COMPARATIVE ANALYSIS OF COMPTON SCATTERING CROSS SECTION DERIVED WITH CLASSICAL ELECTRODYNAMICS AND WITH USE OF QUANTUM APPROACH.

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Abstract

In the paper the expression for cross section of Compton scattering derived with classical electrodynamics approach is presented. The comparative analysis of the Compton cross section value calculated with the presented expression and with expression derived with quantum approach was carried out for the case of head on collision and low photon beam intensity. Results of the analysis show the good agreement of both approaches. It proves legitimacy of classical electromagnetic approach use for analysis of particle beam dynamics and estimation of generated x-ray beam parameters in laser electron storage rings.

INTRODUCTION

In the last decade the study of laser-electron interactions attracts attention of the many physicists for the two main reasons. Firstly, the design and development of a new type of X-ray source – Laser Electron Storage Ring – are begun [1-3]. Secondly, the future generation of accelerator facility for fundamental investigations in nuclear physics is under development. These newest colliders will give rich opportunities to study *γγ* and *γe* reactions [4]. Despite that idea of both facility types were formulated quite long ago [3,6,7] the practical consideration of the facilities realization could began only recently because the huge progress in laser equipment development. If only ten years ago physicists could hope to get a conversion factor $k=N\gamma/Ne$ of about 10^{-10} only, today the opportunity to reach the value of this factor equal to 1 in new *γγ* colliders becomes a reality.

The detail description of the electron-photon interaction has been done by the many authors in quantum mechanical approach as well as in classical electrodynamics approach [8,9]. Good results were obtained with numerical simulations [10,11]. But new wave of the interest to the subject was stipulated by the investigations of so-called "non-linear" Compton back scattering in quantum electrodynamics terms or "high harmonic" generations in classical electrodynamics [12, 13]. The reason for this was the same – huge increasing of the conversion factor *k*.

The new accelerator facility mentioned above will operate in wide range of *k* between 10^{-5} and 10 and with tiny value of electron beam emittance (10^{-15} mm mrad). Hence, the physicists need universal and accurate instrument for description electron beam dynamics as well as the parameters of generated radiation. Quantum dynamics gives a great exact description of the scattering process but the description and further simulation of nonlinear processes is very difficult and time consuming. It seems that classical electrodynamics approach is a promising method for description of spontaneous radiation (linear Compton scattering) and stipulated radiation (high modes of radiation). But the question is in the range of reliability of classical approach.

The electrodynamics approach can be applied for the system where the following condition is satisfied:

$$N_{\gamma i} \cong \frac{1}{(2\pi)^3}\frac{\varepsilon_0}{\varepsilon_k} >> 1,$$

where $N_{\gamma i}$ is the number of photons in initial income wave, $\varepsilon_{\gamma i} = h\nu$ is average quanta energy of income wave, *h* is Plank constant, *ν* is income wave frequency, ε_0 is the income wave energy included in cubic with the side size equal to the wavelength.

In practice this inequality means that classical approach is valid in the case, when a number of income photons are much bigger then 1. Simultaneously, the quantum consideration of the linear Compton scattering operates with one electron and one photon in an act of interaction.

The purpose of the presented paper to compare results for cross section of Compton scattering derived for the case of interaction of low intensity initial electromagnetic wave with low energy relativistic electron beam (case of Laser Electron Storage Ring) in head on collision with results of calculations with applying of quantum approach for the same conditions of interaction.

CLASSICAL APPROACH

To derive expression for cross section of Compton scattering we considered a relativistic electron with energy E_0, moving in laboratory coordinate frame under angles $\alpha_1, \alpha_2, \alpha_3$ toward plane, linear polarized, electromagnetic wave with field intensity *E* and frequency *ν*. [14]. As a result of a Lorentz force equation integration, the electron trajectories were obtained. The velocity components of the electron that moves in conditions mentioned above can be written in the following form:

$$\beta_{xt} = \frac{1-F^2+a^2-B^2}{1-F^2+a^2+B^2},$$

$$\beta_{zt} = \frac{2BF}{1-F^2+a^2+B^2},\ \beta_{yt} = \frac{2aF}{1-F^2+a^2+B^2} \qquad (1)$$

