# EFFECTS OF ALIGNMENT ERROR OF MAIN SUPERCONDUCTING CAVITIES ON ERLS AND THEIR CORRECTION

N. Nakamura, ISSP, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, 277-8581 Chiba, Japan K. Harada, M. Shimada, Y. Kobayashi, S. Sakanaka, KEK, Tsukuba, 305-0801 Ibaraki, Japan R. Hajima, JAEA, Tokai, 319-1195 Ibaraki, Japan

Abstract

Effects of alignment error of main SC cavities were studied with analytical considerations and numerical simulations for the compact ERL. It was found that the cavity alignment error can cause large orbit distortion, significant emittance growth and considerable bunch lengthening. However the effects of the cavity alignment error at least up to 1 mm can be almost compensated for the compact ERL by orbit correction.

#### INTRODUCTION

In ERLs, many superconducting(SC) cavities accelerate and decelerate low-emittance and short-bunch beams with high-gradient standing-wave RF fields. If alignment error of the cavities is considerable, the cavities can harmfully affect the beam orbit and quality because they have strong effects on transverse motion of electrons that are much deviated from the cavity central axis. Achieving high alignment accuracy of the cavities is not easy compared with the other ERL elements such as magnets and monitors because the cavities are contained in cryomodules. Therefore we have studied effects of misalignment of main SC cavities on the ERL beam with analytical approaches and numerical simulations, using a compact ERL model. In this paper, we present the effects of alignment error of main superconducting cavities on ERLs and their correction.

### **OVERVIEW OF THE COMPACT ERL**

Figure 1 shows layout of a compact ERL[1]. The compact ERL consists of two TBA(Triple Bend Achromat) arc sections and eight main SC cavities in cryomodules. The injection energy of the beam from the injector is 5 MeV.

The compact ERL has three operation modes: high current(HC), low emittance(LE) and bunch compression (BC) modes. In HC and LE modes, the bunch charge is 77 and 7.7 pC and the initial normalized emittance (just after the merger) 1 and 0.1 mm mrad, respectively. The initial bunch length and momentum spread are 2 ps and 2×10<sup>-3</sup> in both modes. The beam is accelerated on-crest by the eight SC cavities with the accelerating field of 15 MV/m and the beam energy and momentum spread after the acceleration is about 125 MeV and  $2\times10^{-4}$ . The  $R_{56}$  value of the TBA arc sections is set at 0 m and the sextupole magnets in the TBA arc sections are turned off. On the other hand, in BC mode, the bunch charge, initial normalized emittance, initial bunch length and initial momentum spread are 77 pC, 1 mm mrad, 1 ps and  $2\times10^{-3}$ . The beam is accelerated off-crest by the main SC cavities

and compressed by the 1<sup>st</sup> TBA arc section with non-zero  $R_{56}$ . Since the RF phase of the main SC cavities  $\phi_{RF}$  is set at about 15 degrees, the beam energy and momentum spread after the acceleration is about 121 MeV and  $2.3\times10^{-3}$ . The  $R_{56}$  value of the 1<sup>st</sup> TBA arc section is set at 0.131 m and the fields of the sextupole magnets in the 1<sup>st</sup> TBA arc section are optimized to minimize the bunch length at the exit of the 1<sup>st</sup> TBA arc section[1].

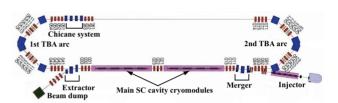


Figure 1: Layout of a 1-loop compact ERL.

# TRANSVERSE MOTION IN SC CAVITIES

For an ultra-relativistic electron( $\beta=v/c\approx1$ ), the averaged equation of transverse motion in a SC cavity is expressed as[2]

$$x'' + \left(\frac{\gamma'}{\gamma}\right)x' + \frac{1}{8}\left(\frac{\gamma'}{\gamma}\right)^2 x = 0 \tag{1}$$

$$\gamma' = \frac{eE_{rf}}{mc^2} \tag{2}$$

Here x, x', e, m, v, c, p,  $\gamma$  and  $E_{rf}$  are the transverse position and angle (derivative of x respect to the longitudinal position s), electron charge and mass, velocities of the electron and light, momentum and Lorentz factor of the electron, and the average RF field of the SC cavity. The second and third terms of Eq. (1) mean momentum change of the electron due to acceleration ( $\gamma$ >0) or deceleration( $\gamma$ <0) by the SC cavity and focusing force due to alternating transverse RF fields, respectively. Furthermore the beam is kicked at the entrance and exit of the SC cavity as[2]

$$\Delta x'_{i(f)} = \mp \frac{1}{2} \frac{\gamma'}{\gamma_{i(f)}} x_{i(f)} \tag{3}$$

where  $\gamma_i$  and  $\gamma_f$  are the Lorentz factors at the entrance and exit of the SC cavity. Eq. (3) means that each of edge fields at the entrance and exit works as an axisymmetric transverse focusing or defocusing lens with the focusing strength (the inverse of the focal length)  $k=\pm \gamma/2\gamma_{i(f)}$ .

