ANALYSIS OF RESONANT TE WAVE MODULATION SIGNALS FOR ELECTRON CLOUD MEASUREMENTS

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Abstract

Recent TE wave measurements of the electron cloud density (ECD) in the beampipe at Cesr-TA and DAΦNE have shown that, especially near cutoff, the microwave excitation takes place by coupling to a standing wave, rather than to a propagating TE mode. With the beampipe acting as a resonant cavity, the effect of the periodic electron cloud density is a modulation of the cavity's resonant frequency. As a result, the measured sidebands are a combination of amplitude, phase, and frequency modulation, as the periodic cloud density modulates this resonant frequency. The quality factor Q of the resonance will determine its response to transients in the electron cloud density, and the resulting effect on modulation sidebands. In order to estimate the peak electron cloud density and its spatial distribution, knowledge of the Q and the standing wave pattern need to be determined, either by experimental measurements or simulation codes. In this paper we analyze the dependence of the modulation sidebands on the electron cloud density in two different regimes, when the cloud rise/decay time is much longer, or much shorter than the filling time of the resonance.

INTRODUCTION

The fundamental principles of the TE wave technique for measuring the electron cloud density in particle accelerators have been laid out in several publications [1-3]. In all those instances the received signal has been analyzed under the assumption that an electromagnetic wave, the beampipe's fundamental mode, is propagating from the beam position monitor (BPM) used as an input coupler, to the BPM used as a receiver. The phase velocity of the propagating wave is affected by the electron cloud encountered during its passage and information about the cloud density is deducted by measuring the modulation index of the resulting phase modulation.

Recent measurements on Cesr-TA [4] and DAΦNE [5] have shown that in many instances and especially at lower frequencies, the electromagnetic transmission between the two BPM's takes place through coupling to a standing wave which is excited in a region of the vacuum chamber comprising them.

In such case the analytical model to extract the electron cloud density from the spectrum of the received signal becomes more complex: Instead of just a phase modulation (PM), the changing electron cloud density also induces an amplitude modulation (AM) and a frequency modulation (FM). Furthermore, the standing wave quality factor Q_0, compared to the electron cloud rise/decay times, establishes two different regimes depending on whether the electron cloud density changes much faster or much slower than the resonance time constant $\tau = 2Q_0/\omega_0$.

In this paper we discuss how the electron cloud presence changes the standing wave resonant frequency and calculate the resulting modulation indexes with their dependence on the average cloud density.

E-CLOUD INDUCED FREQUENCY SHIFT

The effects of the presence of a low-energy electron plasma on the resonant frequency of a cavity, or of a standing wave, have been discussed in detail in [6]. Essentially, a uniform electron cloud distribution can be modeled as a dielectric with a frequency dependent relative permittivity ε_r given by

$$\varepsilon_r(\omega) \approx 1 - \frac{\omega_p^2}{\omega^2} \tag{1}$$

where the plasma frequency ω_p is related to the electron density per cubic meter n_e by

$$\frac{\omega_p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{e^2}{m_e\varepsilon_0}n_e} \approx 9\sqrt{n_e} \tag{2}$$

In the absence of external magnetic fields and assuming that the electron cloud uniformly fills the volume of beampipe occupied by the standing wave, it is easy to show that the wave resonant frequency is shifted by

$$\delta\omega = \frac{\omega_p^2}{2\omega_0} \tag{3}$$

where ω_0 is the resonant value in the absence of the cloud. Considering the resonance quality factor not dependent on the electron cloud is an excellent approximation with the cloud densities and energy spectra found in accelerators.

The effect of such a shift can be easily understood by looking at Fig.1. An external excitation (our signal source) on resonance when there is no electron cloud, produces a response (our detected signal) at the same frequency and in phase with the excitation. If suddenly the resonant frequency is shifted, amplitude and phase of the response change. After a transient lasting for a time

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interval of the order of 3τ, the response is well approximated by its asymptotic value: An oscillation at the excitation frequency, but with amplitude and phase given by

$$A_1 = \frac{A_s\beta_{in}\beta_{out}}{\sqrt{\left(\omega_1^2-\omega_0^2\right)^2+\frac{\omega_1^2\omega_0^2}{Q_0^2}}} \qquad (4)$$

$$\phi_1 = \tan^{-1}\left(Q_0\frac{\omega_1^2-\omega_0^2}{\omega_1\omega_0}\right)$$

where the betas are the input and output coupling factors and A_s is the excitation amplitude. The new resonance frequency ω_1 can be written as $\omega_1 = \omega_0 + \delta\omega$.

