HOM SPECTRUM AND Q-FACTOR ESTIMATIONS OF THE HIGH-BETA CERN-SPL-CAVITIES*

H.-W. Glock, T. Galek, G. Pöplau, U. van Rienen, Universität Rostock, IAE, IEF, Germany

Abstract

Beam energy deposited in Higher-Order-Modes may affect both beam stability and cryo power requirements of superconducting proton linacs like those which are objective of the CERN SPL study. We report on numerical studies of the high-beta cavity type, analyzing it's HOM spectrum. These especially include Q-factor estimations based on scattering spectra calculations and an improved pole-fitting algorithm. The procedure is described in detail. Results are shown for a sample coaxial-type HOM-coupler without a fundamental mode filter.

SPL CAVITY

The SPL- ("Superconducting Proton Linac"-) Design Study [1], mainly conducted by CERN, aims for elliptically shaped superconducting five-cell cavities. The concept foresees two cavity types, both operating at 704.4 MHz, with design velocities of β=0.65 and β=1, only the latter one being under discussion here (compare Figure 1). It's gradient should reach 25 MV/m and the accelerating mode an unloaded $Q_0=10^{10}$. Each cavity has an individual coaxial-type power coupler on one and a tuning mechanism on the other end. Somewhat special, even though not finally decided, are beam tube tapers on both sides reducing the beam tube radius from 70 mm (main coupler end) and 65 mm, respectively, to 40 mm in the cavity-cavity-connection. This reduces both fundamental and higher order mode coupling between adjacent cavities and allows to neglect any cavity interaction up to 2.5 times the fundamental mode's frequency. A field-pattern based analysis of the beam relevance of certain modes is published at this conference [2].

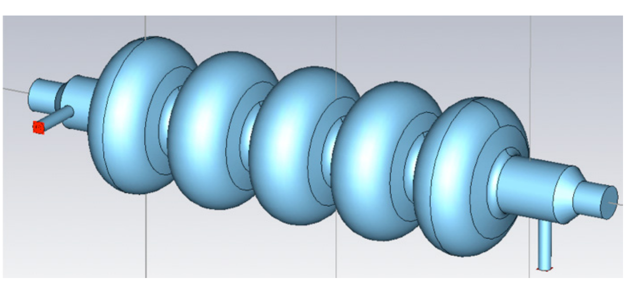

Figure 1a: Computation model of the SPL-β=1-cavity, equipped with shortened outer beam pipes of 40 mm radius and with two coaxial HOM coupler without fundamental mode filters (comp. Figure 1b).

*Work supported by German Federal Ministry of Education and Research (BMBF) under contract 05H09HR5

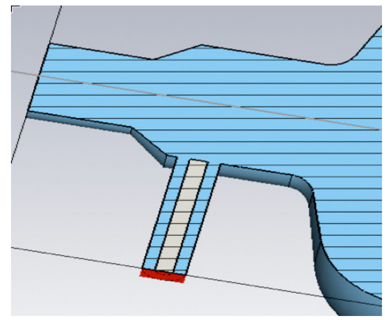

Figure 1b: Cross section of the coaxial HOM coupler, shown with an antenna tip of the inner conductor (light grey) on beam pipe radius. In order to adjust the fundamental mode loading the length of the inner conductor may be reduced if damping of higher order modes is kept strong enough.

POLE FITTING ALGORITHM

In [3] we described a procedure to extract pole parameters from a measured or numerically calculated scattering parameter spectrum $S(\omega)$. Usually the (circular) frequency ω *is* taken out of a set of discrete values as found in the spectrum given. It starts with the approach:

$$S(\omega) = \sum_k \frac{a_k}{\omega - p_k} , \qquad (1)$$

which is (except from the convention to expand the fraction in (1) by a factor of i) well known in system theory. Every individual resonance contributes one term to the sum in (1), characterised by two complex-valued parameters a_k and p_k, the former expressing a weighting factor and the latter the location of the resonance in the complex-valued frequency plane. A pole with a purely real-valued p would correspond with the absence of any losses and an infinitely high quality factor, which is reflected in