where, $F = \frac{eE}{mc(2\pi\nu)} Sin[2\pi\nu(t-\frac{x}{c})+\delta]+C$,

$$C = \frac{\beta_{zt}(t_0)}{\sqrt{1-\beta(t_0)}} - \frac{eE}{mc(2\pi\nu)} Sin[2\pi\nu(t_0-\frac{x(t_0)}{c})+\delta],$$

$$a = \frac{\beta_{yt}(0)}{\left(1-\beta^2(0)\right)},\ B = \frac{1-\beta_{xt}(0)}{\left(1-\beta_{xt}^2(0)-\beta_{zt}^2(0)\right)^{1/2}},$$

c is velocity of light, e is electron charge, $m = m_0 / (1-\beta^2)^{1/2}$, $\beta = v/c$, m_0 is the rest mass of the electron, $x_0, y_0, z_0, \beta_{xt}(0), \beta_{yt}(0), \beta_{zt}(0)$ are initial coordinates of the electron, t *and* t_0 are the current time and initial moment of time, δ initial electron phase.

After electron velocity component integrations we could write expressions for the electron coordinates [14].

Expressions (1) completely describe an electron motion in the field of electromagnetic wave. As it was mentioned in [14] these expressions are almost periodical functions of time and can be expanded in generalized Fourier series. This fact can be used for the further analysis of electron beam trajectory and is very useful for obtaining electron radiation characteristics.

Magnetic component of the field radiated by the electron on far distance (distance is longer then radiation wavelength) can be expressed in the following form [15]:

$$\vec{H} = \sum_f H_{\Omega f} e^{-i\Omega_f t}, \quad (2)$$

where $\vec{H}_{\Omega f} = \frac{2ei\Omega_f e^{ikR_0}}{c^2 R_0} \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i\Omega_f (t - \frac{\vec{n}\vec{r}_0}{c})} [\vec{n} d\vec{r}(t)]$,

R_0 is distance from point of origin to point of observation, $\vec{n}$ is unit vector in the same direction, Ω_f is frequency of the radiation harmonic, $\vec{k} = (\Omega_f / c)\,\vec{n}$ is the wave vector.

The magnetic fields $\vec{H}_{\Omega f}$ components can be written as the following:

$$(H_{\Omega_f})_x = -\Omega_f \eta_f \lim_{s\to\infty} \frac{1}{s} \int_{s_0}^{s} (\beta_{ys} Cos[\gamma_2] - \beta_{zs} Cos[\gamma_3]) e^{i\Omega_f (\Lambda s + \Psi(s))} ds;$$

$$(H_{\Omega_f})_z = \Omega_f \eta_f \lim_{s\to\infty} \frac{1}{s} \int_{s_0}^{s} (\beta_{ys} Cos[\gamma_1] - \beta_{xs} Cos[\gamma_3]) e^{i\Omega_f (\Lambda s + \Psi(s))} ds; \quad (3)$$

$$(H_{\Omega_f})_y = -\Omega_f \eta_f \lim_{s\to\infty} \frac{1}{s} \int_{s_0}^{s} (\beta_{zs} Cos[\gamma_1] - \beta_{xs} Cos[\gamma_2]) e^{i\Omega_f (\Lambda s + \Psi(s))} ds;$$

where $\eta_f = \frac{2eie^{ikR_0} e^{i\Lambda_f M}}{cR_0}$, $\Omega_f = \frac{2\pi v}{\Lambda} f$,

$$\Lambda = \left(1 + \frac{\beta_{xt}(t_0)}{1-\beta_{xt}(t_0)}(1 - Cos[\gamma_1]) - \frac{\beta_{zt}(t_0)}{1-\beta_{xt}(t_0)} Cos[\gamma_2] - \frac{\beta_{yt}(t_0)}{1-\beta_{xt}(t_0)} Cos[\gamma_3]\right)$$
$$+ P\left[1 - \frac{\beta_{zt}(t_0)}{1-\beta_{xt}(t_0)} Sin[g_0](1 - Cos[\gamma_1]) + Sin[g_0] Cos[\gamma_2]\right]$$
$$+ P^2 \left[\frac{1}{4} Cos[2g_0] + Sin^2[g_0]\right](1 - Cos[\gamma_1]),$$

