

# EMITTANCE GROWTH ESTIMATION DUE TO INTRABEAM SCATTERING IN HEFEI ADVANCED LIGHT SOURCE(HALS) STORAGE RING

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## Abstract

Hefei Advanced Light Source(HALS) will be a high brightness light source with about 0.2nm·rad emittance at 1.5GeV and about 400m circumference. To enhance brilliance and transverse coherence, very low beam emittance is required and becomes one of the critical issues of HALS storage ring. Intra-beam scattering(IBS) is usually thought a fundamental limitation to achieve low emittance. Here we preliminarily estimate the emittance growth due to IBS for the temporary lattice design of HALS based on Piwinski and Bjorken-Mtingwa theories, and discuss the effects of implementation of damping wigglers and higher harmonic cavity on the emittance growth due to IBS.

## INTRODUCTION

The Hefei Advanced Light Source (HALS) is an advanced VUV and soft X-ray source operating with high-intensity, low-emittance beam. The purpose of HALS is to provide synchrotron radiation with high brilliance and better coherence in the VUV and soft X-ray range to synchrotron users. Considering the radiation spectrum and achievable magnetic strength parameters of undulator, the beam energy of the storage ring is set to 1.5GeV.

To enhance brilliance and coherence, very low emittance is required. More dipoles and stronger focusing are needed in ring lattice design to achieve lower emittance. Considering the achieved bare lattice emittance and parameters of straight sections, the magnet lattice composed of  $18 \times$  FBA (Five Bend Acromat) is the first option now, which provides natural emittance of about 0.27nm·rad. Figure 1 shows the beta and dispersion functions of one cell<sup>[1]</sup>. The main parameters of HALS are listed in table 1.

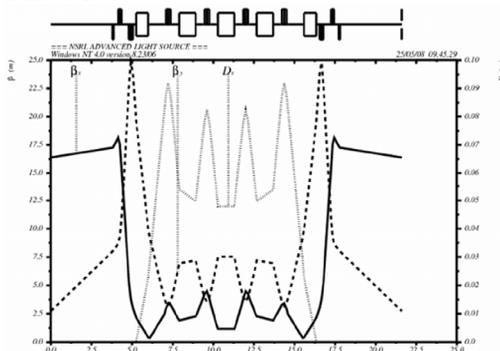


Figure 1: Beta and dispersion functions of one cell.

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Table 1: Main Parameters of HALS Storage Ring

| Parameters                | Values      |
|---------------------------|-------------|
| Circumference             | 388m        |
| Energy                    | 1.5GeV      |
| Transverse tunes          | 29.32/10.28 |
| Natural chromaticities    | -55/-51     |
| Momentum Compaction       | 0.00047     |
| Harmonic number           | 648         |
| Emittance of bare lattice | 0.27nm·rad  |
| Coupling factor           | 0.1         |
| Energy spread             | 0.00022     |
| Natural bunch length      | 0.0269cm    |

## ESTIMATION OF IBS EFFECT

Intra-beam scattering (IBS) includes multiple IBS and single-event IBS. Multiple small-angle Coulomb scattering can lead to an increase in the six dimensional emittance of the particle bunch. Its effects can be neglected at most third generation light sources, but for the electron storage rings which operate in the regime of low-energy, ultra low-emittance and high-luminosity, it severely limits the minimum emittance to be achieved. In the HALS, design constraints place the ring in a parameter regime where intrabeam scattering is likely to be a limitation on the achievable emittance, and hence on the final brilliance and coherence.

Intrabeam scattering theory is well discovered in many publications. The emittance growth rate due to intrabeam scattering can be calculated using full Piwinski or Bjorken-Mtingwa theories, which are difficult because of the complicated integrals needed to be performed. Some high energy approximations are derived to make the calculation more efficient. Here we adopt the Bjorken-Mtingwa's theory<sup>[2]</sup> and Bane's approximation<sup>[3]</sup>, and also the completely integrated modified Piwinski (CIMP) model<sup>[4][5]</sup> to estimate the emittance growth. All approximations are valid in the parameter regime that the ring performs.

