NEW APPROACH TO OPTIMIZATION OF RFQ RADIAL MATCHING SECTION*

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Abstract

New approach to define geometry of the radial matching section in RFQ accelerator is suggested. Approach is based on the application methods of the control theory. In paper special functionals are introduced which allow optimize radial section parameters with taking into account space charge. This approach gives wider opportunities for the design of the radial matching section because it does not have certain prescribed laws of variation of focusing strength along the section.

INTRODUCTION

Acceptance at the entrance of the RFQ accelerator, as it is known, depends on the time of arrival and rotates with the frequency of the RF field. On the other hand, the input beam has constant emittance, not changing in time. Thus, there is a problem of the beam matching to the accelerating RFQ channel. It was suggested in [1] to provide the transverse matching of the beam with the accelerating channel using radial matching sections, and particular laws of change of the focusing strength in these sections have been considered. Later, this problem was addressed by different authors [2-4]. In this paper the new approach [5,6] for the solution of the problem is considered. Parameters of the radial matching section are defined by solving the optimization problem described in the article.

PROBLEM STATEMENT

For the radial matching section of the accelerator, charged particle dynamics in the (x, y) plane which is perpendicular to the longitudinal axis in the case of microcanonical charge distribution can be described by the following system of equations [7]:

$$\frac{d\xi}{d\tau} = A_x\xi , \qquad \frac{d\eta}{d\tau} = A_y\eta , \tag{1}$$

where $\xi = (\xi_1, \xi_2)$, $\xi_1 = x$, $\xi_2 = \frac{dx}{d\tau}$, $\eta = (\eta_1, \eta_2)$, $\eta_1 = y$, $\eta_2 = \frac{dy}{d\tau}$, and the matrices A_x and A_y have the form

$$A_x = \begin{pmatrix} 0 & 1 \\ Q_x & 0 \end{pmatrix}; \qquad A_y = \begin{pmatrix} 0 & 1 \\ Q_y & 0 \end{pmatrix}. \tag{2}$$

Here

$$Q_x(\tau, \varphi_0, r_x, r_y) = \frac{eU}{W_0 a^2}\cos(\theta\tau + \varphi_0) - \frac{eI}{2W_0\pi\varepsilon_0 v r_x r_y}\left(1 - \frac{r_x - r_y}{r_x + r_y}\right) \tag{3}$$

$$Q_y(\tau, \varphi_0, r_x, r_y) = -\frac{eU}{W_0 a^2}\cos(\theta\tau + \varphi_0) - \frac{eI}{2W_0\pi\varepsilon_0 v r_x r_y}\left(1 + \frac{r_x - r_y}{r_x + r_y}\right). \tag{4}$$

where $\tau = ct$, $\theta = 2\pi\omega/c$, U is the intervane voltage, W_0 is the charged particle rest energy, ω is the accelerating field frequency, φ_0 is the initial phase, c is the velocity of light, a is the radius of the channel, $v = \dot{z}$ is the longitudinal velocity of a particle which is constant along the matching section, r_x and r_y are the beam envelopes, I is the beam current.

Let the set of conditions for system (1) at some instant τ fill the ellipses

$$\xi^* G_x \xi \le 1 , \qquad \eta^* G_y \eta \le 1 , \tag{5}$$

in the planes $\left(x, \frac{dx}{d\tau}\right)$ and $\left(y, \frac{dy}{d\tau}\right)$ correspondingly.

Then, the matrices G_x and G_y satisfy the following system of matrix equations

$$\frac{d}{d\tau}G_x = -A_x^* G_x - G_x A_x , \quad \frac{d}{d\tau}G_y = -A_y^* G_y - G_y A_y . \tag{6}$$

The system of equations (6) should be solved on the interval from the entrance into the regular part of the accelerator to the entrance into the radial matching section, i.e. from $\tau = \mathrm{T}$ to $\tau = 0$. Initial conditions for the system (6) are the matrices of ellipses defining acceptances of the regular part of the accelerator, depending on an initial phase φ_0:

$$G_x(\mathrm{T}, \varphi_0) = G_{x,\mathrm{T}}(\varphi_0), \quad G_y(\mathrm{T}, \varphi_0) = G_{y,\mathrm{T}}(\varphi_0). \tag{7}$$

The optimization problem for the radial matching section is to find a function $a(\tau)$, i.e. law of the radius change along the matching sections, providing under the conditions (7) the maximum possible overlapping of

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families of ellipses at the entrance of the radial matching section.

