VARIOUS APPROACHES TO ELECTROMAGNETIC FIELD SIMULATIONS FOR RF CAVITIES*

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Abstract

The Superconducting Proton Linac (SPL) cavity is mainly designed and conducted by CERN. It is a part of the planned injector upgrade of the Large Hadron Collider (LHC). The SPL cavity is used to accelerate the ion beam from 160 MeV to 5GeV and served as a driver for neutrino facilities and radioactive beam facilities.

In the Superconducting Proton Linac (SPL) cavity, it is very important to calculate the eigenmodes precisely, because many higher order modes (HOMs) can lead to particle beam instabilities. We used and compared three different ways to calculate the eigenmodes in the SPL cavity: field simulation with hexahedron mesh in time domain, field simulation with hexahedron mesh in frequency domain and field simulation with tetrahedral mesh and higher order curvilinear elements. In this paper the principles of the three numerical methods will be introduced and compared. Finally the calculated results will be presented.

INTRODUCTION

The Superconducting Proton Linac (SPL) [1] at CERN uses two families ($\beta = 0.65$ and $\beta = 1$) of elliptical five cell superconducting cavities. Both families operate at 704.4 MHz. In this paper only the family ($\beta = 1$) will be researched. As shown in earlier studies [2], many higher order modes (HOMs) can cause particle beam instabilities. For this reason the properties of higher order modes should be exactly analysed. Therefore the frequencies of HOMs must be calculated as accurately as possible. In addition, the detailed field distribution of HOMs is also essential to calculate the shunt impedance of the individual mode. In this paper the principles of the three numerical methods will be introduced and compared.

FIELD SIMULATION WITH HEXAHEDRON MESH IN TIME DOMAIN

The first method field simulation with hexahedral grid in time domain should be carried out with Transient Solver from CST MICROWAVE STUDIO [3]. In order to excite the electromagnetic field in the cavity a broadband high-frequency pulse was coupled into the cavity through a coaxial-type power coupler, furthermore a discrete port was put in the first cell of the SPL cavity. Some probes in x-, y- and z-direction were placed in the fifth cell of the cavity, so that the electromagnetic field in time domain can be detected and recorded (see Fig. 1). After a Fourier transformation of the recorded field date the eigenmode spectrum in frequency domain can be achieved. It is obvious by comparing the results from frequency and time domain, that many eigenmodes frequencies are located at the maximums from the eigenmode spectrum. For example the fundamental passband TM_{010} is shown in Fig. 2. Therefore the frequencies of the eigenmodes can be measured from the eigenmode spectrum. Before the second simulation in Transient Solver the electric field monitors for the measured frequencies should be defined, so that the field distribution of the eigenmodes can be achieved after the second simulation and the corresponding shunt impedance can be determined.

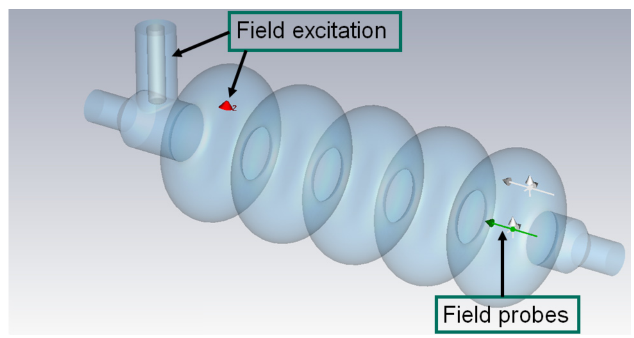

Figure 1: Power coupler and discrete port in cell 1, Field detection in cell 5.