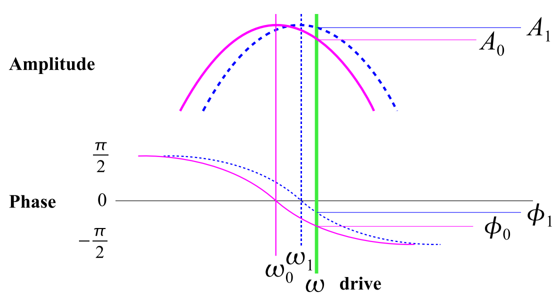

Figure 1: Amplitude and phase change of the response to a fixed frequency excitation caused by a shift in the resonant frequency.

For the sake of simplicity we will assume that the drive frequency $\omega = \omega_0$, and include the coupling factors in the amplitude coefficient A_s.

From Eq.(4) it is easy to understand the origin of AM and PM signals, in the form of modulation sidebands, on the detected waveform. Periodically changing ECD at the beam revolution frequency will modulate amplitude and phase of the waveform.

To understand the source of the FM component we need to look at the expression of the transient in the received signal $s_R(t)$, when the ECD changes, shifting the resonance frequency:

$$s_R(t) \approx A_1\left[\sin(\omega_0 t+\phi_1) - e^{-t/\tau}\sin(\omega_1 t+\phi_1)\right] + A_0 e^{-t/\tau}\sin(\omega_1 t+\varphi) \qquad (5)$$

Assuming an instantaneous change of the resonant frequency from ω_0 to ω_1, φ is simply the phase at which this change took place, so that $s_R(t=0) = A_0\sin(\varphi)$. We will show that the frequency of such a signal as described in Eq.(5) shifts from roughly from ω_1 to ω_0 inducing an FM component too.

AMPLITUDE MODULATION

The maximum modulation depth in the received signal takes place when the ECD maintains its maximum value and minimum (assumed to be zero for simplicity) values for a time t equal to at least 3τ. In such case the amplitude of the received signal can settle at both its extreme A_1 and A_0 yielding the largest modulation index. When this condition is not verified, gap or bunch train too short, the oscillations in amplitude are smaller and the ECD will be underestimated.

The amplitude modulation index can be calculated as:

$$k_{AM} = 1 - \frac{A_1}{A_0} \qquad (6)$$

In practical cases standing waves resonate between hundreds of MHz and the beampipe cutoff, usually between 1 and 2 GHz. Additionally $\delta\omega << \omega_0$ and of the order of tens of MHz at most. The quality factors measured range from a few 100's to 3000. Lower values are hard to measure and higher values are generally avoided in the accelerator design phase. With these values in mind and from Eq.(4), we can approximate A_1 as

$$A_1 \approx \frac{A_sQ_0}{\omega_0^2}\frac{1}{\sqrt{1+4Q_0^2\frac{\delta\omega^2}{\omega_0^2}}} \approx \frac{A_0}{1+2Q_0^2\frac{\delta\omega^2}{\omega_0^2}} \qquad (7)$$

and from Eqs.(3) and (6) we can derive:

$$k_{AM} \approx \frac{Q_0^2}{2}\frac{\omega_p^4}{\omega_0^4} \approx 3\cdot 10^3 Q_0^2\frac{n_e^2}{f_0^4} \qquad (8)$$

where $f_0 = \omega_0 / 2\pi$

PHASE MODULATION

Analogous considerations on the time scale of the cloud density changes apply to the phase modulation index.
The maximum value of phase change for a given ECD is

$$k_{PM} = \phi_1 - \phi_0 = \phi_1 \qquad (9)$$

since we have assumed an excitation on resonance when no electron cloud is present.
With the same approximations used in calculating k_{AM} we obtain:

$$k_{PM} \approx Q_0\frac{\omega_p^2}{\omega_0^2} \approx 81\cdot Q_0\frac{n_e}{f_0^2} \qquad (10)$$

Comparing Eqs.(8) and (10) we can see that

$$k_{AM} \approx \frac{1}{2}k_{PM}^2 \qquad (11)$$

Therefore, for low values of the ECD (small modulation indexes) the AM component is smaller than the PM component. When the density increases the reverse is true.