Let's consider the functions

$$\Phi_x(\varphi_0) = \mathrm{Sp}(G_x(0,\varphi_0) - B_x)^2, \qquad (8)$$

$$\Phi_y(\varphi_0) = \mathrm{Sp}(G_y(0,\varphi_0) - B_y)^2, \qquad (9)$$

where B_x, B_y are given matrices, Sp is the trace of the corresponding matrix. Functions $\Phi_x(\varphi_0)$ and $\Phi_y(\varphi_0)$ characterize deviations of ellipses G_x and G_y at $\tau = 0$ from the given ellipses B_x and B_y, accordingly.

METHOD OF SOLUTION

Here two approaches based on introduction of two different functionals are considered.

Introduce the functional

$$J(a) = c_1 \int_{\varphi_1}^{\varphi_2} \Phi_x(\varphi_0) d\varphi_0 + c_2 \int_{\varphi_1}^{\varphi_2} \Phi_y(\varphi_0) d\varphi_0, \qquad (10)$$

estimating the degree of mutual overlapping of ellipses corresponding to various initial phases at the entrance of the matching section. Here φ_1 and φ_2 are limits of variation of initial phase φ_0; c_1, c_2 are some positive constants.

Note that the optimization of the functional (10) over the control function $a(\tau)$ can be viewed as a nonstandard problem of the optimal control theory[5].

Let's also consider the following functional which characterizes the quality of the matching section by mismatch of ellipses $G_x(0,\varphi_0)$ and $G_y(0,\varphi_0)$ with given ellipses B_x and B_y:

$$J(a) = \max_{\varphi_0} \lambda_x^{-1}(\varphi_0) + \max_{\varphi_0} \lambda_y^{-1}(\varphi_0), \qquad (11)$$

where

$$\lambda_x^{-1}(\varphi_0) = \lambda^{-1}(G_x(0,\varphi_0), B_x), \qquad (12)$$

$$\lambda_y^{-1}(\varphi_0) = \lambda^{-1}(G_y(0,\varphi_0), B_y). \qquad (13)$$

Here $\lambda = \min(\lambda_1, \lambda_2)$ is a minimum eigenvalue of a cluster of quadratic forms generated by a pair of ellipses with the matrices G and B:

$$\chi(\lambda) = \det(G - \lambda B) = 0, \; \chi(\lambda_1) = \chi(\lambda_2) = 0. \qquad (14)$$

The value of the inverse minimum eigenvalue characterizes the degree of mismatch pairs of ellipses. In the case of fully identical ellipses, this value is equal to unity. So always $\lambda^{-1} \geq 1$.

Matrices B_x and B_y describing the desired phase portrait of the beam at the beginning of the matching section.

The problem of minimizing the functional (11) is the minimax optimization problem [6].

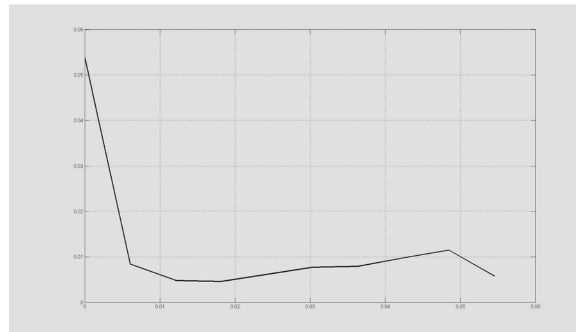

Figure 1: Radius of channel in radial matching section.

