LARGE SCALE SIMULATION OF SYNCHROTRON RADIATION USING A LIENARD-WIECHERT APPROACH

R. D. Ryne*, C. E. Mitchell, J. Qiang
Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
B. Carlsten, N. Yampolsky, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Abstract

Synchrotron radiation is a key phenomenon affecting lepton accelerators. Large-scale parallel modeling provides a means to explore properties of synchrotron radiation that would be impossible to study through analytical methods alone. We have performed first-principles simulations of synchrotron radiation fields, using a Lienard-Wiechert approach, with the same number of simulation particles as would be found in bunches with charge up to 1 nC. The results shed light on the limits of the widely used one-dimensional model, the enhancement of coherent synchrotron radiation due to microbunching, and the noise in the radiation field due to discreteness effects in bunches with billions of electrons.

INTRODUCTION

Large-scale simulation using parallel computers is essential to the development of future light sources. Thanks to initiatives like the U.S. D.O.E. SciDAC program [1], the capability for 3D parallel space-charge modeling has become widespread throughout the accelerator design community. However, the inclusion of synchrotron radiation effects is in a much less advanced state. Synchrotron radiation is arguably the least well modeled collective phenomenon in parallel beam dynamics codes. Most codes use 1D models. The code CSRtrack [2] has 3D capability, but it uses an averaging procedure that affects its ability to model phenomena associated with the discrete-particle nature of the bunch. And while some codes have a 3D point-to-point model, the computational requirements are so high that self-consistent simulations can be performed only with far fewer simulation particles than in a physical beam bunch, which, as will be shown below, can lead to huge inaccuracies.

Here we will demonstrate that much can be learned from a 3D Lienard-Wiechert code that is *not* self-consistent, but that is of sufficiently large scale that it allows for exploring single-particle effects and allows for exploring phenomena at very small scales, of order nanometers or less.

COMPUTATIONAL MODEL

We begin with the Lienard-Wiechert fields:

$$\begin{aligned}\vec{E} &= \left[\frac{q}{\gamma^2\kappa^3 R^2}\left(\hat{n}-\vec{\beta}\right)+\frac{q}{\kappa^3 Rc}\hat{n}\times\left\{\left(\hat{n}-\vec{\beta}\right)\times\frac{\partial\vec{\beta}}{\partial t}\right\}\right]\\ \vec{B} &= \left[\hat{n}\times\vec{E}\right]\end{aligned} \quad (1)$$

where square brackets denote that the quantity inside is to be evaluated at the retarded time. In the above, $\vec{\beta}$ denotes a particle's velocity vector divided by the speed of light, $\vec{R}$ is a vector pointing from a radiating particle's position at the retarded time to the observation point at time t, $\hat{n}$ is the unit vector $\hat{n}=\vec{R}/|R|$, and $\kappa=1-\hat{n}\cdot\vec{\beta}$. For the purpose of this study we will focus on the motion of an electron bunch in a uniform, constant magnetic field, $\vec{B}(x,y,z,t)=B_0\hat{y}$. This describes the field in the interior of an ideal dipole magnet. The bend radius is $\rho=1$ m. A steady state model is assumed, i.e. we neglect transitions into and out of the magnet.

The Lienard-Wiechert fields are calculated at the moment the bunch centroid is moving in the z-direction. Once the retarded time is calculated, the fields follow immediately from Eq. (1) by superposition of contributions from all the particles. To compute the retarded time, we solve $(x-x_r)^2+(y-y_r)^2+(z-z_r)^2=c^2(t-t_r)^2$, where (x,y,z) denotes an observation point at time t, and where (x_r,y_r,z_r) denotes a particle's position at the retarded time t_r. The equation is solved by a 2-step process. First, a bisection search is used to find a value of t near the retarded time, then a Newton search is used to compute t_r. For this study, t_r is calculated to a relative accuracy of 10^{-10}.

LIMITS OF THE 1D MODEL

We explored the limits of the 1D model by performing simulations with 6.24 billion particles (1 nC bunch charge) for 4 different distributions. Each was a 3D, zero emittance Gaussian bunch with longitudinal rms size $\sigma_z = 10\ \mu m$. We use the names "cigar," "ball," "disk," and "plate" to describe bunches with transverse rms sizes $\sigma_\perp=\sigma_x=\sigma_y=1, 10, 100, 500\ \mu m$, respectively. Fig. 1 shows the on-axis z-directed radiative field for these 4 cases at 100 MeV. The 1 GeV case is shown in Fig. 2. The 1D model is likely to be valid so long as the transverse rms size, $\sigma_\perp$, is well below $\rho(\sigma_z/\rho)^{2/3}$, or about 500 μm for a 10 μm long bunch and $\rho=1$ m. This is consistent with our simulation results which show that the 1D model is an excellent approximation for all cases except the plate. The simulations show that the radiation field is very smooth at 100 MeV, but it is significantly noisier at 1 GeV. Since we are using the "real-world" number of simulation particles as would be found in a 1 nC electron bunch, and since we are summing the contributions from all the electrons at each observation point, this noise is physical, not numerical. The increased noise at 1 GeV compared to 100 MeV is due to the smaller opening angle of the radiation field at 1 GeV.

