A GLOBAL OPTIMIZATION APPROACH BASED ON SYMBOLIC PRESENTATION OF A BEAM PROPAGATOR *

S. Andrianov, A. Ivanov, M. Kosovtsov, E. Podzyvalov,
St. Petersburg State University, St. Petersburg

Abstract

It is known that modern systems of beam lines consist of huge control elements even in the case of small machines. The problem of the beam line design leads us to formulate this problem as a global optimization ones. This approach allows us defining a family of appropriate solutions. On the next steps a researcher should narrow this optimal solutions set using additional methods and concepts. The symbolic presentation of necessary information plays leading role on all steps of the suggested approach. The corresponding implementation presented in the paper allows us to find the optimal sets in parameters spaces in a proper way. The corresponding applied software was used for solution of some practical problems. The described ideology implies to use distributed and parallel technologies for necessary computing and will be integrated in the Virtual Accelerator concept.

INTRODUCTION

Modern tools for physical systems modeling are highly specialized and solve some control problems.

On the first step the problem of the optimal structures retrieval is solved for definition of necessary requirements. Secondly, physical accelerator control is realized by system with ensemble of controllers providing optimum operating of beam lines systems. Necessities controllers number their location and type are also depends on different requirements for the beam lines features. Successful solution of these two problems can be achieved using only corresponding powerful software package. It is known that the accelerator optimization problems are multivariable and multicriteria ones and require significant computing resources and computational time. Increasing difficulty of similar accelerator systems and their high price require solutions of optimization problems on all stages of the beam lines design and controlling.

In this paper there is offered a global optimization method based on the symbol presentation for particle motion equations and corresponding optimization criteria [1]. In particular, some problems parameters can be evaluated using corresponding analytical presentations. For this purpose we use the matrix formalism for Lie algebraic tools [1]. A similar approach permits to use well known advantages of linear algebra. This method allows us to search the optimal matching channel structure under corresponding constraints on beam characteristics [2].

* The work is supported by Federal Targeted Programme "Scientific and Scientific-Pedagogical Personnel of the Innovative Russia in 2009-2013" (Governmental Contract No. P 793)

MATCHING CHANNEL MODELLING

In this article we consider the matching channel structure described in [3]. This channel is designed to align two preacceleration rings of particles (Booster and Nuclotron) of the NICA (see references in [3]).

Let us point as main beam characteristics the following: transportation conditions performing, apperture restrictions and demands on the beam dispersion (see [3]). In the present paper there is offered a method for solution the dispersion problem as the main reason of channel mismatching (from result of the work [3] obviously that restrictions on dispersion are not executed in process of the horizontal transportation of the bunch). The main control elements

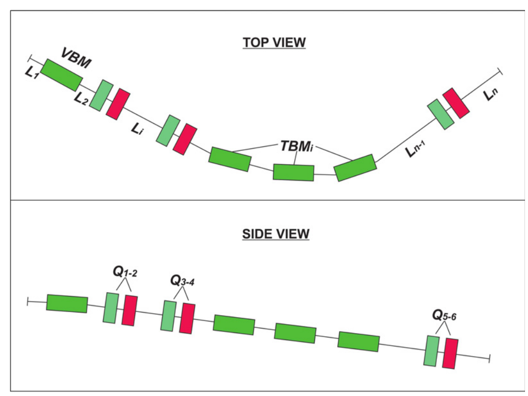

Figure 1: Matching channel structure.

submitted in fig. 1 are free gaps L_i, dipoles thumb magnets VBM and TBM_i turning by vertically and horizontally accordingly and quadrupoles Q_i.

This controlling elements set is given by the project NICA developers and considered in presented paper for further analysis.

Particles Motion Equations

It is known that particle beam motion is described using the well known Newton–Lorentz equation [4]:

$$\frac{d\mathbf{X}}{ds} = \mathbf{F}(\mathbf{X}, s),$$

where $\mathbf{F}(\mathbf{X}, s) = \mathbf{F}(\mathbf{X}, \mathbf{E}, \mathbf{B}, s)$ is a Newton–Lorentz force, and $\mathbf{E}$ – a electrical field intensity vector, $\mathbf{B}$ – a magnetic induction vector, s – an arc length along the reference orbit and $\mathbf{X} = \{x, x', y, y'\}^*$ in two dimensional transversal phase coordinates.

On the first step a researcher uses linear approximation for the motion equations in the following linear form:

$$\frac{d\mathbf{X}}{ds} = \mathbb{P}^{11}(s)\mathbf{X}, \quad \mathbf{X}(s) = \mathbb{R}^{11}(s|s_0)\mathbf{X}_0.$$

On the next steps one should take into account the corresponding higher orders nonlinearity (see [5]).

