MODELING SPACE CHARGE IN AN FFAG WITH ZGOUBI

Sam Tygier*, Robert B. Appleby, Hywel Owen, University of Manchester, UK and the Cockcroft Institute, UK, Roger John Barlow, University of Huddersfield, UK

Abstract

The Zgoubi particle tracker uses a ray tracing algorithm that can accurately track particles with large offset from any reference momentum and trajectory, making it suitable for FFAGs. In high current FFAGs, for example an ADSR driver, space charge has a significant effect on the beam. A transverse space charge model was added to Zgoubi using the interface pyZgoubi. The magnets are sliced and a space charge kick is applied between each slice. Results are presented for an ADSR driver lattice.

INTRODUCTION

While there are several codes that can model Fixed-Field Alternating-Gradient accelerators (FFAGs) [1] and several that can model space charge, their intersection is limited. Space charge modelling was added to the existing code Zgoubi [2, 3]. Zgoubi is a particle tracking code originally developed in the 1970s for designing spectrometers. It is therefore well suited to the large, shaped magnets and wide energy ranges seen in FFAGs, and has gained widespread use. It tracks using 6th order integration with fields from analytic magnet models or from simulated or measured field maps.

FFAGs have been proposed for high current applications for example Accelerator Driven Subcritical Reactors (ADSRs), where their rapid cycling would give an advantage over synchrotrons, and their energy reach gives an advantage over a cyclotron [4, 5]. At these high currents space charge will be a significant and disruptive force.

This paper describes the implementation of a space charge model added to Zgoubi and results for an FFAG lattice simulated with this code.

IMPLEMENTATION

PyZgoubi [6] is a interface to Zgoubi written in Python, which allows scripting Zgoubi and makes it easy to interface to other programs or functions. A space charge model was added to PyZgoubi. It takes an accelerator lattice and splits the elements into slices. Particles are tracked through a slice using Zgoubi and then their coordinates returned to PyZgoubi. Then a space charge kick is applied to the particles before tracking them through the next slice. Communication of particle coordinates between PyZgoubi and Zgoubi is done with binary files, but these can be in shared memory (RAM disk) for increased performance. Multiple instances of Zgoubi can be run in parallel to share the work of a large number of particles.

* sam.tygier@hep.manchester.ac.uk

Three transverse space charge algorithms have been implemented, in C using openMP for multi-threading. A linear method, equivalent to the KV envelope equations [7] on a beam of uniform charge density; an N-body method sums up the Coulomb force between each pair of particles; and a method that decomposes the beam into concentric elliptic rings, which is suitable for elliptically symmetric beams [8]. The N-body method has no symmetry assumptions, but is too slow for normal use. It also suffers from numerical instability due to summing a large number of contributions to the total force. It is however useful to verify the fields of the other methods. Longitudinal effects are not yet considered, so the model is only useful for studying long bunches where transverse space charge dominates.

The concentric rings method creates an ellipse in transverse space that encloses the whole bunch. It then creates concentric elliptic rings of the same aspect ratio and equal width inside, and counts the number of particles in each. The multipole field due to each ring of charge is found. These fields are then summed to find the force on any particle in the beam.

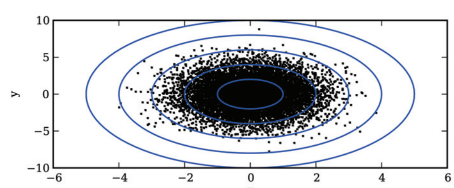

Figure 1: The number of particles in each ring in transverse real space (x-y) is counted.

Close to the longer side of the ellipse the multipole field expansion does not converge, so a correction is applied to make the field smoothly meet the exact KV envelope value on the surface. An alternative method is to find the potential in elliptic coordinates, this may be able to replace the multipole method [9].

A beam of particles can be tracked through a lattice including the space charge effects. It is also possible to track a witness bunch which feels the space charge force, but does not contribute to it. This is useful for measuring the space charge fields at test points. It can also be used to track pairs of particles with small initial offsets that can be used to find the transfer matrix of the lattice. This is useful for measuring changes in the Courant-Snyder parameters and the tune.

Benchmarking

The models have been compared against KV envelope equations by tracking a uniform elliptical bunch along a

ISBN 978-3-95450-115-1

drift. It shows good agreement to the analytic solution, although N-body model has some divergence due to numerical instability. While it is possible to improve the N-body method by adjusting the potential between very close particles, it would still not be a practical method for large numbers of particles due the slow O(N^2) algorithm.

It has also been benchmarked against the KVBL [10] envelope code on a synchrotron lattice, to show that it can replicate analytic results.

