# CHALLENGES OF QUASIPERIODIC APPLE UNDULATORS

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### Abstract

In many synchrotron radiation facilities APPLE undulators have become workhorses for the production of variably polarized light. In helical mode higher harmonics are not produced. In the linear modes (horizontal, vertical, inclined) higher harmonics may contaminate the first harmonic and spoil the quality of experimental data. Planar undulators employing a quasiperiodic magnetic structure have been built and are successfully operated at several facilities. The implementation of a quasiperiodic lattice in an APPLE undulator is more complicated since the device is operated in various polarization modes. A detailed analysis of the magnetic and spectral performance of a quasiperiodic APPLE is presented.

## **INTRODUCTION**

The APS-upgrade design [1] includes an APPLE undulator which is intended to be operated in the range 2.4-27keV. In circular mode higher undulator harmonics are absent whereas a significant contamination of the spectra is expected when operating the 1<sup>st</sup> harmonic in linear modes (horizontal, vertical and inclined). An efficient suppression of higher harmonics in the range 2.4-9keV would be beneficial and the potential of a quasiperiodic structure has been evaluated for this case. Above 9keV the 3<sup>rd</sup> harmonic, and above 22keV the 5<sup>th</sup> harmonic will be used. For these cases the spectral contamination is small since the high energy photons are poorly transmitted by the beamline components.

## **QUASIPERIODIC LAYOUT**

In analogy to quasiperiodic crystals which produce sharp and intense diffraction patterns Sasaki proposed the quasiperiodic undulator design [2-4]. Energy shifted but sharp and intense harmonics are expected. The higher harmonics are efficiently blocked by the monochromator. Several planar undulators employing this scheme have been built and in a few cases the scheme has been adapted to APPLE undulators [5-8].

Quasiperiodic crystals can be described as a projection of a high dimensional periodic crystal onto a lower dimensional lattice. In analogy a quasiperiodic undulator can be constructed from the projection of a two dimensional lattice with two lattice parameters onto a straight line [2-4]. The ratio of the two lattice parameters is given by r and the inclination of the projection axis is defined by the angle  $\alpha$ . Then, the locations of the periodic lattice distortions are evaluated via:

$$z_m = m + (r \cdot \tan(\alpha) - 1) \cdot \left\lfloor \frac{1}{1 + \eta}m + 1 \right\rfloor$$

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#### $\eta = r/\tan(\alpha)$

where the bracket represents the biggest integer number smaller than the bracket argument. The parameter  $\eta$ defines the locations of the quasiperiodicities. Larger values of  $\eta$  imply less locations of quasiperiodicity. Comparing a quasiperiodic and a periodic structure the phase step ratio is given by:

$$\frac{\Delta \Phi_{quasiperiodic}}{\Delta \Phi_{periodic}} = r \cdot \tan(\alpha)$$

The phase step can be realized by either retracting Amagnets (longitudinally magnetized) or pairs of Bmagnets (vertically magnetized) where the 1<sup>st</sup> option is mechanically simpler and, in addition, it provides a better localization of the field distortion. A-magnets can be retracted vertically, horizontally or moved in both directions. The amount of retraction has to provide the desired phase step. Alternatively, additional magnets can be implemented at both sides of the structure [7-8].

The phase steps in a quasiperiodic APPLE II undulator have to be optimized simultaneously for various polarization modes. They depend upon the magnet row phasing and they can be completely different in amplitude and fine structure in the horizontal, vertical or inclined operation mode. An optimized phase step in one mode may have detrimental effects on the spectral performance in another mode. In the following we will concentrate on the horizontal and the vertical linear modes.

The phase step can be either positive or negative and it depends upon the specific magnet configuration. For the evaluation of the phase step we use the following two approximations which are generally true for particles in synchrotron radiation light sources:

$$\sqrt{1 + {x'}^2 + {y'}^2} \approx 1 + \frac{{x'}^2}{2} + \frac{{y'}^2}{2}$$
$$\frac{1}{\beta} \approx 1 + \frac{1}{2\gamma^2}$$

