END-TO-END G4BEAMLINE SIMULATION OF AN INVERSE CYCLOTRON FOR MUON COOLING*

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Abstract

The production of intense high energy muon beams for the next generation of particle physics experiments is an active area of interest primarily due to the muon's large mass (compared to electrons) and pointlike structure (unlike protons). The muon production and the subsequent preparation into a beam are challenging due the large emittance of the initial muon beam and the short mean muon lifetime. Most muon cooling channels being developed are single-pass structures due to the difficulty of injecting large emittance beams into a circular device. Inverse cyclotrons can potentially solve the injection problem and also reduce the muon beam emittance by a large factor. An end-to-end (injection to extraction) simulation of an inverse cyclotron for muon cooling is presented performed with G4Beamline, a GEANT-based particle tracking simulation program.

INTRODUCTION

Facilities employing intense high energy muon beams, such as neutrino factories [1], muon colliders [2, 3], and neutrinoless muon to electron conversion experiments [4] are current research and development programs for possible next generation particle physics experiments studying neutrinos, the Higgs particle, possible supersymmetric particles, and lepton number violating decays. Muon beams are produced from the decays of pions which, in turn, are produced from collisions of high-energy protons and a high-density target. A tertiary muon beam has a large emittance which must be reduced for a useful beam for experiments. Also, the short lifetime of the muon ($\sim 2.2\ \mu$s) makes the rapid ionization cooling technique necessary as opposed to standard cooling techniques for stable electrons or anti-protons.

Most muon cooling channels are single-pass structures requiring strong radio-frequency gradients and strong transverse focusing to achieve rapid cooling. Multi-pass sychrotron ring-like structures would be much less expensive than linear designs, but injecting large emittance initial muon beams is a limiting difficulty [5]. An inverse cyclotron, a cyclotron in which particles are injected at large radius and lose energy by passing through a moderator material and thus spiral in toward the cyclotron center, is such a multi-pass device with potentially large beam admittance [6].

AN INVERSE SECTORED CYCLOTRON

An inverse three-sectored cyclotron with some novel features is presented. After being slowed down to roughly tens of eV kinetic energy by helium, muons are trapped in vacuum above the helium by an electric trap which employs a field that rises with time. Stochastic energy loss and multiple scattering as well as decay losses are simulated in this design. The initial beam energy in this design, about 13.5 MeV, is larger than the 3 - 7 MeV initial energies of some designs based on frictional cooling [7], but smaller than the roughly 100 MeV initial kinetic energies of conventional single-pass designs [3].

Simulations of this muon inverse cyclotron are done by G4Beamline [8]. Fig. 1 shows a picture of the inverse cyclotron. The initial beam is 1000 muons with average initial starting position of (x, z) = (132 mm, 1 mm), average initial kinetic energy of 13.5 MeV, and average initial time of -3 ns. The initial Gaussian spreads are 24 mm in radius and z, 4.1 MeV/c and 4.0 MeV/c in p_r and p_z, 2.0 MeV in kinetic energy, and 123 ns in time. Muons are slowed down, cooled, trapped, and reaccelerated as follows:

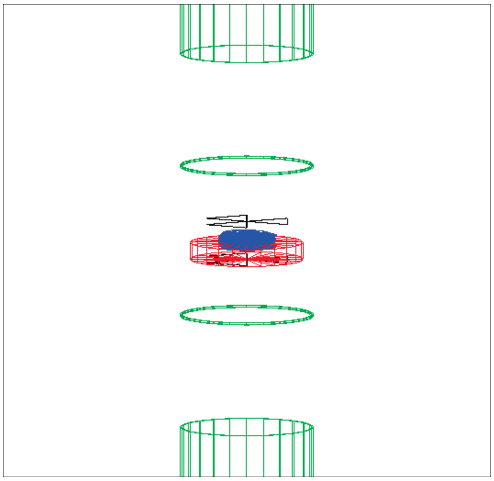

Figure 1: G4Beamline visualization of the inverse cyclotron showing the triangular current sectors providing the three-sectored field, the two coils providing a magnetic bottle, the upper solenoid guiding the muons after extraction, the lower solenoid with idential current as the upper one to maintain magnetic field symmetry with respect to $z = 0$, and the helium moderator (red). A muon orbit (blue) that is trapped is also shown.