* RDRyne@lbl.gov

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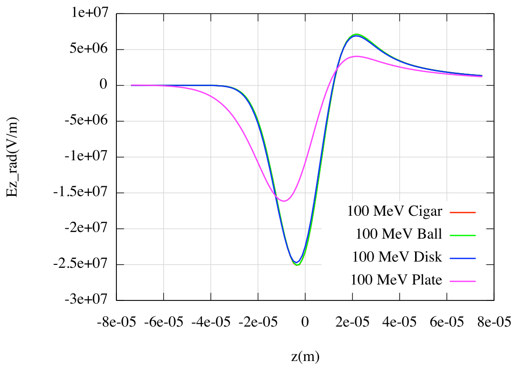

Figure 1: On-axis radiation electric field, $E_{z,rad}$, for a 100 MeV, 10-micron long Gaussian bunch modeled using 6.24 billion electrons. The labels "cigar," "ball," "disk," and "plate" describe bunches with $\sigma_x = \sigma_y = 1, 10, 100, 500\ \mu m$, respectively. Bend radius $\rho = 1$ m. The fields for all cases except the "plate" are nearly indistinguishable. The 1-D approximation is very robust.

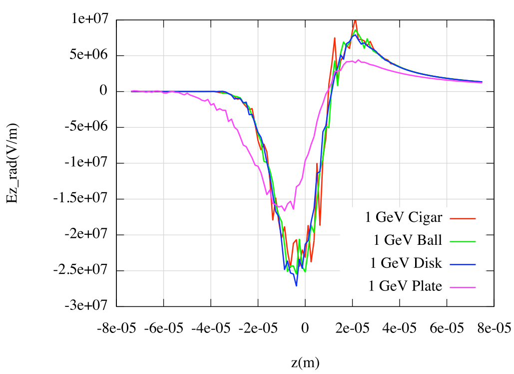

Figure 2: $E_{z,rad}$ for a 1 GeV, 10-micron long Gaussian bunch modeled using 6.24 billion electrons. Due to the smaller opening angle of the radiation at high energy, the shot noise is much larger at 1 GeV than at 100 MeV.

We also performed simulations using a tilted disk, with $\sigma_x = \sigma_y = \sqrt{2} \times 10\ \mu m$ and $\sigma_z = 1 \mu m$, with a tilt angle of 45 degrees so that the projection of the disk onto the the z-axis was 10 μm. The radiation field was nearly identical to that of the 10 μm ball.

CSR ENHANCEMENT FROM MICROBUNCHING

Due to the nature of the 1D single-particle wake, very large radiation fields can be produced at high γ. To study this, we imposed microbunched structure on the cigar-shaped Gaussian bunch. Fig. 3 shows the longitudinal radiation field at 100 MeV and 1 GeV, with and without microbunching, for sinusoidal microbunches at a wavelength $\lambda_{\mu b} = 100$ nm with a depth of modulation of 90%. At 1 GeV there is a roughly 10x enhancement (*i.e.,* a 10x increase in $E_{z,rad}$ compared with no microbunching), while there is very little enhancement at 100 MeV. This is consistent with theory since the single-particle wake peak is within $z \approx \rho/\gamma^3$, and since $\lambda_{\mu b} >> \rho/\gamma^3 \approx 0.1$ nm at 1 GeV, and $\lambda_{\mu b} \approx \rho/\gamma^3 \approx 100$ nm at 100 MeV. At 1 GeV we observed that, if the microbunches are tilted a small amount so that the projected density has less modulation, the enhancement rapidly decreases.

We also performed 1D simulations of microbunched beams using a new integrated Green function (IGF) approach [3]. 1D CSR calculations usually involve performing a discrete convolution based on values of the Green function (*i.e.,* the single-particle wake) and the line change density at grid points. In the IGF approach, one uses an effective Green function that is related to definite integrals of the wake and certain basis functions. The result is that the accuracy of the discrete convolution depends only on resolving the density. Applying this method to the 1D CSR problem results in an efficient and robust algorithm that does not require resolving the short-range peak of the single-particle wake. The 1D, 1GeV, IGF results are also shown in Fig. 3, and are seen to be in good agreement with the 3D Lienard-Wiechert results.

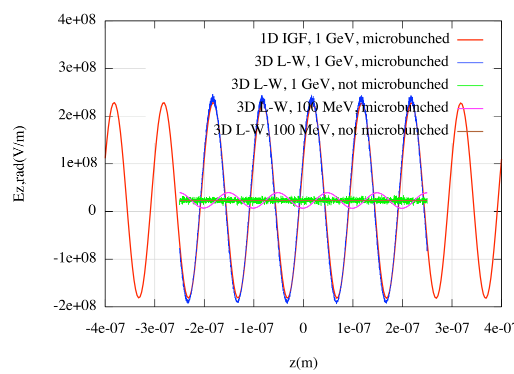

Figure 3: $E_{z,rad}$ in a cigar-shaped Gaussian bunch at 100 MeV and 1 GeV, with and without sinusoidal microbunching at $\lambda_{\mu b} = 100$ nm. The 3D Lienard-Wiechert code used 624 million particles. The field in the 1 GeV microbunched case is approximately 10x greater than for the non-microbunched case. Also shown is the result from a 1D integrated Green function (IGF) calculation [3].