The equilibrium emittance will meet when:

$$\frac{d\epsilon_x}{dt} \equiv -\frac{2}{\tau_x}(\epsilon_x - \epsilon_{x0}) + \frac{2\epsilon_x}{T_x} = 0 \quad (1)$$

$$\frac{d\epsilon_y}{dt} \equiv -\frac{2}{\tau_y}(\epsilon_y - \epsilon_{y0}) + \frac{2\epsilon_y}{T_y} = 0$$

$$\frac{d(\sigma_p^2)}{dt} \equiv -\frac{2}{\tau_p}(\sigma_p^2 - \sigma_{p0}^2) + \frac{2\sigma_p^2}{T_p} = 0$$

Where  $T_x$ ,  $T_y$  and  $T_p$  are the horizontal, vertical and longitudinal IBS growth rates,  $\tau_x$ ,  $\tau_y$  and  $\tau_p$  are the radiation damping times,  $\epsilon_{x0}$ ,  $\epsilon_{y0}$ ,  $\sigma_{p0}$  are zero-current

transverse emittances and relative energy spread. Because the IBS growth times  $T_{x,y,p}$  depend on beam emittances, energy spread, and bunch length which increase due to IBS, it's necessary of using iterative procedures to solve the coupled equations above.

In Bane's high energy approximation, the emittance growth rate is:

$$\frac{1}{T_p} \approx \frac{r_0^2 c N (\log)}{32 \gamma^3 \varepsilon_x^{3/4} \varepsilon_y^{3/4} \sigma_s \sigma_p^3} \langle \sigma_H g_{\text{bane}}(a/b) (\beta_x \beta_y)^{-1/4} \rangle \quad (2)$$

$$\frac{1}{T_{x,y}} \approx \frac{\sigma_p^2 \langle H_{x,y} \rangle}{\varepsilon_{x,y}} \frac{1}{T_p}$$

And in completely integrated modified Piwinski (CIMP) approximation model,

$$\frac{1}{T_p} \approx \frac{r_0^2 c N (\log)}{32 \pi^{1/2} \beta^3 \gamma^4 \varepsilon_x \varepsilon_y \sigma_s \sigma_p} \left\langle \frac{\sigma_H^2}{\sigma_p^2} \left( \frac{g_{\text{CIMP}}(b/a)}{a} + \frac{g_{\text{CIMP}}(a/b)}{b} \right) \right\rangle$$

$$\frac{1}{T_x} \approx \frac{r_0^2 c N (\log)}{32 \pi^{1/2} \beta^3 \gamma^4 \varepsilon_x \varepsilon_y \sigma_s \sigma_p} \left\langle -a g_{\text{CIMP}} \left( \frac{b}{a} \right) + \frac{H_x \sigma_H^2}{\varepsilon_x} \left( \frac{g_{\text{CIMP}}(b/a)}{a} + \frac{g_{\text{CIMP}}(a/b)}{b} \right) \right\rangle$$

$$\frac{1}{T_y} \approx \frac{r_0^2 c N (\log)}{32 \pi^{1/2} \beta^3 \gamma^4 \varepsilon_x \varepsilon_y \sigma_s \sigma_p} \left\langle -b g_{\text{CIMP}} \left( \frac{a}{b} \right) + \frac{H_y \sigma_H^2}{\varepsilon_y} \left( \frac{g_{\text{CIMP}}(b/a)}{a} + \frac{g_{\text{CIMP}}(a/b)}{b} \right) \right\rangle$$

(3) where  $r_0$  is the classical radius of the electron;  $N$  is the number of particles in the bunch; The brackets indicate an average over the lattice. Other quantities are defined as follows:

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{H_x}{\varepsilon_x} + \frac{H_y}{\varepsilon_y}, a = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_x}{\varepsilon_x}}, b = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_y}{\varepsilon_y}}$$

The dispersion invariant is defined as

$$H_{x,y} = [\eta_{x,y}^2 + (\beta_{x,y} \eta'_{x,y} - \frac{1}{2} \beta'_{x,y} \eta_{x,y})^2] / \beta_{x,y}$$

For the vertical emittance, we need to verify how the emittance is generated. The vertical dispersion in HALS ideal design lattice is zero, but magnet misalignments will lead to vertical dispersion in real storage ring. Here we treat the vertical emittance as generated mainly by the transverse coupling, the contribution of vertical dispersion is ignored. So the vertical emittance is simply  $\varepsilon_y = k \varepsilon_x$ , in HALS design,  $k$  is fixed to be 0.1.