ADSR LATTICE

An ADSR with a $k_{\mathrm{eff}} = 0.98$ and an electrical output of 600 MW will require a 10 mA proton beam at an energy of 1 GeV, a power of 10 MW. It is likely that several accelerators would work in parallel to share the current requirement and increase reliability. This is beyond the capability of any existing accelerator, although large linac based spallation sources such as SNS and ESS are in the MW range.

A non-scaling FFAG lattice made of quadrupoles and dipoles with a circumference of 30 m was used for simulations. One 1 m cell is shown in figure 2. It accelerates protons from 35 to 400 MeV. A second FFAG ring would accelerate the protons to 1 GeV, but at the higher energies space charge would be less of an issue. The orbit moves out wards by about 0.5 m during acceleration resulting approximately 10 % change in path length. The speed changes considerably, from about 0.3 c to 0.7 c, and so the time of flight and therefore RF frequency vary by over a factor of 2. This means that the beam must be pulsed, resulting in high peak currents.

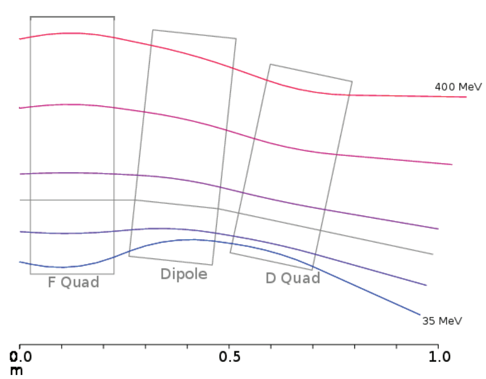

Figure 2: Non-scaling FFAG lattice, with closed orbits from 35 through 400 MeV.

All simulations presented were carried out with 50k macro particles.

EFFECTS OF SPACE CHARGE

Space charge causes several effects to the beam. Its defocusing effect can be measured at low peak currents. It increases the β function and decreases the tune. This tune shift is important because it can move the beam onto a resonance. In a synchrotron having the tune move to a resonance will cause large disruption to the beam, but due to the rapid acceleration in an FFAG it is possible to pass over resonances. The FFAG EMMA accelerates quickly through 4 integer resonances [11]. Studies of disruption at slower acceleration rates is ongoing.

As the current is increased the space charge forces can blow up the bunch size, cause halos to form, and push the bunch appart completely.

Simulation Results

The lattice described above was simulated using PyZgoubi for a range of peak currents.

In a non-scaling FFAG there is already a tune shift because the optics are not scaled with beam momentum, which can move the beam across resonances. The space charge tune depression is strongest at low energies, whereas without space charge the tune would fall with increasing energy. While it is not possible to use this to cancel out tune shift, it does reduce the total shift during acceleration, as shown in figure 3.

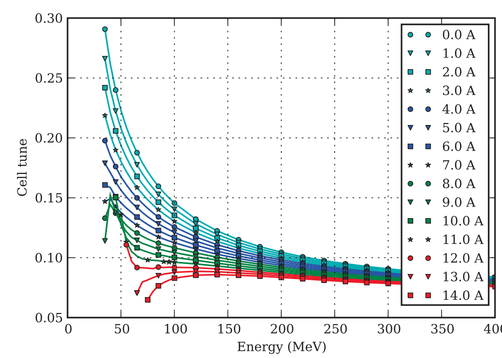

Figure 3: Cell tune as a function of energy and peak current for 10 mm mrad beam.

Due to the rapid acceleration in an FFAG it is not possible to adjust to the quadrupole strengths as function of beam energy to compensate for tune shift, as can be done in a synchrotron.

At high currents of around 10 A, the space charge defocusing is enough to cancel out the focusing from the quadrupoles, so the lattice becomes unstable. However at lower currents the beam can still be unstable over a large number of turns. Beams were tracked for 1000 cells of the lattice, and their size and emittance was recorded. Figure 4 shows the growth of a beam with a current of 2 A. There is already considerable growth, from 10 to about 40 mm mrad. There is much less increase in the RMS emittance as the core of the beam does not grow much. However in a MW accelerator even a halo of 1 % of the beam carries enough energy to cause considerable damage and activation.

A similar effect is seen if an initial distribution of a waterbag [12] is used. The expansion happens much faster as shown in figure 5, this is because the waterbag is not stationary under transverse space charge forces. The beam still expands to roughly the same final emittance.