Beam Lines Characteristics

There are several ways to write the equations of motion of a particle beam as an ensemble. In the present paper we use the envelope $\mathbb{S}(s)$ – matrix presentation [4]:

$$\mathbb{S}(s) = \int\limits_{\mathfrak{M}(s)} f(\mathbf{X}, s)\mathbf{X}(s)\mathbf{X}^*(s)d\mathbf{X},$$

where $\mathfrak{M}(s)$ – a current phase manifold occupied by beam particles, $f(\mathbf{X}, s)$ – a distribution function.

For all forms of the envelope matrixes can be written the corresponding motion equations [1]:

$$\mathbb{S}(\boldsymbol{s}) = \mathbb{R}(s|s_0)\mathbb{S}_\mathbf{0}\mathbb{R}^*(s|s_0). \qquad (1)$$

The envelope matrix (1) involves the main beam characteristics and matches with the matrix formalism ideology [1].

A GLOBAL OPTIMIZATION APPROACH

In this paper we consider a global optimization algorithm using some specific optimization techniques. It is known that the problem of extremum seeking for multivariable functionals is a very difficult problem even for modern supercomputers and data centers.

Let us describe the basic features used global optimization mechanisms some in more details. The problem of matching channel structure optimization includes functional requirements on the particles beam in the following general form:

$$\mathrm{J}[\mathbf{A}] = \int\limits_{s_0}^{s_T} \int\limits_{\mathfrak{M}(s)} g_1(\mathbf{A}, \mathbf{X}, \tau)d\mathbf{X}d\tau + \int\limits_{\mathfrak{M}_T} g_2(\mathbf{A}, \mathbf{X}, T)d\mathbf{X},$$

where $\mathfrak{M}$(s) – a current phase set, occupied by beam particles, $\mathbf{A}$ – control parameters vector. The function g_1 describes the functional criteria distribution inside the system (if necessary) and the function g_2 – the terminal beam requirements. The choice of the functions g_1, g_2 is determined by the specific task and requirements to the system.

The first integral can be represented as a finite sum of partial functionals and can be written in the form:

$$\mathrm{J}[\mathbf{A}] = \sum_{i=1}^{p} \alpha_i \mathrm{J}_i[\mathbf{A}], \qquad (2)$$

where $\mathrm{J}_i[\mathbf{A}]$ – partial functional responsibility for certain characteristics of the beam, α_i – weight coefficients determining the contribution of a functional, i. e. its importance. Functional (2) can be used to find complex orbit accelerators structures.

Optimization Algorithm

The proposed approach allows to find optimal structures and has some different features from the standard global optimization algorithms [6]. Modern directions in beam particles control offer to include all available parameters of the global optimization problem and allows to search only the global extremum of the problem [7] or some Pareto optimal solutions [8, 9]. These approaches are effective for systems with rather few parameters number. For more complex systems the process of optimal solutions searching and corresponding analysis becomes very difficult using direct methods. Therefore, in the present paper is proposed an ideology which allows to detect the set of optimal solutions, analyze them both for linear and nonlinear models and solve the tolerance problem (see fig. 2).

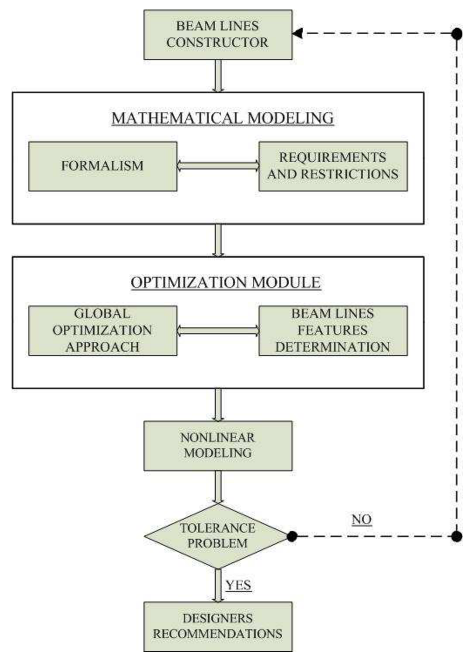

Figure 2: Optimization algorithm scheme.

Let us present the algorithm for solution of the matching problem as sequence of the following stages:

- *initial stage* constructs the linear approximation of the control particles beam structures relying on developers personal experience and preference;
- *mathematical modeling* chooses mathematical tools (in our case matrix formalism [1]) and introduce the basic functional requirements and constraints;
- *optimization stage* defines a set of feasible solutions for given constraints on the beam characteristics using the global optimization algorithms;
- *nonlinear modeling* introduces a variety of nonlinear effects which can influence on the constructed sets of feasible solutions;

- *tolerance problem* investigates the solutions stability and other necessary control system properties;
- *final stage* allows to choose the globally optimal solution (or solutions sets) satisfying the basic requirements.

It happens that some of the local extremums will be much easier to implement in practice than the global. Therefore, the final choice of the optimal solution lies on the developer of the system.