$$Q_k = -\frac{\mathrm{Re}\{p_k\}}{2\,\mathrm{Im}\{p_k\}} , \qquad (2)$$

that correlates the quality factor Q_k with the pole location p_k. If a system has well separated resonances it is appropriate to describe a multi-resonance spectrum *close to a single resonance* $\mathrm{Re}\{p_k\}$ like

$$S(\omega) \approx \frac{a_k}{\omega - p_k} + R_k , \qquad (3)$$

which summarises the contribution of all other resonances in a constant addend R_k. Relation (3) can be applied to a set of at least three points of the given dependence $S(\omega_j)$

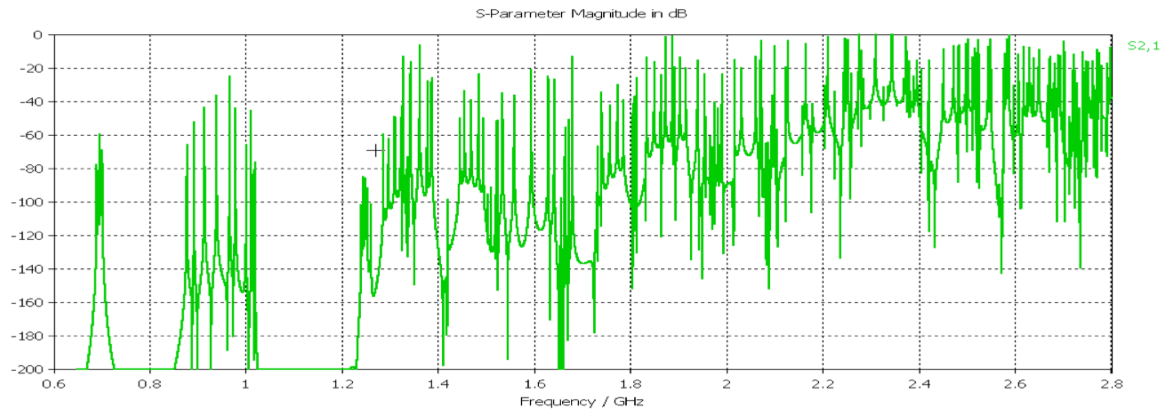

Figure 2: Transmission spectrum computed with the "FastResonant" solver of CSTStudio© [4] for the setup shown in Figure 1, using an antenna tip depth of 0. It contains more than 400 resonances with widely spread Q-values.

resulting in an exactly determined system or any bigger set leading to an overdetermined, explicitly solvable system. This strategy was used in [3] to analyse spectra of 18 well separated modes, but it failed for a significant amount of resonances of the spectrum shown in Figure 2.

In order to improve the algorithm an iterative procedure was implemented, starting from the difference between "correct" parameters from Equation (1) (to be found by the procedure) and "approximate" ones (that may be computed like previously described and denoted here with tildes). With the definitions:

$$\tilde{S}(\omega) = \sum_k \frac{\tilde{a}_k}{\omega - p_k}; \quad \tilde{a}_k = a_k + \Delta a_k; \qquad W_k(\omega) = \frac{1}{\omega - \tilde{p}_k}; \quad \tilde{p}_k = p_k + \Delta p_k \tag{4}$$

and after some elementary calculations the current approximation (Equations (4)) can be correlated with the ideal, error free fit (Equation (1)) like:

$$\sum_k W_k(\omega)\cdot \Delta a_k^{(m)} + \sum_k W_k^2(\omega)\cdot[(\tilde{a}_k - \Delta a_k^{(m)})\cdot \Delta p_k^{(m)}] = \tilde{S}(\omega) - S(\omega) + \sum_k (\tilde{a}_k - \Delta a_k^{(m-1)})\cdot \frac{(\Delta p_k^{(m-1)})^2}{(\omega - \tilde{p}_k)^2}\cdot \frac{1}{\omega - (\tilde{p}_k - \Delta p_k^{(m-1)})} \tag{5}$$