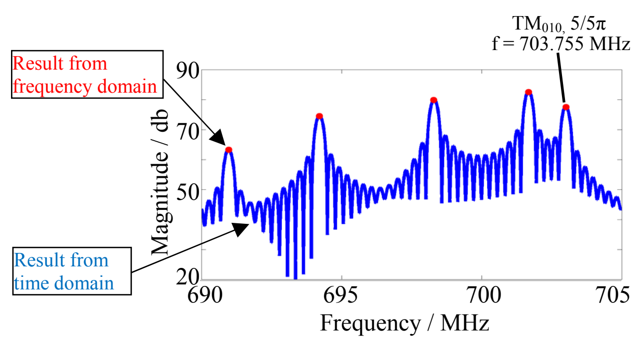

Figure 2: Eigenmode spectrum of TM_{010} passband.

The field simulation from Fig. 2 was run with very high resolution (49.630.173 hexahedral elements FPBA), but the calculated frequency for $TM_{010,}$ π mode was only 703.755 MHz. The difference to the design frequency (704.4 MHz) is a little bigger. Therefore much more hexahedral elements are required to get the accurate eigenmodes frequencies and the corresponding electromagnetic field date, which will lead to a time consuming simulation process. For example it took about 50 hours with the GPU cluster to get the results in Fig. 2.

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Table 1: Electromagnetic Field Simulation Results (Hexahedral Mesh and Tetrahedral Mesh)

Mode	Hexahedral Elements (CST MWS 2011) Meshcells PBA / million	Frequency / MHz	Shunt Impedance / Ω	Curvilinear Tetrahedral Elements Meshcells / million	Frequency / MHz	Shunt Impedance / Ω
$TM_{010,\ 1/5}\ \pi$	80	692.416	0.002	6.2	692.446	0.002
$TM_{010,\ 2/5}\ \pi$	80	695.643	0.037	6.2	695.676	0.037
$TM_{010,\ 3/5}\ \pi$	80	699.710	0.010	6.2	699.744	0.011
$TM_{010,\ 4/5}\ \pi$	80	703.065	0.071	6.2	703.101	0.067
$TM_{010,\ 5/5}\ \pi$	80	704.361	565.468	6.2	704.398	565.452

FIELD SIMULATION WITH HEXAHEDRON MESH IN FREQUENCY DOMAIN

Compared to the first method the simulation with hexahedral grid in frequency domain is a considerably simpler way to simulate the field in SPL cavity. The eigenmodes frequencies and shunt impedances can be calculated with hexahedral discretization directly by Eigenmode Solver in CST MICROWAVE STUDIO. The results from the Eigenmode Solver involve the frequencies and the field distribution of the eigenmodes. From that the shunt impedance of the individual eigenmode can be calculated. The frequencies as well as the shunt impedance of the modes in the fundamental passband are listed in Table 1. But similar to the first method a large number of hexahedral grid points for the elliptical SPL cavities are required to achieve stable simulation results, because a lower number of hexahedral elements cannot represent the contours of the elliptical resonator precisely. This can result in a slow convergence of the Eigenmode Solver (Fig. 3), which lead to an extremely time consuming simulation. As can be seen from the Fig. 3, the convergent frequency (80 million hexahedral elements PBA) for $TM_{010,}\ \pi$ mode is 704.36 MHz. The convergent value is not very satisfied. In addition it took almost 670 hours to run the computation task with 80 million hexahedral elements. Therefore the simulation with hexahedral elements results in a demanding computation task. To solve this problem, accurate curvilinear finite elements, which are able to capture the non-flat shape of typically applied superconducting resonator structures, are used [4].

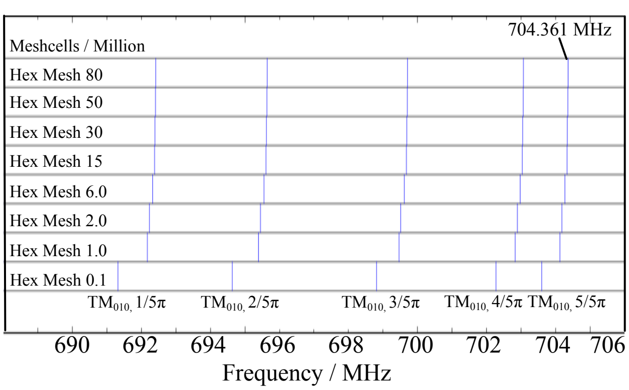

Figure 3: Convergence analysis of various numbers of hexahedral elements based on the resonance frequency of the fundamental modes (TM_{010}) in a SPL cavity.