PROPERTIES OF THE SHOT NOISE

As seen above, strong shot noise effects are evident in bunches with energy 1 GeV. Fig. 4 shows the spatial dependence of the shot noise by exhibiting the longitudinal radiation field along 3 lines: one line is displaced $-2\ \sigma_x$ "inside" of the bend, one line going through the bunch centroid, and one line is displaced 2 σ_x "outside" of the bend. The shot noise increases from inside to outside the bend. This is due to the fact that, at large γ, the Lienard-Wiechert fields have small denominators when the vector from the retarded source position to the observation point at the observation time is parallel to the velocity vector of the source

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charge at the retarded time. Geometrically, this alignment is more likely to occur for observation points on the outside of the bend.

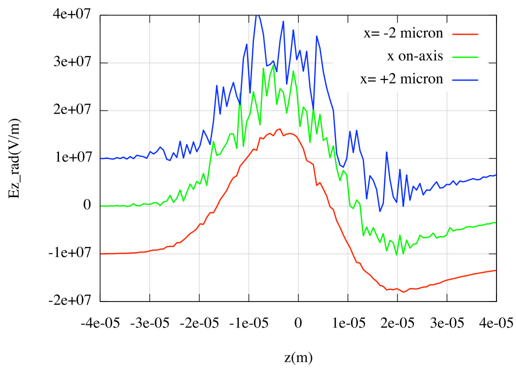

Figure 4: $E_{z,rad}$ for the 10 micron "cigar" case at 1 GeV, modeled using 6.24 billion electrons. The field is shown along lines at x=-2 μm, x=0, and x=2 μm. The curves at ± 2 μm are displaced to make them easier to see on the plot. Note that the shot noise is much stronger on the outside of the bend than on the inside of the bend.

In order to study the statistical properties of the shot noise, we simultaneously performed 12000 simulations with different initial conditions, each with 624 million particles (100 pC bunch charge), and computed the radiation field at the bunch centroid. Then we made histograms of the resulting data. Fig. 5 shows the scaled probability density of the longitudinal radiation electric field at 10, 100, and 1000 MeV, all with $\rho = 1$ m. For these parameters the shot noise appears to be roughly Gaussian (though it was non-Gaussian at higher energy), and the rms width increases rapidly with energy. Table 1 shows the mean radiation field and its standard deviation, $\sigma_{Ez,rad}$, for these 3 cases. Note that, at 1 GeV, $\sigma_{Ez,rad}$ is so large that it is of the same order as the mean value. Based on this data $\sigma_{Ez,rad}$ appears to grow as γ^2 while the mean remains roughly constant at high energies.

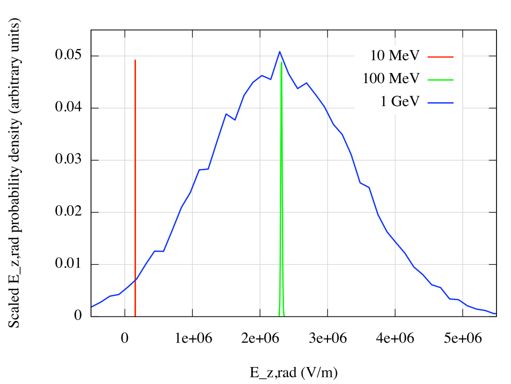

Figure 5: Histograms of the longitudinal radiation field at the bunch centroid, deposited in 64 bins, as obtained from 12000 simulations, each with 624 million particles.

Table 1: $E_{z,rad}$ Statistics (12000 Realizations)

E (MeV)	Mean (MV/m)	$\sigma_{Ez,rad}$ (MV/m)
10	-0.155	0.00033
100	-2.32	0.011
1000	-2.30	1.105

DISCUSSION AND CONCLUSIONS

Using first-principles, large-scale simulation we have explored three properties of synchrotron radiation in electron bunches. First, we explored the limits of the 1D model and found it to be very robust. Second, we observed very large CSR enhancement in microbunched beams. The enhancement exhibited a strong dependence both on electron energy and on the microbunching wavelength. Third, we found surprisingly large amounts of shot noise at energies of 1 GeV, even in bunches with 6 billion electrons.

Regarding the shot noise, it is important to emphasize that the calculations described here are purely classical. In reality, the emission of synchrotron radiation is a quantum process, and accurate predictions will require a quantum treatment depending on the problem parameters. However, preliminary semi-classical calculations indicate that the classical results presented here are trustworthy up to approximately 1 GeV. At energies of a few GeV and above, the semi-classical calculations indicate that the quantum shot noise will be *greater* than that predicted by the classical Lienard-Wiechert calculation.

Finally, it is worth mentioning that the study presented here focused just on field calculations, not on the influence of the fields on the particle dynamics. It is possible that the observations of strong microbunching enhancement and strong shot noise will be short-lived, and may have less impact on the beam quality than one might expect given the amplitude of the fields exhibited here. This will be explored in the future.

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