The iteration rule in Equation (5) essentially takes all contributions of exponents of $W_k(\omega)$ higher than 2 (which may be expressed as an infinite geometric series) and evaluates them based on the previous iteration's (m-1) results. Together they give the right hand side of known (in the context of the iteration) quantities if evaluated for a set of frequencies ω_j. The left hand side then consists of a linear combination of the unknown quantities

$$\Delta a_k\,,\, \Delta p_k\,,\, \Delta a_k \cdot \Delta p_k$$

with known coefficients. Therefore the unknowns can be computed explicitly if the number of equations, i.e. frequency points under consideration, at least reach the number of poles to be characterised. This demands that each pole is found with more than three sample frequencies. The frequency points taken from a given spectrum are selected automatically around the peaks.

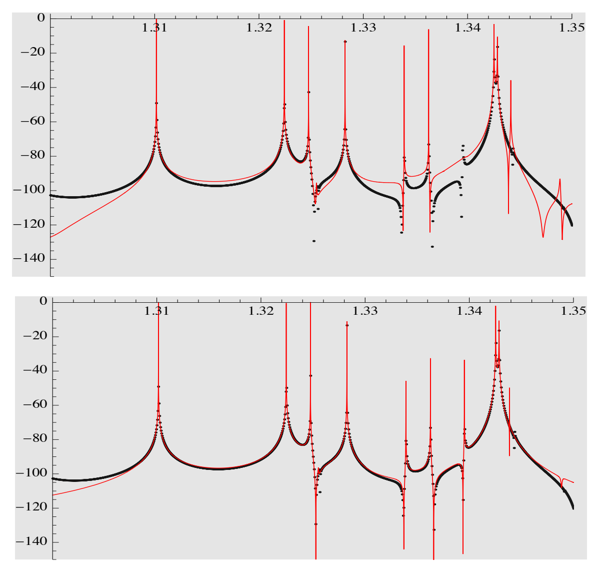

Figure 3: Pole fitting (red lines) applied on a small fraction of the spectrum (dots given as $S_{coax\text{-}coax}$(f/GHz) in dB) shown in Figure 2. Original (upper plot) and improved procedure. The deviations at the spectrum's limits are due to the neighbouring poles, here missing.

The procedure described above significantly improves the fit results (compare Figure 3). Convergence often is reached within three to four steps. Even though the fitting process is not totally reliable (e.g. compare the rightmost pole in Figure 3), which is understood as consequence of poor start values delivered from the former method.

Q-VALUE DISTRIBUTION

In order to compute the Q-values of all modes found in the spectrum of the SPL-cavity (compare Figure 2), the pole fitting algorithm was applied to subsequent sections of it's entire range. Figure 4 shows the distribution found, Figure 5 displays in detail the quality of the fitting result in the range of the fundamental mode passband. The numerical computation of the spectrum was performed with a much stronger coupling of the coax couplers, adjusted by their inner conductor length, than allowed for the fundamental mode Q. The latter is expected to reach an unloaded value of 10^{10} and parasitical losses into any HOM coupling or absorbing system should not significantly affect this, which leads to a HOM coupler loaded Q not worse than 10^{11} ... 10^{12}. This computational approach became necessary in order to avoid resonances of extreme high Q during the numerical calculation, that would be insufficiently sampled by a spectrum with a reasonable number of frequency points. This effect may be estimated in Figure 5 even for Q=10^8: There are only few - but still sufficiently numerous - sampling points along the resonance peaks.

An additional scattering analysis was performed for the coax-cavity transition without cavity cells in order to compute the dependency of the coupling strength, especially to TM_0-like field patterns, on the antenna tip distance to the beam pipe outer radius. Shortening the inner conductor results in an additional segment of a hollow circular waveguide, then operated far bellow it's cut-off frequency. The transmission shown in Figure 6 confirms this, since the exponential decay of field strength (and coupling) with increasing length represents a straight line using a logarithmic scaling. Thus the dominating effect of the wave guide field evanescence over other more subtle effects of non-homogeneous field distributions in the coax-cavity transition is demonstrated.

