X-RAY SPECTRA RECONSTRUCTION OF THOMSON SCATTERING SOURCE FROM ANALYSIS OF ATTENUATION DATA*

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Abstract

Thomson Scattering sources have attracted a lot of interest as the new generation of bright X-ray sources. Traditional spectra measurement methods, applied to measured Thomson Scattering source, are troublesome as the X-ray beam is too intense to cause pile up problems. In this case, the analysis of attenuation data, which can provide some information about the spectral distribution, as not affected by the rate of incidence of photons, is a good candidate to measure the X-ray spectrum of the Thomson Scattering. We introduce an iterative statistical algorithm (Expectation-Maximization) to reconstruct the spectra from the attenuation data and a numerical experiment is carried out to test the spectrum reconstruction process. This method performs well to reconstruct the local spectra at small collecting angles. In order to reconstruct spectra at large collecting angles, a new method to process the attenuation data was proposed based on the spectral property of Thomson Scattering source. At collecting angles less than 3.5 mrad, our method performs well and we can also still use it to evaluate the FWHM of the spectra at larger angles.

INTRODUCTION

Thomson Scattering sources (also called Inverse Compton Scattering), which can be bright X-ray sources typically produce photons, have attracted a lot of interest as the technologies for producing low-emittance high-brightness relativistic electron sources and ultra-short high-power lasers have progressed. The X-rays that are generated by the interactions between laser and electron, exhibit high directivity, and have a polarized tunable quasi-monochromatic spectrum. The knowledge of the spectrum of an X-ray source is a key point for the development of any kind of application, for example in imaging both contrast and absorbed dose strongly depend on energy. However, direct methods performing a standard spectrometric measurement based on single photon energy measurement to detect the X-ray spectrum of Thomson Scattering sources have always been considered troublesome to implement because the beam is too intense to cause pulse pile up problems. Thomson Scattering source can produce up to 10^8 photons, bunched in 10ps long pulse [1]. An alternative way to measure the spectrum might request the measurement to be integral-type, which will not be affected by the high rate of incidence of photons. The analysis of attenuation data (transmission curves), which can provide some information about the spectral distribution of an X-ray source, as not affected by the rate of incidence of photons, is a good candidate to measure the X-ray spectrum of the Thomson Scattering X-ray source. Although there are several problems with this method, such as low accuracy, non-unique solution to ill-condition system and instability with different measurement error [2], this method can still give good estimation and reconstruction of spectra with some improvements based on the property of the measured spectra. In this article, we introduce an iterative statistical algorithm (Expectation-Maximization) [3] to reconstruct the spectra from the attenuation data on simulated measurement. Results show that this method can give good approximations for the mean energy of the spectra, while it is not sensitive to the specific spectral distribution and the energy broadening. In order to reconstruct the shape of the spectra, especially the energy broadening, we present a new method based on the Expectation-Maximization algorithm. The simulated experiments based upon the Monte Carlo justify the reliability and feasibility of this method.

THE EXPECTATION-MAXIMIZATION METHOD

The Inverse Problem

The relative transmission function $T(x)$ responding to an attenuator of thickness x is related with the spectrum of an X-ray beam in the following way[2]:

$$T(x)=\frac{S(x)}{S(0)}=\int_0^{E_{\max}} e^{-\mu(E)x}F(E)dE \tag{1}$$

where $S(x)$ is the signal measured with an absorber of thickness x, and $S(0)$ is the signal measured without any absorber. $\mu(E)$ is the total attenuation coefficient of the attenuator in which are included both photoelectric and Compton effects for photons of energy E $F(E)=\Phi(E)\cdot C(E)$, where $\Phi(E)$ represent the spectrum distribution of photons and $C(E)$ represent the energy response determined by the way to measure the signal $S(x)$. $E_{\max}$ is the maximum energy of the spectrum.

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When we treat the problem from its numerical point of view by discretizing the integral, the Eq. (1) turns out a linear system of the form

$$T = A \bullet F \qquad (2)$$

where $T \in \mathbb{R}^M$ represent the attenuation curve as a function of the attenuator and $F \in \mathbb{R}^N$ is the discretized spectrum. Both these functions are represented by vectors and $A \in \mathbb{R}^{M\times N}$ is a matrix which contains the properties of the attenuators with $A_{mn} = e^{-\mu_n x_m}$ where μ_n is the total attenuation coefficient of the attenuator computed at the energy corresponding to the n-th energy bin and t_m is the thickness of the attenuator related to the m-th transmission measurement ($m = 1,..,M$ and n=1,...,N). The vector F can be regarded as the point-by-point product of Φ and C which represent, respectively, the discretization of $\Phi(E)$ and $C(E)$. Once F is computed it is possible to correct for the coefficients contained in C and a spectrum in terms of number of photons per energy bin is obtained.

