

$$\nabla^2\phi = -\frac{\rho(\vec{x},t)}{\epsilon_0}, \quad (3)$$

where

$$\rho(\vec{x},t) = Z|e|\delta(\vec{x}-\vec{x}_{ion}) + e\tilde{n}(\vec{x},t), \quad (4)$$

$\tilde{n}(\vec{x},t) = \int f(\vec{x},\vec{p},t)d\vec{p}$, and the ion located at $\vec{x}_{ion}$ has charge $Z|e|$.

We now split the electron density

$$f = f_0 + f_1, \quad (5)$$

where f_0 describes the bulk beam behavior without the ion, and f_1 is a small perturbation which describes the electron shielding response to the ion. f_0 satisfies the Vlasov equation (1) and ϕ_0 satisfies a self consistent Poisson equation (3), except that the ion is not present in Equation (4). We note that in general f_0 need not be a steady-state solution or radially symmetric.

A convenient way to obtain a steady-state f_0 is to start from a time independent Hamiltonian. We assume the simplest form for the external field, uniform radial focusing

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$\vec{E}_{ext} = E_0' r\hat{r}$ where $r = \sqrt{x^2+y^2}$ is the radial coordinate. Assuming the potential ϕ_0 is time independent,

$$H = \frac{p_r^2}{2m_e} + e\left(\phi_0(r) - \frac{E_0' r^2}{2}\right), \tag{6}$$

where m_e is the electron mass, and recall that $e<0$. Since the Hamiltonian is time-independent, any function of H is also time-independent, and this can be used to form the steady-state density function

$$f_0 = \frac{n_0}{2\pi m_e \sigma^2} \exp\left[-\frac{H(\vec{x},\vec{p})}{m_e\sigma^2}\right], \tag{7}$$

where n_0 is the maximum density value in the center of the beam, and σ is the x or y RMS velocity of the Gaussian particle distribution. The self-consistent Poisson equation for $\phi_0(r)$ is

$$\nabla^2\phi_0 = \frac{-en_0}{\epsilon_0}\exp\left[\frac{-e}{m_e\sigma^2}\left(\phi_0 - \frac{E_0' r^2}{2}\right)\right]. \tag{8}$$

These are the same equilibrium solutions to the Vlasov-Poisson equations in Ref. [5]. Equation (8) can be easily solved numerically using a standard ODE solver.

Figure 1 shows equilibrium density curves as the external focusing parameter E_0' doubles in magnitude. The central density value n_0 as well as the RMS velocity σ have been adjusted so that each beam in Figure 1 has the same total charge and emittance. Note that the central density value doubles while the radius decreases by a factor of $\sqrt{2}$. The initial values are chosen to correspond to the Coherent Cooling proof-of-principle experiment: $n_0 = 5.48\times10^{16} e/m^3$, $\sigma = 8.79\times10^5 m/s$. These give a base plasma frequency of $w_p = 1.32\times10^{10}$ rad/sec and a Debye radius of $\sigma/w_p = 66.5\mu m$. Note, however, that the beam profiles in Figure 1 have variable density, so the plasma frequency and Debye radius will vary.

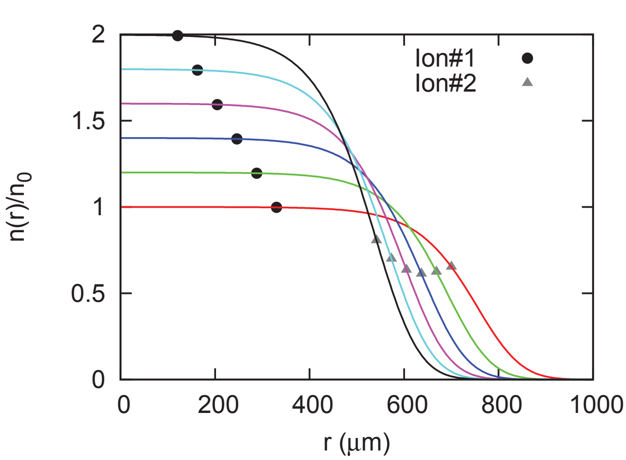

Figure 1: Density curves for a 2D beam with radial symmetry. Symbols show the position and local density seen by the two ions in the simulations.

The perturbation density f_1 satisfies

$$\frac{Df_1}{Dt} = e\nabla_x\phi_1\cdot\nabla_p f_0, \tag{9}$$

where $\phi_1(\vec{x},t)$ is the self-consistent potential for the perturbation f_1,

$$\nabla^2\phi_1 = -\frac{\rho_1(\vec{x},t)}{\epsilon_0}, \tag{10}$$

where

$$\rho_1(\vec{x},t) = Z|e|\delta(\vec{x}-\vec{x}_{ion}) + e\tilde{n}_1(\vec{x},t), \tag{11}$$

and $\tilde{n}_1(\vec{x},t) = \int f_1(\vec{x},\vec{p},t)d\vec{p}$. In this case it is easy to calculate $\nabla_p f_0$ directly from (7)

$$\nabla_p f_0 = -\frac{f_0 p_r}{m_e^2\sigma^2}\hat{r}. \tag{12}$$

The perturbation f_1 is modeled using delta-f PIC [4]. Suppose the i'th delta-f PIC particle has position $\vec{x}_i$, velocity $\vec{v}_i$ and weight w_i. These particles represent the perturbation f_1, as defined by

$$f_1(\vec{x},\vec{v},t) = \sum_i w_i\delta(\vec{x}-\vec{x}_i)\delta(\vec{v}-\vec{v}_i). \tag{13}$$

The initial weight of the delta-f particles is zero, and usually delta-f particles are distributed uniformly in phase space. The delta-f particles move in response to all fields, and their weights evolve according to Equation (9), which specifies

$$\frac{dw_i}{dt} = \frac{e}{g}\nabla_x\phi_1(\vec{x}_i,\vec{v}_i,t)\cdot\nabla_p f_0(\vec{x}_i,\vec{v}_i), \tag{14}$$

where g is the loading distribution of the delta-f particles. For uniform loading $g = n_{ptcl}/V$ where n_{ptcl} is the nominal plasma density and V is the loading volume in velocity space.