CONCLUSION AND OUTLOOK

In consequence a pure coaxial coupler could be adjusted to a sufficiently weak fundamental mode damping (Q=10^{12} would correspond to ~ 40 mm tip distance) but this would (almost) similarly raise higher order mode's Qs, thus leading to a large crowd of modes with Q-values significantly above 10^6, which is estimated as acceptable limit for most operating conditions [5]. A similar result was reported for pipe absorber set-ups [6]. Therefore further studies will focus on coupler structures with fundamental mode rejection. Additional focus will be spent on full, black-box-like reliability of the pole fitting algorithm, which is not reached yet.

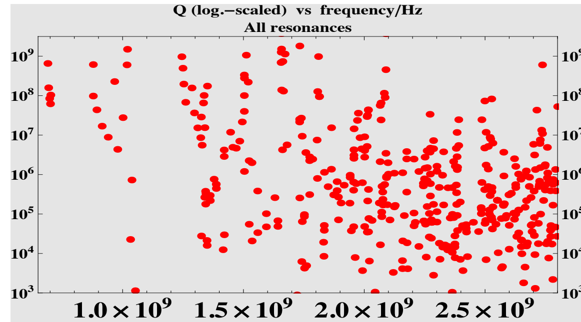

Figure 4: Q value distribution of the SPL-cavity with coaxial couplers as shown in Figure 1, computed with the pole fitting algorithm. The fundamental mode (belonging to the leftmost groups of dots) reaches almost exactly Q=10^8, which is coupled by far too strongly.

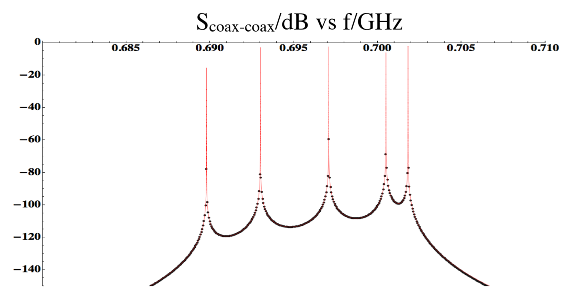

Figure 5: Detail of the pole fitting result (red line) for the fundamental mode passband.

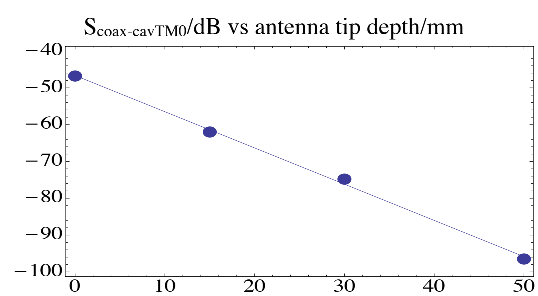

Figure 6: Transmission of the coax-cavity transition, computed as coax-TM_0 scattering parameter at 704.4 MHz for different antenna tip distances, measured from the beam pipe outer radius. Blue dots are numerically calculated, straight line is the best linear fit.

REFERENCES

[1] O. Brunner et.al.: "Assessment of the basic parameters of the CERN SPL". PhysRev.ST-AB vol. 12, 070402 (2009)
[2] M. Schuh et.al.: "Higher Order Mode Analysis of the SPL Cavities", this conference, THPE082
[3] D. Hecht et.al.: "Computation of RF properties of long and complex structures", Proc. EPAC2002, pp. 1685 http://cern.ch/AccelConf/e02/PAPERS/WEPRI060.pdf
[4] CSTStudio©, Ver. 2010, CST AG, Darmstadt, Germany
[5] M. Schuh et.al.: "Higher order mode ...", Proc. SRF09, Berlin, pp. 871; http://accelconf.web.cern.ch/AccelConf/srf2009/papers/thppo100.pdf
[6] W. Weingarten, priv. comm.