The function  $g()$  in the two methods are reduced from the scattering function from IBS theory, and both have approximation expressions to simplify calculation, which are

$$g_{\text{bane}}(a) \approx 2a^{(0.021-0.044 \ln a)}, 0.01 < a < 1, g(a) = g(1/a) \quad (4)$$

$$g_{\text{CIMP}}(a) \approx 2.691 \left( 1 - \frac{0.22889}{b} \right) \frac{1}{(1+0.16a)(1+1.35e^{-a/0.2})}, 0.1 < a < 10 \quad [6]$$

(5)

The quantity  $(\log)$  is the "Coulomb log", the logarithm of the ratio of the maximum to the minimum impact parameter in the collision of two electrons in the bunch. It may be estimated as:

$$(\log) \approx f_{\text{CL}} \ln \left( \frac{\sqrt{\beta_y} \varepsilon_y \gamma^2 \varepsilon_x}{r_0 \beta_x} \right)$$

The factor  $f_{\text{CL}} = 1$  for Gaussian beams. But IBS populates the tails of the bunch distribution, and this leads to a reduction in the growth rates of the core emittances,

so a "tail-cut" procedure is adopted in some researches, resulting a reduction of the factor. The value of  $(\log)$  varies from about 10 to 16 in different publications<sup>[7][8]</sup>.

Using these methods above, the equilibrium emittance of HALS design is calculated. It's shown in figure 2 that under conditions that the particle number in bunch is  $10^{10}$  and the  $(\log)$  factor is set to be 13, the natural horizontal emittance will rise to about 1.4nm·rad. The equilibrium horizontal emittance is lower comparing to the result of ZAP program which is about 1.6nm·rad, maybe due to the smaller log factor in our calculation (about 15 given by ZAP result).

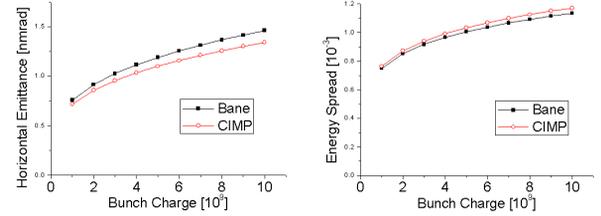


Figure 2: IBS emittance growth estimation in HALS, solid black dot for Bane's method, hollow red dot for CIMP method

## INFLUENCE OF DAMPING WIGGLER

Since the equilibrium emittance of bare design lattice of HALS is much higher than the design goal, some devices need to be employed to fight the emittance growth effects. Damping wigglers will help to shorten damping times and decrease beam emittance simultaneously.

The damping wiggler will be accommodated at the zero-dispersion straight section of HALS to decrease the emittance and provide broadband synchrotron radiation. According to the design, the length to locate the whole damping wiggler section is limited under 30 metres.

Quantum fluctuations and the synchrotron radiation damping in both the bending magnet and wigglers will result in new equilibrium emittance. With wigglers on, the horizontal damping time will be reduced to

$$\tau_x = \frac{2E_0 T_0}{J_x U_0} = \frac{3(B\rho)C}{2\pi r_0 c \gamma^3 B_a (J_{xa} + F_w)} \quad (6)$$

Where  $F_w = \frac{I_{2w}}{I_{2a}} = \frac{L_w B_w^2}{4\pi(B\rho)B_a}$  is the ratio of the radiation loss due to damping wiggler and the dipoles.