In the absence of any perturbation, the Hamiltonian (6) for each delta-f particle is constant in time. If a particle begins with a large value of $H/(m_e\sigma^2)$, by (7) f_0 will be exponentially small and the particle weight will remain small. So we need not include delta-f particles with large values of $H/(m_e\sigma^2)$. However, in practice it is simplest to uniformly populate phase space with delta-f particles in the region of the beam.

Simulation Results

In the modulator, quadrupole magnets decrease the beam radius by a factor of two for passage through the FEL. We model this by increasing the external focusing field linearly in time by a factor of two, giving the beam profiles shown in Figure 1. In these simulations the beam radius decreases by only a factor of $\sqrt{2}$. We assume that at all times the bulk beam remains in equilibrium, which, strictly speaking is only the case if the external focusing field increases slowly compared to a plasma period. This is not the case for our simulations, which take place over half a plasma period.

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Wang and Blaskiewicz [2] derived an exact shielding result assuming uniform electron density, for electrons with a kappa-2 velocity distribution (rather than Maxwellian). Their result for $\tilde{n}_1(\vec{x}, t)$ is written in terms of a single integral over the time interval of the interaction, for this case the integral is Eq. (15) in Ref. [6]. In this integral the plasma frequency and electron temperature are constants, however, by moving them inside the integral we can use this formula to estimate the shielding when the density and electron temperature are changing. We calculate the electron density at the location of the ion, and from this a local plasma frequency which now changes with time. If we move the plasma frequency inside the integral we can calculate $\tilde{n}_1(\vec{x}, t)$ numerically as before. The electron temperature also increases as the beam is squeezed down, this is also incorporated into the integral.

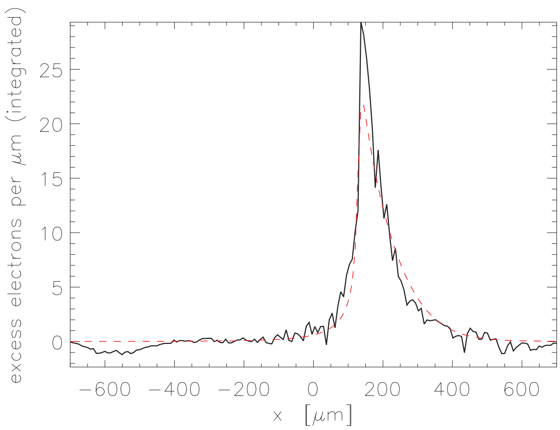

Figure 2: Ion#1 shielding response at $y = 0$ using delta-f (solid) or theory (dashed, red) at half a plasma period.

Ion#1 stays in the region of the beam where the density is relatively flat. Figure 2 shows the electron shielding response of Ion#1, which begins at $x = 330\mu m$, $y = 0$ and moves to the left (toward the beam center) at the thermal speed σ. The ion is almost unaffected by the external focusing field due to its large mass, and travels at a constant velocity. The figure shows the $y = 0$ slice of $\tilde{n}_1$ at half a plasma period. The theoretical curve (dashed) agrees well with the delta-f simulation. Note the small negative areas of the curve near the left and right edges of the beam. These regions hold a small negative perturbation, but when integrated over all slices their contribution is significant. In fact the total integral of $\tilde{n}_1$ over the beam is zero, which is to be expected since the beam is finite. The theoretical curve is always positive, and the integral over all x and y is approximately $2Z$, because it assumes an infinite domain of constant density.

Ion#2 begins at $x = 700\mu m, y = 0$ and moves left at $3/4$ the thermal speed σ. This position and velocity ensure that it remains near the edge of the beam where the density is dropping rapidly (Figure 1). Figure 3 shows that the theory predicts the amplitude of the wake fairly accurately, but it is too wide. The response is much lower than predicted to the right of the ion, because the density goes to zero in this region. This is expected to produce a very weak response because the plasma frequency goes to zero with the density.

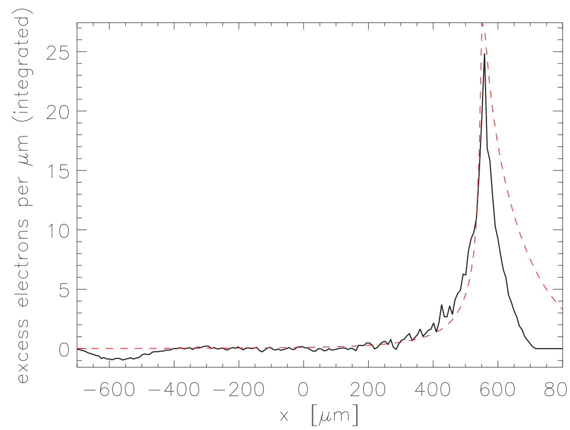

Figure 3: Ion#2 shielding response iat $y = 0$ using delta-f (solid) or theory (dashed, red), at half a plasma period.

Summary

The constant-density theory of Wang and Blaskiewicz [2] is able to predict wakes accurately when the density is relatively uniform near the ion but changing in time. Near the edge of the beam, where the density is changing rapidly, delta-f PIC results show a weaker shielding response toward the edge of the beam compared the the constant-density theory.

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