Generally, the linear system described by Eq. (1) translates, after the discretization, into an ill-conditioning linear system in Eq. (2). Here the Expectation-Maximization (EM) algorithm is used to obtain an approximation to the exact solution of the system Eq. (2) [4]. Expectation -Maximization algorithm is an iterative statistical algorithm with the form [1,5]:

$$F_n^{(k+1)} = F_n^{(k)} f_n, \quad f_n = \frac{\sum_m A_{nm}{}^t \dfrac{T_m}{\sum_{n'} A_{mn'} F_{n'}^{(k)}}}{\sum_m A_{nm}{}^t} \qquad (3)$$

Numerical Simulation

A numerical experiment is carried out to test the spectrum reconstruction process. We consider a calculated spectrum as the target spectrum. Starting from this distribution we compute the exact attenuation curve with Eq. (2). Then we add a random perturbation representing the noise inherent during the measure process to the exact attenuation data as the measured data. The spectrum is computed in the energy range 20-70keV with 0.1keV energy bins. The attenuator is Silicon and the thicknesses are between 0 and 2.4cm with an equal interval of 0.12cm. We add 5% Gaussian noise to the attenuation data. In order to indicate the degree of convergence, we also define the normalized mean absolute distance between the attenuation data computed using the estimated and the true spectra as follows [2]:

$$d = \frac{1}{M}\sum_{m=1}^{M}\frac{\left|T_m - T_{m,cal}\right|}{T_m} \qquad (4)$$

Figure 1 shows the results of reconstruction for two different true target spectra. In Fig. 1(a), the true target spectrum is a symmetrical Gaussian distribution with peak energy 50keV and the standard deviation 0.03 times of the peak energy and in Fig. 2(c) the true target spectrum is asymmetrical with peak energy 50keV and standard deviation 0.04 (left side) times and 0.01 (right side) times of the peak energy. The initial spectrum for the iterations is a flat spectrum.

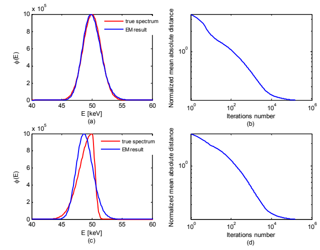

Figure 1: (a) and (c) are EM reconstruction results (blue line) and true spectra (red line); (b) and (d) are the converge curves with the iteration number respectively; the initial spectrum for iteration is flat spectrum and the Gaussian noises are both 5%.

It is observed that for symmetrical Gaussian spectrum, the EM algorithm can give an estimated spectrum which is close to the true spectrum in shape. However, the EM iteration cannot recover the asymmetrical spectrum in Fig. 2(c). The peak energy (48.7keV) of the estimated spectrum of EM is the mean energy (48.8keV) rather than the peak energy (50keV) of the true spectrum. The EM algorithm generates a spectrum whose reproduced attenuation data are consistent with the measured attenuation data with the maximum likelihood. The full width at half maximum (FWHM) of the EM result corresponds to the symmetrical spectrum with the mean energy as the peak energy, may not reflect the actual FWHM and the shape of the true spectrum. When the EM algorithm is applied to reconstruct the spectra of Thomson Scattering source, we can get the mean energy and estimated FWHM for the true target spectra.

SPECTRA FOR THOMSON SCATTERING SOURCE

The previously mentioned EM method can be used to reconstruct the local spectra for the Thomson Scattering source as the local spectra with small collecting angle (about 0.5 mrad) are close to Gaussian distribution. During the simulation process, Monte Carlo method is used to simulate the Thomson Scattering and calculate the attenuation curve through silicon with different thickness to justify the reliability and feasibility of this method. Figure 2 shows the results for the reconstruction of local spectra for different two points in the X-ray spot.

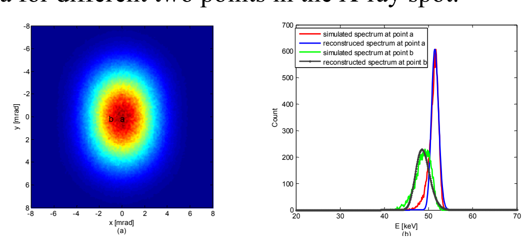

Figure 2: (a) X-ray profile simulated by CAIN and the two points selected to reconstruct local spectra. (b) Simulated and reconstructed spectra at two different points with 0.5 mrad collecting angles and 5% Gaussian random error.