1. The muons are slowed down by 2.24 bar helium offset from $z = 0$ so that the top surface of the helium

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is at $z = -10$ mm. While the muons are slowed down, a trapping electric field pointed into the $z = 0$ midplane is applied. Single-turn injection by passing muons through outer lithium hydride wedges works and improvement is being explored.

2. Some muons are slowed to roughly 20 eV kinetic energy in the helium (reaching frictional cooling equilibrium between the electric field force and the retarding force from passing through helium), pulled out by the electric field (positive for $z < 0$), and then trapped in vacuum above the helium by the electric field which increases with time.
3. When the magnitude of the trapping field (pointed into the $z = 0$ midplane) reaches 4 MV/m, the field transitions to a uniform 1.8 MV/m extraction field pointed in the $+z$ direction in 50 ns.
4. The extracted muons are accelerated to K.E $\sim$ 100 MeV by a 1.8 MV/m electric field over 56 m through a guiding solenoid providing a 1.8 T field.

The static magnetic field consists of a three-sectored field, a magnetic bottle, a guiding solenoidal field and a second solenoidal field. The sectored field is generated by six triangular current paths, three in the $z = +102$ mm plane and three in the $z = -102$ mm plane; the current in the sectors is 0.0788 MA. The bottle field is generated from two current-carrying 350 mm radius coils in the $z = \pm 400$ mm planes; the currents in the coils are 1.164 MA. These currents generate a 1.460 T field at the inverse cyclotron origin. The guiding solenoidal and balancing fields are provided by solenoids with length$= 60,000$ mm, radius$= 400$ mm, and total current$= 85,944$ MA. The centers of the guiding solenoids are at $z = \pm 31,000$ mm.

We have studied the properties of the equilibrium orbit by the simulation code CYCLOPS [9]. The magnet field grid data in polar coordinates is generated by G4Beamline and imported into CYCLOPS in which the closed orbits at different energies are searched for by numerical iteration. Once the closed orbit is identified, it then calculates the betatron tunes, flutter factor, cycling frequencies, average orbit radius, and other quantities.

The CYCLOPS results of the flutter and betatron tunes of the inverse cyclotron are shown in Fig. 2 and Fig. 3, respectively. Stable closed orbits are found for energies less than 18.5 MeV. The flutter factor is small so that $\nu_r^2 + \nu_z^2$ is approximately 1. During the deceleration from 13.5 MeV, the beam will cross two of the 4th order resonances: $4\nu_r = 3$ and $3\nu_r + \nu_z = 3$, which are close to each other. A rough estimate shows that the tune shifting rate is about 0.002/turn for the radial tune and 0.005/turn for the vertical tune. With such a fast crossing speed, the systematic 4th order resonance is unlikely to be a problem.

The electric field which traps and extracts muons has the following spatial and time dependence.

- Trapping field for $0 < t < 1200$ ns: $|E_z| = 0.0033$ (MV/(m ns))t. E_z points into the $z = 0$ midplane.

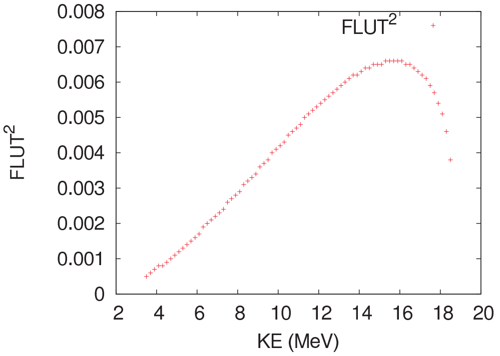

Figure 2: The square of the midplane magnetic field flutter as a function of kinetic energy. The equilibrium orbits and corresponding flutter were determined by the CYCLOPS simulation code.