If misalignment of the SC cavities exists, the beam moving on the central orbit is deflected at the first turn in the acceleration by the misaligned SC cavities and as a result orbit distortion is generated. At the second turn, the beam also suffers orbit change in the deceleration by the SC cavities according to Eqs. (1) and (3). The change of the transverse motion shown in Eqs. (1) and (3) is relatively large for the low momentum(energy) beam. For example, the kick angle in Eq. (3) due to the alignment error of 1 mm reaches about 1.5 mrad for the beam energy of 5 MeV and the accelerating field of 15 MV/m.

# EMITTANCE GROWTH DUE TO CHROMATIC EFFECTS

Chromatic effects of focusing and defocusing elements such as bodies and edges of SC cavities and quadrupole magnets can cause emittance growth. Normalized rms emittance  $\varepsilon_n$  is generally defined by

$$\varepsilon_{n} = \gamma \beta \sqrt{\left\langle \left(x - \langle x \rangle\right)^{2} \right\rangle \left\langle \left(x' - \langle x' \rangle\right)^{2} \right\rangle - \left\langle \left(x - \langle x \rangle\right) \left(x' - \langle x' \rangle\right) \right\rangle^{2}}$$
 (4)

Here  $\langle x \rangle$  means the average of the transverse position x over the beam. When an electron has the initial position  $x_0$  and angle  $x_0$  with the Lorentz factor  $\gamma$  and the relative velocity  $\beta$ , the transverse position x and angle x' after a thin focusing (or defocusing) element with focusing strength k are given by

$$x = x_0$$

$$x' = x'_0 - \frac{kx_0}{(1+\delta)^a}$$

$$\approx x'_0 - kx_0(1-a\delta) \quad \left(\delta = \frac{\Delta p}{p} << 1\right)$$
(6)

Here  $\Delta p/p$  is the relative momentum deviation from the reference momentum and the focusing strength is assumed to have momentum dependence as  $k \propto p^{-a}$ . When the initial position and angle are uncorrelated with the momentum, the normalized emittance of Eq. (4) just after the element is obtained from Eqs. (5) and (6) as follows:

$$\varepsilon_n = \sqrt{\varepsilon_{n0}^2 + a^2 \gamma^2 \beta^2 k^2 \sigma_{x0}^2 \sigma_{\delta}^2 \langle x_0^2 \rangle}$$
 (7)

Here  $\varepsilon_{n0}$ ,  $\sigma_{x0}$ , and  $\sigma_{\delta}$  are the initial normalized emittance, initial beam size and initial momentum spread. The average of the square of the initial position  $\langle x_0^2 \rangle$  is approximately equal to  $\sigma_{x0}^2$  for  $|\langle x_0 \rangle| << \sigma_{x0}$  and to  $\langle x_0 \rangle^2$  for  $|\langle x_0 \rangle| >> \sigma_{x0}$ . Therefore the normalized emittance can significantly increase if the cavity misalignment is large or causes large orbit distortion at the focusing and defocusing elements. The emittance growth behaves as Eq. (7) with a=1 for edges of SC cavities and quadrupole magnets. For example, the emittance increase is estimated from Eq. (7) to be more than 50 % of the initial emittance  $\varepsilon_{n0}=0.1$  mm mrad for  $\langle x_0 \rangle=5$  mm, k=5 m<sup>-1</sup> and the typical beam parameter values after acceleration in LE mode of the compact ERL,  $\gamma\beta=244.6$ ,  $\sigma_{x0}=0.1$  mm and  $\sigma_{\delta}=2x10^{-4}$ .

#### **SIMULATION**

In order to evaluate the effects of the alignment error of the main SC cavities accurately, simulations for the compact ERL were performed by using elegant[3]. In the simulations presented here, the alignment error was assumed to be the horizontal or vertical position error of +1 mm (or -1 mm) for all the cavities, because almost all the effects were found to be the strongest when all the cavities have the same position error (+1 mm or -1 mm).

#### Orbit Distortion and Correction

Figure 2 shows simulated horizontal/vertical orbit distortions due to horizontal/vertical alignment error of  $\Delta X/\Delta Y$ =+1 mm for all the cavities in LE and BC modes. The maximum orbit distortion reaches about 30 mm and 15 mm for LE and BC modes, respectively. Such orbit distortion can cause serious beam loss for a comparable physical aperture size.

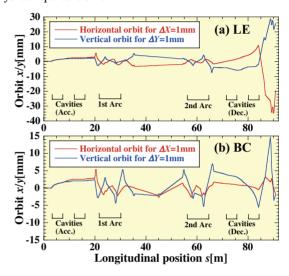


Figure 2: Horizontal/vertical orbit distortions due to horizontal/vertical alignment error of  $\Delta X/\Delta Y$ =+1 mm for all the cavities in (a) LE and (b) BC modes.