With these approximations and with  $\beta=v/c$ ,  $\lambda=wavelength$  of light,  $B\rho=stiffness$  of electron beam,  $B^{fit}$  and  $B^{res}$  being the fitted purely sinusoidal field and the error field, respectively, the phase error of an elliptical device is given by the expression:

$$\Delta \Phi(z) = \frac{\pi}{\beta \lambda (B\varrho)^2} \Big( \Delta \phi_x(z) + \Delta \phi_y(z) \Big)$$
  
$$\Delta \phi_{x/y} = 2 \int_0^z \left[ \int_0^{z'} B_{x/y}^{fit} dz'' \cdot \int_0^{z'} B_{x/y}^{res} dz'' \right] dz' + \int_0^z \left[ \int_0^{z'} B_{x/y}^{res} dz'' \cdot \int_0^{z'} B_{x/y}^{res} dz'' \right] dz'$$

From the longitudinal distribution of  $\Delta \Phi$  an rms-value is derived which is usually given in degrees. All terms of the phase error expression include B<sup>res</sup>. The contribution due to the slippage effect has already been subtracted, thus, a perfect undulator is described by  $\Delta \Phi(z)=0$ .

The phase expression above consists of two contributions depending upon i) the product of the integrals of the main field and residuals and ii) the product of the integral of the residuals with itself. The 1<sup>st</sup> term can be either positive or negative. If the main field is weakened (negative residuals) the term is negative, otherwise positive. The 2<sup>nd</sup> term is always positive.

For small amounts of retraction the 1<sup>st</sup> phase contribution dominates and lowers the phase ( $\Delta\Phi$  is negative). The 2<sup>nd</sup> contribution increases simultaneously and eventually it dominates. At this point the value of the phase step starts to increase. Thus, the relation between the phase advance and the amount of retraction is highly non-linear and it has to be determined numerically. Figures 1 and 2 show the phase steps for A-magnets being retracted in vertical and horizontal direction.



| Figure 1 (left): Phase steps due to a horizontal retraction of four A-magnets (solid lines) and two A-magnets (dashed lines) in hor. linear mode. Figure 2 (right): Phase steps due to a vertical retraction of four A-magnets (solid lines) and two A-magnets (dashed lines) (hor. mode). Magenta curve: phase step of scaled 5.6mm retraction fingerprint (phase over longitudinal coordinate). The bullets are identical configurations on different axes.

Ideally, the phase steps in horizontal linear and vertical linear mode have the same absolute amount, where the signs are permitted to be different. The non-linear behaviour of the phase step dependence upon row phase (figures 1-2) proofs the existence of such configurations. A closer look, however, demonstrates the importance of the phase step fine structure on-top of the averaged phase step value: A phase step of 69° can be produced with two different 4-magnet retractions of 5.6mm and 12.4mm as shown in figures 2. However, the different phase step fine structures (figure 3) cause the spectra to look quite different (figures 4-5). For a retraction of 12.4mm the 1<sup>st</sup> and 3<sup>rd</sup> harmonic are similar to the 5.6mm retraction case, the 5<sup>th</sup> harmonic, however, clearly suffers from the larger retraction. A sharp single peak is split into a bunch of peaks (lowering the usable flux), and one of them contaminates the 1<sup>st</sup> harmonic. The larger retraction of 12.4mm leads to a broadening of the field imperfection which causes a smear of the phase step. Interestingly, the error field broadening is not the only reason for the phase smear. Scaling the 5.6mm retraction field imperfection by a factor of 3.6 (figure 2, magenta) which gives also a phase step of  $69.7^{\circ}$  yields an even larger phase step blurring (figure 3). For larger field imperfections the  $2^{nd}$  phase term cannot be neglected anymore. The  $2^{nd}$  phase error contribution has another longitudinal dependence as compared to the  $1^{st}$  term and a ringing or smearing of the phase step structure occurs. This leads to a degradation of the spectral performance. Thus, small retractions are always preferable.



Figure 3: Phase step for vertical retractions of four Amagnets by 5.6mm and 12.4mm, respectively. Horizontal linear mode, gap = 9.8mm.



Figure 4 (left): Spectra of a periodic structure (black) and a quasiperiodic structure with 4 A-magnets vertically retracted by 5.6mm and the phase step fine structure of figure 3. The black bullets show the harmonics of a periodic device whereas the white bullets indicate the integer multiples of the 1<sup>st</sup> harmonic in the quasiperiodic case. Figure 5 (right): Spectra of a periodic structure (black) and a quasiperiodic structure with 4 A-magnets vertically retracted by 12.4mm and the phase step fine structure of figure 3.