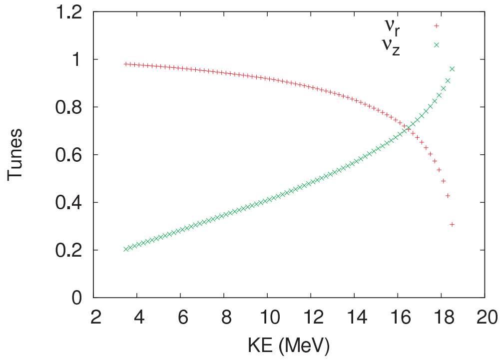

Figure 3: The axial (z) and radial (r) tunes as functions of kinetic energy. The equilibrium orbits and corresponding flutter were determined by the CYCLOPS simulation code.

- Transition field for $1200 < t < 1250$ ns: For $z < 0$, E_z changes from +4 MV/(m ns) to +1.8 MV/(m ns), and for $z > 0$, E_z changes from -4 MV/(m ns) to +1.8 MV/(m ns). The electric field rate of change is linear above and below the midplane.
- Extraction field: $t > 1250$ ns, E_z = 1.8 MV/m, all z.

RESULTS AND DISCUSSION

The initial normalized transverse and longitudinal emittances of the initial 1000 muon beam are 0.93 mm-rad and 699 mm, respectively. Of the 1000 muons, 121 are slowed down by the helium, trapped in the vacuum and reaccelerated to 100 MeV kinetic energy in less than 1600 ns. The initial normalized transverse and longitudinal emittances of 121 extracted muons are 0.87 mm-rad and 546 mm. The primary loss mechanisms are muons getting stopped in helium, muons getting trapped at high kinetic energies, and decay losses. The tranverse emittance is decreased only to about 0.6 mm-rad while the longitudinal emittance is reduced to about 0.2 mm, about 3000-fold. Table 1 shows the kinematic spreads of the input and extracted beams.

Figure 4 shows the 2D time-kinetic energy distribution of extracted muons at $z = 56$ m after being accelerated up

Table 1: Kinematic spreads of input and output beams

Input Beam	Extracted Beam (initial spreads)	Extracted Beam (final spreads)
1000 μ^+	121 μ^+	121 μ^+
$\sigma_r = 24$ $\sigma_{p_r} = 4.1$ $\sigma_{r,p_r} = -0.4$	$\sigma_r = 21$ $\sigma_{p_r} = 4.0$ $\sigma_{r,p_r} = -2.4$	$\sigma_x = 47$ mm $\sigma_{p_x} = 1.6$ MeV/c $\sigma_{x,p_x} = 33$ mm-MeV/c
$\sigma_z = 24$ $\sigma_{p_z} = 4.0$ $\sigma_{z,p_z} = 0.5$	$\sigma_z = 25$ $\sigma_{p_z} = 4.1$ $\sigma_{z,p_z} = -16$	$\sigma_y = 48$ mm $\sigma_{p_y} = 1.5$ MeV/c $\sigma_{y,p_y} = 31$ mm-MeV/c
$\sigma_t = 123$ $\sigma_{KE} = 2.0$ $\sigma_{t,KE} = -3.9$	$\sigma_t = 113$ $\sigma_{KE} = 1.7$ $\sigma_{t,KE} = 18$	$\sigma_t = 2.1$ ns $\sigma_{KE} = 0.035$ MeV $\sigma_{t,KE} = 0.027$ ns-MeV
$\epsilon_{r,N} = 0.94$ $\epsilon_{z,N} = 0.92$ $\epsilon_{L,N} = 699$	$\epsilon_{r,N} = 0.79$ $\epsilon_{z,N} = 0.95$ $\epsilon_{L,N} = 546$	$\epsilon_{x,N} = 0.63$ mm-rad $\epsilon_{y,N} = 0.63$ mm-rad $\epsilon_{L,N} = 0.21$ mm

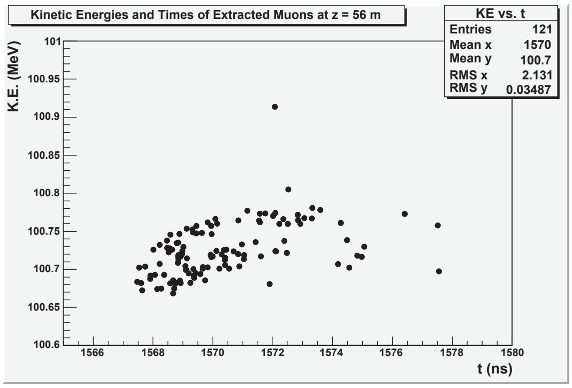

Figure 4: Time and kinetic energy of muons extracted and accelerated to 100 MeV and located at $z = 56$ m.

to 100 MeV. Fig. 5 plots the emittance decrease.