FIELD SIMULATION WITH TETRAHEDRAL MESH AND HIGHER ORDER CURVILINEAR ELEMENTS

A popular discretization method is finite element method (FEM), which is employing a plain tetrahedral mesh (Fig. 4 (a)) [4]. But in case curved material interfaces (for example, elliptical SPL cavity) have to be considered, the curvilinear elements (Fig. 4 (b)) can be applied to increase demand for high precision modeling. The precise modeling employs huge memory resources and requires long simulation times, therefore it is difficult to run such large computation on classical workstation. To keep the simulation time on an acceptable level, the simulation is run on a distributed memory architecture using the MPI parallelization strategy. The geometric modeling of the SPL cavity structure with the tetrahedral meshing is performed within the CST MICROWAVE STUDIO. Then the necessary information is passed to the FEM program by means of ASCII or binary file transfer. The entire FEM program is written in C++ by Wolfgang Ackermann at Computational Electromagnetics Laboratory (TEMF), Technische Universitaet Darmstadt.

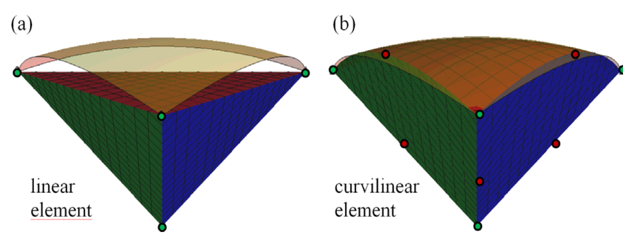

Figure 4: Linear versus curvilinear tetrahedral element [4].

In Fig. 5, the results of the convergence for tetrahedral mesh are displayed. As expected, Fig. 5 shows us a very fast convergence of the eigenmodes frequencies in TM_{010} passband. The convergent value of the frequency for TM_{010}, π mode is also precise (704.397 MHz). Another advantage for employing the tetrahedral mesh is the acceptable simulation time. Using the MPI parallelization strategy the time needed to run the simulation is much more efficient than hexahedral mesh. The mode frequencies and the R/Q of the TM_{010} modes in the five cell SPL cavities obtained with symmetric tetrahedral elements are listed in Table 1.

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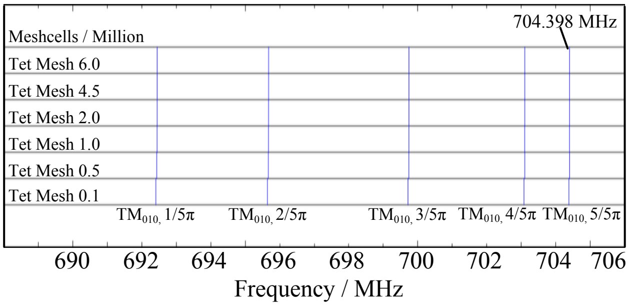

Figure 5: Convergence analysis of various numbers of tetrahedral elements based on the resonance frequency of the fundamental modes (TM_{010}) in a SPL cavity.

For the field simulation in SPL cavity apart from the eigenmode frequencies also the field distribution demands high precision. Due to the asymmetrical main input power coupler the transverse components of the electric strength along the particle beam axis are up to three orders of magnitude smaller (Fig. 6). Such transversal field, which can cause inaccurate value of shunt impedance, should be suppressed. To overcome such difficulties, symmetric mesh cells should be involved in the field evaluation process [4]. The tetrahedral grids should be symmetric to the particle beam axis (Fig. 7). As is shown in Fig. 6, if the field is evaluated on symmetric mesh, the transverse field components in resonant region are completely eliminated. The transverse components only appeared in the beam pipe, where the power coupler is assembled, because the mesh in this area is non-symmetric.