Matching Channel Optimization Results

To resolve the existing disagreement by requirements for the dispersion functions described in [3] there is considered the possibility of search for the lengths of the initial and final intervals of the system [1]. The focusing requirements "from point to point" allows us remove undesirable effects of beam size spreading (see [10]).

For this purpose the matching channel should be presented as three blocks complex (see the scheme on fig. 3).

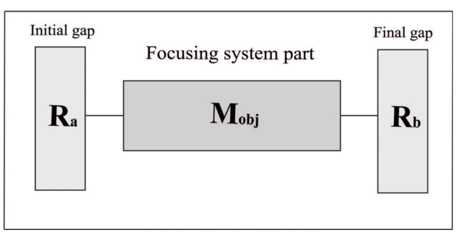

Figure 3: Matching channel scheme.

In this case we calculate the general system matriciant $\mathbb{R}_{\mathrm{tot}}$ in the following form

$$\mathbb{R}_{\mathrm{tot}} = \mathbb{R}_b \cdot \mathbb{M}_{\mathrm{obj}} \cdot \mathbb{R}_a,$$

where $\mathbb{R}_b$ and $\mathbb{R}_a$ are respectively matriciant of the final and initial free gaps, $\mathbb{M}_{\mathrm{obj}}$ – matriciant focusing part of the system (so called "objective"). The minimization conditions of the dispersion function in this case are characterized by the following equation:

$$b = -\frac{r_{13}}{r_{23}} = -\frac{r_{46}}{r_{56}} = 0, => r_{13} = r_{46} = 0,$$

where r_{ij} – elements of the matrix $\mathbb{R}_{\mathrm{tot}}$. Here r_{13} describes an influence of δp_x (the extended phase vector $\mathbf{X} = \{x, x', \delta p_x, y, y', \delta p_y\}^*$), r_{46} is a coefficient under δp_y. Note that the elements r_{11} and r_{44} corresponds to the maximum beam envelope along the horizontal and vertical axes respectively.

The proposed approach for dispersion function minimization () should be transformed for obtaining the set of feasible solutions in according to:

$$\parallel r_{13} \parallel + \parallel r_{46} \parallel \leqslant \varepsilon, \qquad (3)$$

where ε – an adjustable parameter for our researching process. The set of solutions satisfying the equation (3) will minimize the dispersion functions.

CONCLUSION

In the paper there is introduced the global optimization approach of searching for the matching channel structure based on the matrix formalism. It allows to write some limitations in analytical forms and including them to the matching channel model without significant complication. For the complex control systems increases the necessity of parallel and distributed computing. Global search algorithms (genetic algorithms) are used as an implementation tools of the optimization approach described above and much more effective than commonly used gradient methods.

The developed software packages allows us to realized all above mentioned steps with ability to optimize particular control elements and find the sets of appropriate optimal problem parameters values.

REFERENCES

[1] S. N. Andrianov, "Dynamical Modeling of Beam Particle Control Systems", St. Petersburg State University, St. Petersburg, 2004 (in Russian).

[2] S. N. Andrianov, E. A. Podzyvalov, A. N. Ivanov, "Methods and Instruments for Beam Lines Global Optimization", PHYSCON'11, Leon, Spain, September 2011 (will be pablished).

[3] A. V. Tuzikov, V. A. Mikhailov, "Matching Channel Booster–Nuclotron of the NICA", Letters Nuclei. 2010. Vol.7. N.7 (163), p. 781–787 (in Russian).

[4] A. Chao, M. Tigner, "Handbook of Accelerator Physics and Engineering", World Scientific, 1999.

[5] S. N. Andrianov, N. S. Edamenko, E. A. Podzyvalov, "Some Problems of Global Optimization for Beam Lines", in J. of Math. Phys., Catania, Italy, 2009, `http://lib.physcon.ru/download/p1998.pdf`.

[6] T. Weise, "Global Optimization Algorithms: Theory and Application", 2-nd Edition.

[7] E. Schuster, C. K. Allen, "Optimized beam matching using extremum seeking", PAC'05, Knoxville, Tennessee, USA, FPAT092, p. 4269–4271 (2005), `http://www.JACoW.org`.

[8] L. Yang, D. Robin, "Global Optimization of the Magnetic Lattice Using Genetic Algorithms", EPAC'08, Italy, Genova, THPC033, p. 3050–3052 (2008), `http://www.JACoW.org`.

[9] W. W. Gao, W. Li, L. Wang, "Low Emittance Lattice Optimization Using Multiobjective Genetic Algorithm", IPAC10, Kyoto, Japan, THPE001, p. 4515–4517 (2010), `http://www.JACoW.org`.

[10] E. A. Podzyvalov, "Globally Optimal Method of Matching Channel Structure Searching", CPS'11, SPb, Russia, p. 171–177 (2011) (in Russian).