These results are dependent on the simulation of multiple scattering and straggling processes at kinetic energies in the keV range. These simulations were made with G4Beamline version 2.06 which uses GEANT4 version geant4-09-03-patch1. When another G4Beamline version with a correspondingly different GEANT4 version is used, muons that would otherwise be pulled out of the helium by the rising electric field, trapped, and extracted are instead stopped in the helium. Correspondence about the validity of GEANT4 simulation of low energy physics is ongoing [10].

Improvements over previous inverse cyclotron designs include the use of an electric field trap which traps muons in vacuum instead of trapping in material, a more realistic magnetic field generated solely from electric currents, and inclusion of energy straggling, multiple scattering, and decay losses in the simulation.

More work needs to be done on a realistic helium container, increasing initial beam energy, increasing admittance, decreasing transverse emittance, increasing efficiency, and using a skew quadrupole triplet to deal with canonical angular momentum [11]. Optimization of the magnetic field through CYCLOPS may improve focusing.

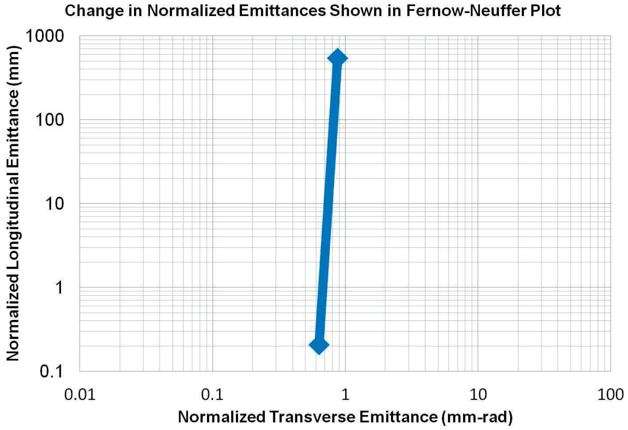

Figure 5: Normalized emittance decrease.

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REFERENCES

[1] D. Koshkarev, CERN-ISR-DI-74-62;
M. Alsharo'a *et al.*, Phys. Rev. ST AB **6** (2003) 081001.

[2] D. Neuffer, AIP Conf. Proc. **156** (1987) 201;
R. Fernow and J. Gallardo, Phys. Rev. **E52** (1995) 1039;
J. Gallardo *et al.*, Snowmass 1996, BNL-52503;
C. Ankenbrandt *et al.*, Phys. Rev. ST AB **2** (1999) 081001;
R. Palmer *et al.*, Phys. Rev. ST AB **12** (2009) 031002;
Y. Torun *et al.*, IPAC-2012-THPPC037;
G. Lyons *et al.*, IPAC-2012-TUPPR008.

[3] R. Palmer *et al.*, arXiv:0711.4275.

[4] R. Kutschke, arXiv:1112.0242.

[5] R. Palmer *et al.*, Phys. Rev. ST AB **8** (2005) 061003.

[6] D. Summers *et al.*, Int. J. Mod. Phys. **A20** (2005) 3851;
K. Paul *et al.*, PAC-2011-MOP051;
T. Hart *et al.*, PAC-2011-WEP190.

[7] Y. Bao, A. Caldwell, D. Greenwald, PAC-2011-MOP017;
T. J. Roberts and D. M. Kaplan, PAC09-WE6PFP096.

[8] T. J. Roberts *et al.*, EPAC-2008-WEPP120.

[9] M. M. Gordon, Part. Accel. **16** (1984) 39.

[10] Private communication with Yu Bao, developer of the sphere cooler, and Vladimir Ivantchenko, GEANT4 developer.

[11] R. Brinkmann *et al.*, Phys.Rev. ST AB **4** (2001) 053501;
B. Carlsten and K. Bishofberger, New J. Phys. **8** (2006) 286.