The effect of damping wigglers on the horizontal emittance is given by Wiedmann<sup>[9]</sup>,

$$\varepsilon_x = \frac{1}{1+F_w} (\varepsilon_{x0} + F_w \varepsilon_{wig}) = \frac{\varepsilon_{x0} + \frac{4C_q}{15\pi J_x} N_p \frac{\beta_x}{\rho_w} \gamma^2 \frac{\rho_0}{\rho_w} \theta_w^3}{1 + \frac{1}{2} N_p \frac{\rho_0}{\rho_w} \theta_w} \quad (7)$$

A more practical form of Wiedmann's equation is<sup>[10]</sup>

$$\frac{\varepsilon_x}{\varepsilon_{x0}} = \frac{1 + 1.21 * 10^{-12} \frac{\beta_x L_w \lambda_w^2 \rho_0 B_w^5}{J_x \varepsilon_{x0} E^3}}{1 + 7.16 * 10^{-3} \frac{L_w \rho_0 B_w^2}{E^2}} \quad (8)$$

Where  $\beta_x$  is the average horizontal beta function in the wiggler,  $\rho_0$  is the radius of the bending magnet,  $\lambda_w$  is

wiggler period length,  $L_w$  and  $B_w$  are the total length and magnetic field value of damping wiggler. The units for them are metre except for  $E$  and  $B_w$  in GeV and T.

Given the ring parameters above, the dependence of the beam emittance on the wiggler period and magnitude is shown in figure 3.

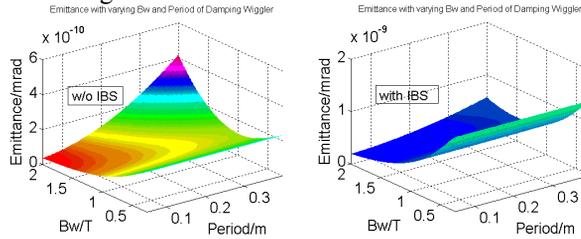


Figure 3: Horizontal beam emittance as function of the damping wiggler parameters with and without IBS.

According to the relationship between emittance reduction and wiggler parameters, the kind of wigglers we will use is discussed. Considering the expense and damping efficiency, we are planning to choose permanent magnet wigglers, 1.5 tesla for the magnetic field and the period length will be 100 mm.

We calculated the new equilibrium emittance including the effects of damping wigglers and intrabeam scattering, assuming that the average beta function in wiggler segment is approximately 10m. we got a result that with damping wigglers installed, beam emittance will be reduced to about 0.4nm·rad.

### INFLUENCE OF HARMONIC CAVITY

Installing harmonic cavity can increase the beam Touschek life time by lengthening the bunch to get a larger beam volume. As the charge density decreases, IBS growth rate is also reduced. So the possibility to mitigate the effect of IBS by use of higher harmonic cavity is considered.

When the voltage slope at the bunch centre is zero by the addition of harmonic cavity, the bunch lengthens and peak charge density decreases while the energy distribution is unaffected<sup>[11]</sup> approximately. So we added the effect of harmonic cavity with different bunch lengthening factor to the calculations above, approximately considering only bunch length is affected.

Table 2: Horizontal Beam Emittance at Different Bunch Lengthening Factor

| Bunch lengthening factor | Horizontal emittance[nm·rad ] DWs off | Horizontal emittance[nm·rad ] DWs on |
|--------------------------|---------------------------------------|--------------------------------------|
| 1.25                     | 1.258                                 | 0.247                                |
| 1.5                      | 1.194                                 | 0.235                                |
| 2                        | 1.101                                 | 0.217                                |
| 3                        | 0.984                                 | 0.195                                |

Shown in table 2 that with proper lengthening factor, the horizontal emittance will be decreased to our design

goal of about 0.2nm·rad by harmonic cavity and damping wigglers.

### CONCLUSION

We calculated the emittance growth due to intrabeam scattering in HALS, which is much larger than the natural emittance of the bare lattice. To fight the large emittance we considered the application of damping wigglers and harmonic cavity, by which the effect of emittance reduction is estimated. Using these devices our design goal of beam emittance can be achieved theoretically. In addition, our estimation is quite conserved, since there are other effects tending to reduce emittance growth. For example, the potential well distortion(PWD), will lead to significant bunch lengthening and so the emittance growth rate is reduced. The microwave instability will also increase the bunch length and energy spread at high bunch current. On the other hand, the parameters of damping wigglers and harmonic cavity in this paper are chosen to optimize the emittance reduction, the final option in practical design of HALS must be made considering the beam dynamics and other physical conditions.

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