The analytic representations [5,6] of the variations of the functionals (10),(11) were used to find geometric parameters of radial matching section of the RFQ accelerator of protons (initial energy 95keV, output energy 5 MeV, intervane voltage 100kV, RF field frequency 352 MHz, initial cell length 6.06 mm). One of the possible choices of the law of variation of the channel radius along the radial matching section is presented in Figure 1. In Figures 2, 3 the RFQ acceptances without radial matching section are shown. The illustration of the radial matching section effect is shown in Figures 4, 5.

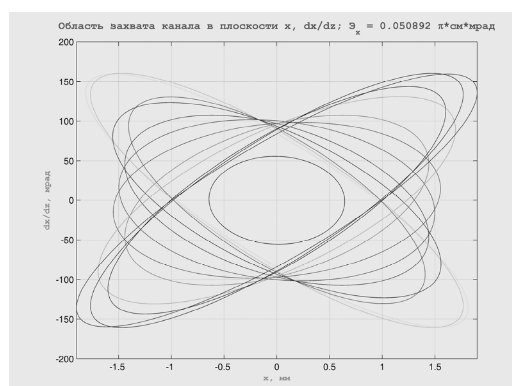

Figure 2: Acceptances in the plane (x, dx/dz) without radial matching section.

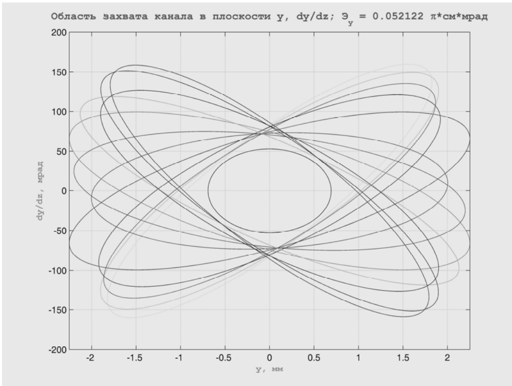

Figure 3: Acceptances in the plane (y, dy/dz) without radial matching section.

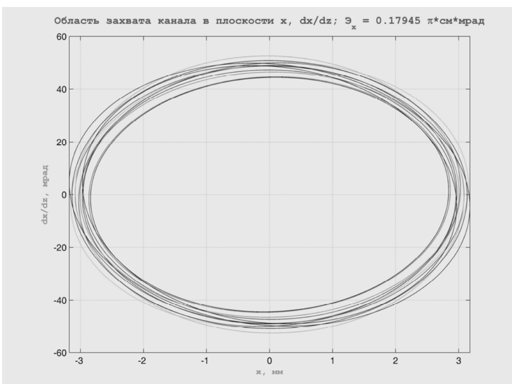

Figure 4: Acceptances in the plane (x, dx/dz) with radial matching section.

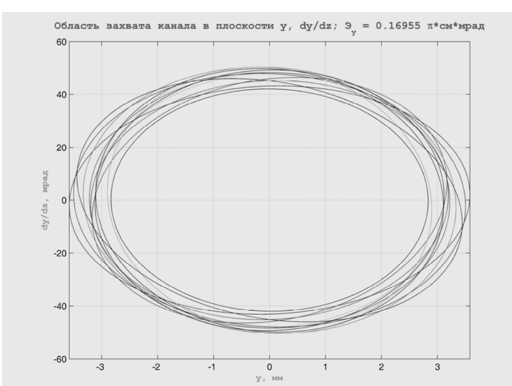

Figure 5: Acceptances in the plane (y, dy/dz) with radial matching section.

CONCLUSION

New mathematical models and methods of the RFQ structure optimization were suggested in the works [10,11]. In this paper the optimization approach with use of two different functionals to find geometric parameters of radial matching section is considered. It should be noted, that the proposed approach can be utilized to optimize the transverse dynamics in accelerators if the dynamics is adequately described by linear equations. In particular, this method can be used to minimize the growth of the effective emittance in the RFQ channel. The nonlinear effects can be taken into account while considering the nonlinear optimization models [8,9,11].

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