FREQUENCY MODULATION

As we already discussed $s_R(t >> \tau) = A_1 \sin(\omega_0 t + \phi_1)$ so that it's asymptotic oscillation frequency is ω_0. Its maximum frequency deviation $\Delta\omega$ is attained right after a change in ECD and then it damps down to zero if the ECD remains constant. The frequency modulation index

$$k_{FM} = \frac{\Delta\omega}{\omega_{FM}} \tag{12}$$

also depends on the frequency of the ECD changes ω_{FM}, usually equal to the beam revolution frequency ω_{rev}, or multiples of it when the fill contains particular symmetries.

Once again we assume small frequency deviations $\delta\omega$ and can rewrite Eq.(5) as

$$s_R(t) \approx 2A_1 \left\{ \cos\left[\left(\omega_0 + \frac{\delta\omega}{2}\right)t + \phi_1\right] \sin\left(\frac{\delta\omega}{2} t\right)\right\} + \\ + A_0 \sin(\omega_1 t + \varphi) \tag{13}$$

when $t \to 0$. In such conditions Eq.(13) becomes

$$s_R(t) \approx 2A_1 \frac{\delta\omega}{2} t \cos\left[\left(\omega_0 + \frac{\delta\omega}{2}\right)t + \tau\delta\omega\right] + \\ + A_0 \sin\left[(\omega_0 + \delta\omega)t + \varphi\right] \tag{14}$$

The amplitude of the second term in the RHS of Eq.(14) is predominant regardless of any particular value of φ, which can be defined only for a step change in frequency, besides. The maximum frequency deviation of the received signal is therefore

$$k_{FM} \approx \frac{\delta\omega}{\omega_{rev}} = \frac{\omega_p^2}{2\omega_0\omega_{rev}} \approx 40 \frac{n_e}{f_0 f_{rev}} \tag{15}$$

From Eqs.(10) and (15), the ratio

$$\frac{k_{PM}}{k_{FM}} \approx 2Q_0 \frac{f_{rev}}{f_0} \tag{16}$$

shows us that for small rings the PM component is still predominant, while for longer rings the FM component can become larger.

CONCLUSIONS

In this paper we have discussed how the electron cloud presence causes standing waves excited in an accelerator vacuum chamber to shift their frequency by an amount proportional to the cloud density. We have shown that this frequency shift induces simultaneous amplitude, phase, and frequency modulations on a signal transmitted between two points in the accelerator by means of coupling to the standing wave. We have calculated the modulation indexes for those modulations as a function of the electron cloud density when the transmitted signal is a continuous wave at the unperturbed resonant frequency and discussed the relative magnitude of those modulation indexes. Our results show that the phase modulation is predominant for smaller cloud densities and the amplitude modulation component becomes larger as the density increases. Frequency modulation of the signal assumes preponderance over the other modulations for larger machines, if the beam pattern doesn't contain too many symmetries.

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REFERENCES

[1] T. Kroyer, F. Caspers, et al. PAC 2005, Knoxville, TN, p. 2212 (2005).
[2] S. De Santis, et al., Phys. Rev. Lett. 100, 094801 (2008).
[3] S. De Santis, J. Sikora, et al., Phys. Rev. ST Accel. Beams 13, 071002 (2010).
[4] J. Sikora, S. De Santis, D. Alesini, et al. MOPPR074, these proceedings.
[5] D. Alesini, S. De Santis, et al. TUOBC03, these proceedings.
[6] J. Sikora, S. De Santis, et al. IPAC 2011, San Sebastian, p. 1434 (2011)