For a one-by-one comparison of the performance in horizontal and vertical linear mode we assume a localized magnet configuration in both cases, i.e. all four magnets are located at one z-position (obviously, these are two different undulator structures). In vertical linear mode a vertical retraction of 4 magnets by 18mm produce the same peak field drop as in the horizontal linear case. The phase step, however, amounts to 30°, only, and the fine structures for the two polarization modes look different (figure 6) which is due to the wider horizontal field error distribution. Aiming for  $69.7^{\circ}$  in the vertical linear mode broadens the field error distribution even further and, consequently, the spectra for both cases will be completely different having a worse performance in the vertical linear case.

Figure 7 shows the field errors in the horizontal and vertical linear mode. Scaling the latter one shows clearly the broader distribution in this case. The error field extends to regions where the main field changes sign. Thus, the 1<sup>st</sup> phase contribution changes sign in longitudinal direction blurring the phase step. A better performance in the vertical linear mode can only be obtained with vertical field errors as produced with a combination of magnet retractions and opposite magnet movements towards the electron beam. However, this option would sacrifice minimum magnetic gap.



Figure 6 (left): Phase steps for magnet configurations at phase = 0mm (black) and phase = 18mm (red). On both cases the magnets are localized at one z-position. Figure 7 (right): The main field (black) is scaled and reversed in sign for comparison. The field errors for a vertical retraction of 4 A-magnets are plotted. Red: horizontal linear mode, blue: vertical linear mode. The dotted blue line is the scaled solid blue line.

Further constraints show up in a real device. With the retraction of four magnets the phase step gets delocalized in either the horizontal linear mode or the vertical linear mode, excluding a sharp phase step in one of the cases. This can only be avoided with the retraction of two magnets on diagonal rows. The retraction of only two magnets has the advantage of leaving the error field pattern independent upon the magnet row phase. The disadvantage is the presence of contributions from both field components to the phase error in all polarization modes. This leads to a partial cancellation of various phase contributions which requires larger retractions with the related disadvantages. Even if the space between the retracted magnets and the vacuum chamber is filled with material of opposite magnetization, which doubles the error fields, the achievable phase steps are smaller as compared to a retraction of four magnets due to partial compensation of contributions with opposite sign.

The error field spread can partly be compensated with the addition of magnet material with the same magnetization at the far end of the retracted magnet. A similar procedure improved the spectrum of a quasiperiodic planar hybrid undulator significantly [9]. A similar improvement could not be found for an APPLE.

A FORTRAN code has been developed for simulating and evaluating quasiperiodic undulator configurations. The code synthesizes arbitrary quasiperiodic structures from a periodic field and a quasiperiodic fingerprint as calculated with an FEM code. A range of  $\eta$ -values and various shapes of quasiperiodicities are evaluated in an automated procedure. The performance in terms of higher order suppression and higher order intensity is evaluated from on-axis spectra. Optimizing the phase step fine structure the following rules have to be regarded: i) localize the error fields to avoid phase step smear; ii) keep the amount of retraction small to reduce impact of  $2^{nd}$  phase error contribution; iii) get same phase steps in horizontal and vertical linear mode (not necessarily with same sign); iv) ignore phase step feature in elliptical mode.

The APPLE II undulator recently installed at HiSOR 8 [8] employs the parameters r = 1.5,  $\tan(\alpha) = 1/\sqrt{15}$ These parameters yield a phase step of 69.7°. A vertical retraction of four magnets by 5.6mm provides this phase change. For a series of  $\eta = r/\tan(\alpha)$  the spectral performance has been analysed resulting in an optimized value of  $\eta = 5.81$ . At a gap of 9.8mm the intensity reduction of the shifted harmonics 1,3,5 in a quasiperiodic undulator is 0.85, 0.73, 0.82, in horizontal linear mode and 0.91, 0.39, 0.40 in the vertical linear mode, respectively. The contamination of the 1<sup>st</sup> harmonic by harmonics 3 and 5 is 0.1 and 0.003 in the horizontal and 0.42, 0.11 in the vertical linear mode. The spectral performance gain is acceptable in the 1<sup>st</sup> case but less pronounced in the 2<sup>nd</sup> case. In particular, the differences in the two modes are unacceptable for linear dichroism measurement of small effects. Many quasiperiodic configurations with similar but not better results exist. This ambiguity is due to the non-perfect phase step and the finite number of periods (50 as compared to many thousands in a quasiperiodic crystal). The difficulties related to a quasiperiodic APPLE II as compared to a planar device can summarized as follows: i) higher complexity due to various polarization modes; ii) contribution of both field components with different signatures to phase step; iii) larger spread of horizontal error fields as compared to vertical error fields.

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