$s_0 = t_0 + \frac{x(t_0)}{c}$ $g_0 = 2\pi v s_0 + \delta$, γ_1, γ_2, γ_3 are direction cosines of an observation point,

$$\Psi(s) = \Psi_x(1 - \cos(\gamma_1)) - \Psi_z \cos(\gamma_2), \quad \Psi_z = \frac{eE}{mc(2\pi v)^2 B} \cos(2\pi v s + \delta),$$

$$\Psi_x = \left(\frac{eE}{mc(2\pi v)B}\right)^2 \frac{\sin 2(2\pi v s + \delta)}{16\pi v} - \frac{eE}{mc(2\pi v)B} \frac{\Lambda_z}{2\pi v} \sin(2\pi v s + \delta)$$

Expanding (3) $e^{i\Omega_f (\Lambda s + \Psi(s))} = \sum_{r=0}^{k} \frac{i\Omega_f (\Lambda s + \Psi(s))}{r!}$ one can find harmonic frequencies Ω_f.

The harmonic radiation energy one can find from the following expression:

$$\varepsilon_{ph} = h\Omega_f. \quad (4)$$

To determine the cross-section of the radiation we use the following well known expression [15]:

$$\mathrm{dI/d\Omega} = \frac{c}{4\pi} |H^2| R_0^2, \quad (5)$$

where I is radiation intensity, Ω is solid angle.

To realize conditions of head-on interaction of electron beam with the low intensity electromagnetic wave we should write expression (5) with $\alpha_1 = 0$, $\alpha_2 = 0$,, $\alpha_3 = \pi/2$ and .parameter $P = \frac{eE}{mc(2\pi v)B} \ll 1$. It is more convenient to write this expression in spherical coordinate system (∂ is azimuthal angle, φ is polar angle). And after integration on polar angle φ, we can find the Compton cross section depending on azimuthal angle ∂:

$$\frac{d\sigma_c}{d\theta} = \frac{\pi r_0^2 \gamma^{-2}}{1 - \beta Cos[\theta]} \left[1 + \left(Cos[\theta] - \frac{\beta Sin^2[\theta]}{1 - \beta Cos[\theta]}\right)^2 Sin[\theta]\right], \quad (6)$$

where r_0 is electron classical radius.

QUANTUM APPROACH

We will study the case of linear Compton scattering and probability for an electron to collide with two or more photons simultaneously will be neglected. Let us write some useful expressions allowing to characterize the properties of the process [8].

An energy of a scattered photon ε_γ of initial energy $\varepsilon_{\gamma 0}$ after scattering under small collision angle α_0 depends on a scattering angle θ_γ, which is measured comparatively initial direction of an E_0 energy electron motion:

$$\varepsilon_\gamma = \varepsilon_{\gamma m} \Big/ \left(1 + \left(\theta_\gamma / \theta_0\right)^2\right), \quad (7)$$

where $\varepsilon_{\gamma m} = 4\gamma^2 \cos^2(\alpha_0/2)\, \varepsilon_{\gamma 0} \Big/ \left(4\gamma \frac{\varepsilon_{\gamma 0}}{mc^2} \cos^2(\alpha_0/2) + 1\right)$ is a maximum energy of a scattered photon, $\theta_0 = \frac{mc^2}{E_0}\sqrt{x+1}$,

$x = 4E_0 \varepsilon_{\gamma 0} \cos^2(\alpha_0/2) \big/ m^2 c^4$.