ISBN 978-3-95450-115-1

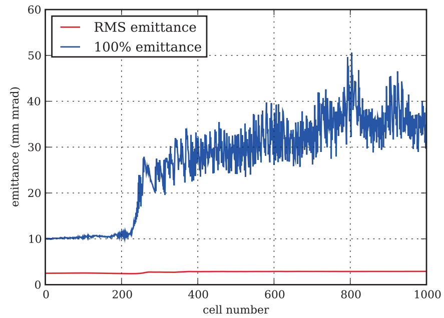

Figure 4: Emittance growth of a 10 mm mrad KV beam at 35 MeV at 2 A.

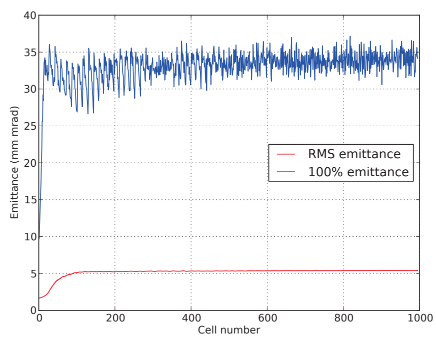

Figure 5: Emittance growth of a 10 mm mrad waterbag beam at 35 MeV at 2 A.

At 1 A no increase is seen after 1000 cells for a KV beam, and only a small growth from 10 to 14 mm mrad is seen for the waterbag beam. This is of the order that it would be cancelled by adiabatic damping.

Increasing the initial emittance of the bunch reduces the space charge effects as the charge density is lower. Doubling the emittance to 20 mm mrad allows the current to be doubled without seeing instability within 1000 cells. So whereas a 2 A 10 mm mrad beam will grow to around 40 mm mrad, the same beam injected at 20 mm mrad will remain constant in size.

With a peak current of 1 A and a bunching factor (ratio between average and peak current) of $\frac{1}{3}$, a high duty factor will be required to achieve the required average current. If acceleration can be achieved in 100 turns, then 3 mA beam could be produced, but if acceleration takes 1000 turns, then the average current is reduced to 0.3 mA. Higher peak currents can be used if a higher emittance is used, but this will make the extraction more difficult as there needs to be separation between the penultimate and final laps to allow for a septum. Another solution is to raise the injection energy, as this will reduce space charge forces.

If pulsing could be avoided by having continuous acceleration, then peak currents could be reduced by a large factor. For an FFAG that would require an isochronous lattice with a constant RF frequency, where a large change in orbit radius was used to cancel the change in particle speed as is done in a cyclotron. This would also introduce space charge effects between bunches on neighbouring orbits.

CONCLUSION

Zgoubi is a widely used code for modelling FFAGs. PyZgoubi adds a flexible layer to it, that can be used to add new features without making intrusive changes to the existing code. By adding space charge modelling to PyZgoubi it is possible to model FFAGs including space charge effects. This allows one to find peak current limits for a lattices, and study the effects on the beam. The model has been checked against theory and an existing code.

Zgoubi and PyZgoubi are freely available on Sourceforge.net. The space charge module for PyZgoubi will be uploaded shortly.

More details of this work can be found in [13].

REFERENCES

[1] E. Keil, CERN-BE-2010-006, CERN, 2010

[2] F. Méot and J. S. Berg, "Zgoubi." http://sourceforge.net/projects/zgoubi/, 9/5/2012.

[3] F. Méot and S. Valero, "Zgoubi users' guide," *CEA DSM DAPNIA-02-395*, 2002.

[4] S. Tygier, R.J. Barlow, H. Owen, TU6PFP030, PAC09, Vancouver, 2009.

[5] R.J. Barlow, S. Tygier, A. Toader, MOPEC047, IPAC10, Kyoto, 2010.

[6] S. Tygier and D. Kelliher, "Pyzgoubi." http://sourceforge.net/projects/pyzgoubi/, 9/5/2012.

[7] I. M. Kapchinskij and V. V. Vladimirskij, Proc. of the Int. Conf. on High-Energy Accelerators and Instrumentation, Geneva, pp. 274–288, CERN, 1959.

[8] S. Machida, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 309, no. 1-2, pp. 43–59, 1991.

[9] R. L. Gluckstern, Fermilab tech. rep. TM-1402. 1986.

[10] C. R. Prior, tech. rep., Rutherford Appleton Laboratory, 1998. RAL-TR-1998-048.

[11] S. Machida et. al. *Nature Physics*, 8, pg 243247, 2012.

[12] M. Reiser, P. O'Shea, S. Bernal, and R. Kishek, *Theory and design of charged particle beams*. Wiley-VCH, 2nd ed., 2008.

[13] S. Tygier, "High Current Proton Fixed-Field Alternating-Gradient Accelerator Designs", PhD Thesis, University of Manchester, 2011.