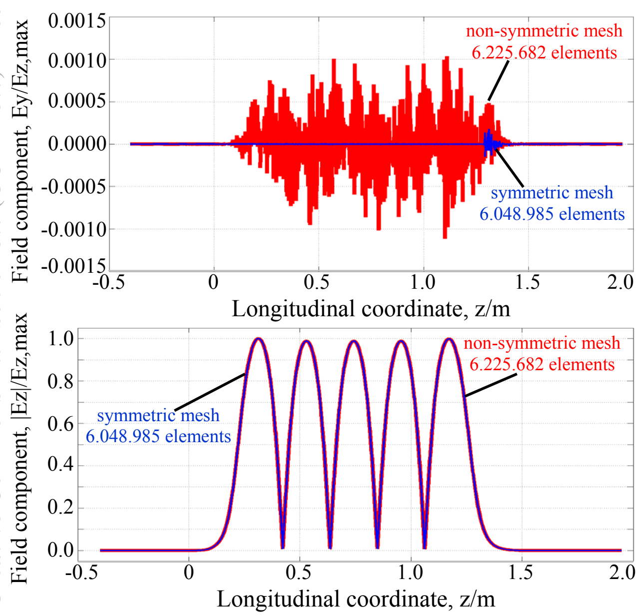

Figure 6: Evaluation of the electric field strength along the cavity axis. All Cartesian components are normalized to the maximum longitudinal field value $E_{z,\,max}$. All calculations are performed using non-symmetrical and symmetrical curvilinear tetrahedral elements.

In addition Fig. 6 shows the quite smooth longitudinal components (E_z) for the non-symmetrical and symmetrical elements, which are applied to accelerate the particle beam. The magnitude difference of E_z between the two mesh types is up to four orders. Generally speaking, compare to the non-symmetric elements the FEM method on the basis of symmetric elements can evaluate the field distribution in the elliptical SPL cavity more precisely.

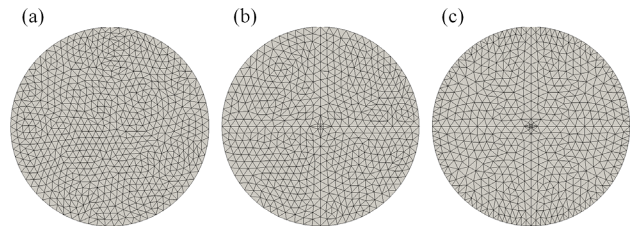

Figure 7: Cross section of a tetrahedral mesh in a plane normal to the longitudinal axis of the resonator for an arbitrary unstructured mesh (a), a mesh aligned at the coordinate planes (b) and a true symmetric mesh (c) [4].

CONCLUSION AND OUTLOOK

In this paper three numerical methods to calculate the electromagnetic field in the elliptical SPL cavity are reported. The numerical methods on the basis of hexahedral elements are not ideal approaches to electromagnetic field simulation for elliptical SPL cavity, because with hexahedral elements it is difficult to illustrate the contours of the elliptical resonator precisely. Furthermore a huge number of the hexahedral elements will lead to a time consuming simulation process and very poor convergence of the simulation results. By comparison a parallel FEM method on the basis of curvilinear tetrahedral elements is able to evaluate the electromagnetic field in the elliptical SPL cavity precisely and efficiently. The superior convergence rate can be preserved. In addition parasitic high frequency oscillations in the transverse field components along the particle beam axis can be eliminated by using a symmetric arrangement of the tetrahedral elements with respect to the particle beam axis in the cavity region [4].

In the future the symmetric tetrahedral mesh will be separately set up in the beam pipe, where the couplers are assembled, so that the transverse components in the beam pipe can be completely suppressed.

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