After substitution $\gamma = \frac{E_0}{mc^2}$ the expression for the energy spectrum of photons scattered in the solid angle equal to 4π is defined by differential cross section of Compton scattering [8] is:

$$\frac{1}{\sigma_c} \frac{\mathrm{d}\sigma_c}{\mathrm{d}y} = \frac{2\sigma_0}{x\sigma_c} \left[\frac{1}{1-y} + 1 - y - 4r(1-r) + 2\lambda_e P_c r x (1-2r)(2-y)\right]; (8)$$

where $y = \frac{\varepsilon_\gamma}{E_0} \le y_m = \frac{x}{x+1}$, $r = \frac{y}{x(1-y)} \le 1$,

$$\sigma_0 = \pi\left(\frac{e^2}{mc^2}\right) = 2.5 * 10^{-29} m^2,$$

λ_e, P_c are directions of electron and photon beam polarization vectors ($\left|\lambda_e\right| \le \frac{1}{2}$; $\left|P_c\right| \le 1$), $\sigma_c = \sigma_c^{np} + 2\lambda_e P_c \sigma_1$,

$$\sigma_c^{np} = \frac{2\sigma_0}{x}\left[\left(1-\frac{4}{x}-\frac{8}{x^2}\right)\ln(x+1)+\frac{1}{2}+\frac{8}{x}-\frac{1}{2(x+1)^2}\right],$$

$$\sigma_1 = \frac{2\sigma_0}{x}\left[\left(1+\frac{2}{x}\right)\ln(x+1)-\frac{5}{2}+\frac{1}{x+1}-\frac{1}{2(x+1)^2}\right].$$

PARAMETERS OF GENERATED RADIATION

Let us compare obtained results for the first radiation harmonic ($f = 1$). The results for the maximum photon energy of the first harmonic of scattered radiation depending on initial electron energy are shown in Fig. 1 for initial electron beam energy equal to 100 MeV. As one can see the coincidence of the results is practically ideal. The natural result is that under zero interaction angle the energy of scattered radiation is equal to initial wave energy. Fig. 2 shows Compton cross section calculated with quantum approach (8) and electrodynamics approach (6) for head on collision, low photon beam intensity at different electron beam energy (38.7, 43, 47.3 MeV) and NdYg laser. As one can see both approach gave the same results.

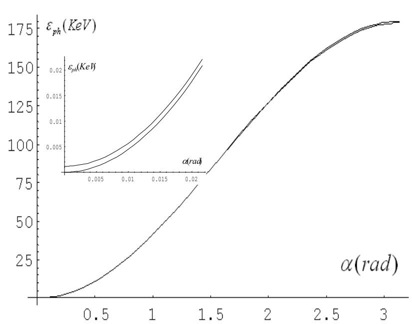

Figure 1. The energy of the first harmonic of scattered radiation depending on interaction angle (E_0=100 MeV).

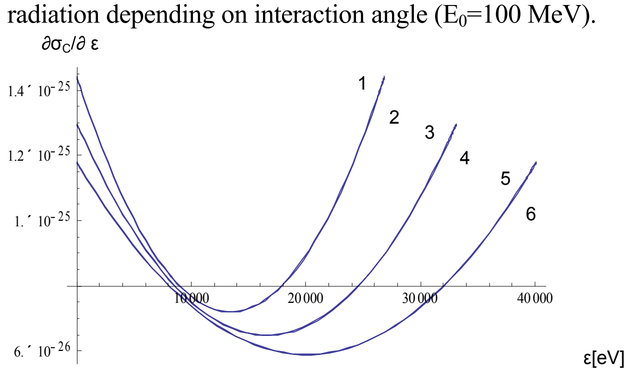

Figure 2. Compton cross section vs radiated photon energy calculated with qantum approach (1- E_0= 38.7 MeV, 3- E_0= 43 MeV,5-E_0= 47.3 MeV) and classical electrodynamics approach (2 - E_0= 38.7 MeV, 4 - 43 MeV MeV,, 6 - 47.3 MeV).

CONCLUSION

In the paper the cross section of Compton scattering for the case of interaction of low intensity initial electromagnetic wave with low energy relativistic electron in head on collision were calculated with quantum and electrodynamics approaches and compared. The results of calculations show the good coincidence. It gives as a confidence in applicability of classical electrodynamics approach for description of low intensity electron photon interactions.

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