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It can be observed that the EM method performs well to reconstruct the local spectra of Thomson Scattering source. However, as the collecting angles increase, the X-ray spectra are not close to Gaussian distribution any more (see Fig. 3).

In order to reconstruct the spectra at different collecting angles, we propose a method to process the attenuation data based on the EM algorithm. As mentioned earlier, the EM algorithm may not perform well in reconstruction of the shape of asymmetric spectra as it did in mean energy. To solve this problem, we propose an assumption that according to the spectral distribution features of Thomson Scattering, as the collecting angles increase, the right half of spectra remains almost unchanged. So we can treat the right-half spectra of the small collecting angles given by EM algorithm as the right half for spectra of other angles within a certain range. Then we can reconstruct the spectra with the attenuation data more accurately as under this assumption, the mean energy will have direct and close relationship with the shape of spectra.

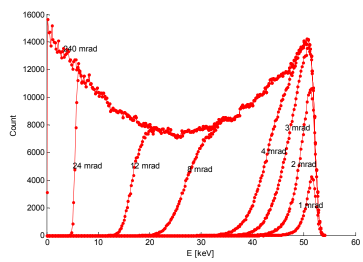

Figure 3: The simulated spectra for the Thomson Scattering source with different collecting angles with CAIN. The electron energy is 46.9MeV with 0.3% broadening and the center-wavelength of the spectra for the laser is 800nm with 2% broadening.

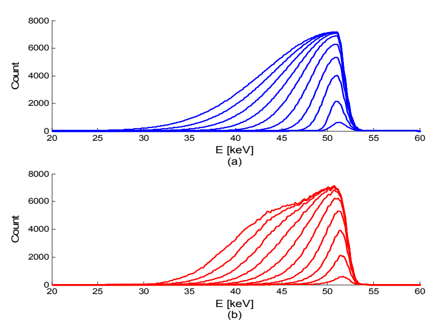

Figure 4: (a): Representative reconstruction results of spectra for collecting angles ranging from 0.5 mrad to 5 mrad with an equal interval of 0.5 mrad (b): The simulated spectra for the Thomson Scattering source with collecting angles of the same range with CAIN.

Figure 4 shows the reconstruction result (top) and the simulation result (bottom) for Thomson Scattering spectra within the collecting angle range from 0.5mrad to 5mrad. The Gaussian random noise added to the measured attenuation data are 5% and with different seed for random, the noise will be slightly different between trials. But this does not affect the conclusion. Here we give a representative result without loss of generality. We regard the left half of the spectra as Gaussian distribution. Then we can find best estimation left-half spectrum which meets the measured attenuation best. The corresponding statistical results about FWHM and collecting angles are given in Fig. 5.

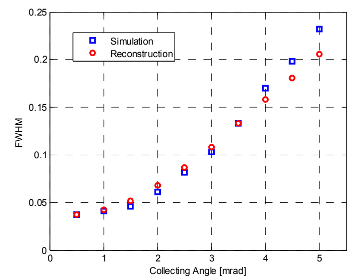

Figure 5: Simulation and reconstruction results of FWHM of spectra for different collecting angles.

We can see that at collecting angle less than 3.5 mrad, the shape and FWHM of spectra between simulation and reconstruction are in good agreement with each other. As the collecting angle increases, the two results have some difference. This may be resulted from the hypothesis that the left-half spectra are Gaussian distribution at these angles. But we can still estimate the shape and the FWHM of spectra through this way.

DISCUSSION AND CONCLUSION

The analysis of attenuation data, as not affected by the rate of incidence of photons, is a good candidate to measure the X-ray spectrum of the Thomson Scattering. The Expectation -Maximization algorithm is one of effective ways to reconstruct the local spectra at small collecting angles which is close to Gaussian distribution from the attenuation data. However, the EM method does not have good performance in reconstructing asymmetric spectra. In order to recover the spectra for different collecting angles in Thomson Scattering source which are asymmetric, we proposed a new method to process the attenuation data based on the EM algorithm. Results show that at collecting angles less than 3.5mrad, our method performs well and at larger angle, we can also still use it to evaluate the FWHM of the spectra.

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