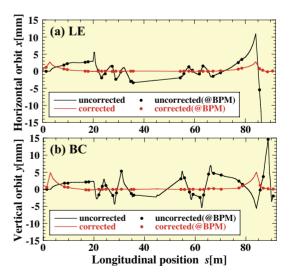


Figure 3: Results of orbit correction: (a) uncorrected and corrected horizontal orbits for the horizontal alignment error of  $\Delta X$ =+1 mm in LE mode and (b) uncorrected and corrected vertical orbits for the vertical alignment error of  $\Delta Y$ =+1 mm in BC mode. Solid circles indicate 28 positions monitored at the 23 BPMs.

Orbit correction using the eigenvector method with constraints[4] was performed for the orbit distortions with twenty-three beam position monitors(BPMs) and nineteen correctors distributed in the compact ERL. Figure 3 shows some results of the orbit correction in LE and BC modes. The beam orbits are almost corrected within ±1 mm except for the local orbit in the SC cavities having no BPMs. More details of the orbit correction method and system are shown in ref. [5].

## Emittance Growth

Simulated normalized emittances (including the momentum dispersion effect) with and without the cavity alignment error of +1 mm are shown by black solid and green broken lines in Fig. 4, respectively. In LE mode, the horizontal emittance with the cavity alignment error significantly increases at four quadrupole magnets(Q41-Q44) just before the 1<sup>st</sup> TBA arc section as shown in Fig. 4a, because the chromatic effects of the quadrupole magnets become considerable by the large orbit distortion at the magnets. The horizontal emittance continues to increase to more than 1.5 times of the initial value in the long straight section and finally exceeds over 1 mm mrad at the beam dump. In HC mode, since the initial emittance is larger by one order of magnitude for the same optics than that in LE mode, the emittance growth due to the cavity misalignment becomes relatively small. In BC mode, the vertical normalized emittance with the cavity alignment error increases by a factor of 25 at maximum in the long straight section and by two orders of magnitude at the beam dump, as shown in Fig. 4b. This emittance growth is mainly caused by the chromatic effects of the quadrupole magnets and the nonlinear fields of the sextupole magnets.

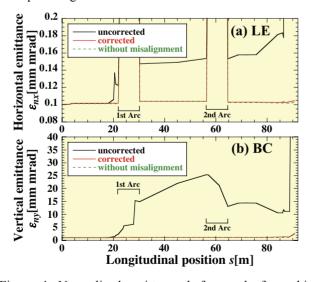


Figure 4: Normalized emittance before and after orbit correction for cavity alignment error of +1 mm and without the cavity alignment error: (a) horizontal emittance for the horizontal error of  $\Delta X$ =+1 mm in LE mode and (b) vertical emittance for the vertical error of  $\Delta Y$ =+1 mm in BC mode.

Red solid lines in Fig. 4a and 4b show the simulated normalized emittances for the corrected orbits in Fig. 3a and 3b. For both cases, the emittance increase is well suppressed by the orbit correction and becomes negligibly small in all the regions of the compact ERL.

# Bunch Lengthening

Figure 5 shows simulated 2D electron distributions of the bunch at the exit of the 1st TBA arc section for horizontal/vertical cavity alignment error of  $\Delta X/\Delta Y=\pm 1$ mm in BC mode. The blue- and green-colored distributions in this figure are for  $\Delta X/\Delta Y = +1$  and -1 mm respectively and the red-colored distribution for without the alignment error. For the horizontal cavity alignment error, the bunch is inclined toward the right or left side on the time-momentum plane and the bunch length increases from 55 fs to more than 100 fs. On the other hand, the distributions for the vertical alignment errors  $\Delta Y = +1$  mm and -1 mm are almost the same and the bunch length increases up to 267 fs. However it is also found from simulation results that the bunch lengthening shown in Fig. 5a and 5b can be easily compensated by orbit correction.

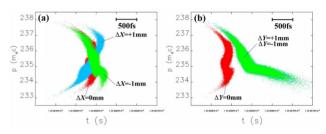


Figure 5 Bunch lengthening due to (a) horizontal and (b) vertical cavity alignment errors of  $\pm 1$  mm in BC mode.

#### **SUMMARY AND CONCLUSIONS**

Simulation results show that misalignment of the main SC cavities in the compact ERL can cause large orbit distortion in all the operation modes, non-negligible emittance growth in LE and BC modes and significant bunch lengthening in BC mode. Accurate alignment is desired especially for cavities accelerating or decelerating a low-momentum(energy) beam. However the alignment error may be tolerable at least up to  $\pm 1$  mm for the compact ERL, because the error effects can be compensated by the orbit correction.

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