3D PIC COMPUTATION OF A TRANSVERSAL TUNE SHIFT CAUSED BY AN ELECTRON CLOUD IN A POSITRON STORAGE RING *

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Abstract

The electron cloud, which is initially presumed as a homogeneous distribution of static electrons, changes its transverse centroid position very fast during the passage of even a single bunch. This is due to the strong focusing transverse field of the highly relativistic positron bunch. As the density of the electrons near the beam axis grows, its impact on the beam becomes stronger. The interaction of the electron cloud with the bunch results in a shift of the betatron tune of the coherent dipole motion of the beam. In this paper we simulated the dipole tune shift of the beam interacting with the electron cloud by taking also in to account the own space-charge forces of the electrons which strongly affect the motion of the electrons during the passage of the bunch. We computed the tune shift for different transverse size of the electron cloud.

INTRODUCTION

In positron storage rings the electrons are initially produced by photoemission and ionization. Caught into the potential of the beam those electrons are accelerated in the transversal plane and after hitting the vacuum chamber walls produce many secondary electrons to build the electron cloud (e-cloud). If the storage ring is operated with high positron currents the density of electrons is growing rapidly until a saturation density has been achieved. Such an e-cloud compared with the positron beam has practically no impulse. On the other hand the transversal components of the electrical field of the accelerated bunch are very strong so that the electrons in the potential of the bunch become very mobile in the plane perpendicular to the bunch movement. As a consequence, the e-cloud density in the transversal plane could change very fast and it is dependent on the parts of the beam already passt through the certain transverse plane. The estimated averaged density of an unperturbed e-cloud in the cross-section of a beam pipe is many orders of magnitude smaller than the positron density in the bunch. However during the beam passage the e-cloud density around and on the beam axes grows rapidly which imposes a dipole kick on the bunch from the e-cloud.

Measurements in positron storage rings operated in a mode of a long bunch trains with short inter bunch distances (very fertile mode to grow a considerable amount of electrons), show a betatron tune shift of the coherent dipole motion of a beam.

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BETATRON TUNE

The betatron tune represents a number of pseudo-harmonic oscillations of a particle in a transverse direction, over the period of a single turn in the storage ring. In vertical direction it is defined as:

$$Q_y = \frac{1}{2\pi} \oint \frac{ds}{\beta_y(s)}, \tag{1}$$

where $\beta_y(s)$ is the beta function which is representing the local wavelength (at longitudinal coordinate s) of the transverse oscillation. In the smooth focusing approximation the unperturbed single particle motion is modelled as a undamped harmonic oscillator with constant wavelength given by the constant beta function $\beta(s) = \overline{\beta}$:

$$y'' + \left(\frac{1}{\overline{\beta}}\right)^2 y = 0. \tag{2}$$

Thus the betatron tune for unperturbed motion Q_{y0} can be obtained by dividing the circumference of the storage ring ($2\pi R$) by the constant wavelength ($2\pi\overline{\beta}$):

$$Q_{y0} = \frac{R}{\overline{\beta}}. \tag{3}$$

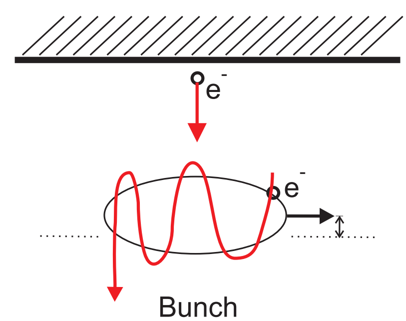

Figure 1: Beam off-set $\triangle y$ in the transversal plane.

In the presence of the e-cloud the motion of the beam particles will be perturbed especially if the center of mass of the beam and the e-cloud doesn't match in the transversal plane. Considering an initial homogeneous distribution of the e-cloud inside the beam pipe a bunch with certain $\triangle y$ offset from the symmetry axis would be perturbed by the e-cloud. By linearizing the perturbation, the equation of motion for the perturbed particle writes:

$$y'' + \left(\frac{Q_{y0}}{R}\right)^2 y = Ky. \tag{4}$$

The right hand side of (4) represents the dipole force F_y acting on a particle with vertical position y relativ to it's energy E_b, thus the coefficient K writes as

$$K = -\frac{1}{E_b}\frac{F_y}{\triangle y}. \tag{5}$$

Equation (4) is still describing an oscillatory motion of a simple oscillator, although now with some different betatron tune Q_y given by

$$Q_y^2 = Q_{y0}^2 - KR^2. \tag{6}$$

For a small perturbation compared with the unperturbed tune Q_{y0}, the tune shift $\triangle Q_y = Q_{y0} - Q_y$ is approximately given by

$$\triangle Q_y \approx \frac{-KR\overline{\beta_y}}{2}. \tag{7}$$

For a practical computation of $\triangle Q_y$ from (7),we take $\overline{\beta_y}$ to be an average beta function of the beam and R is the radius of the ring. The coefficient K has to be computed numerically from (5).

Numerical Computation of the Tune Shift

In order to compute the tune shift caused by the e-cloud we evaluate the vertical force F_y from (5) acting on the bunch particles over one turn along the circumference of the ring $L = 2\pi R$. On the other hand F_y could be expressed as a change of the particles energy ($\triangle E_y$), due to the change of it's vertical impulse $\triangle p_y$ over some section of the ring $\triangle s$ in which the beam interacts with the e-cloud. Plugging $F_y = \triangle E_y/\triangle s$ in to (5) yields:

$$K = -\frac{1}{E_b}\frac{F_y}{\triangle y} = -\frac{1}{E_b}\frac{\triangle E_y}{\triangle s}\frac{1}{\triangle y}, \tag{8}$$

where instead of the energy ratio we could use the impulse ratio so that finally

$$K = -\frac{1}{\triangle s}\frac{\triangle p_y}{p_b}\frac{1}{\triangle y}. \tag{9}$$

By computing the $\triangle p_y$ for the particles of a bunch with a given offset $\triangle y$ and impulse p_b after passing a $\triangle s$ section where it interacts with the e-cloud we get the contribution of the e-cloud situated in that section $\triangle s$ to the $\triangle Q_y$. Another approximation made here is that throughout the ring we consider a constant e-cloud density of $\rho_e = 10^{12}$ $[1/m^3]$. Such an approximation allows the results for the tune shift to be extrapolated for the whole ring. Consequently it is sufficient to perform a 3D PIC simulation of the interaction between the positron bunch and the e-cloud for a small section of the ring $\triangle s$ and then evaluate the $\triangle p_y$ along the bunch length to compute K.

RESULTS

We have simulated the interaction (by MOEVE-PIC Tracking [2]) of a positron bunch from the KEKB factory ($\sigma_x = 0.42$ mm, $\sigma_y = 0.06$ mm, $\sigma_z = 6.00$ mm, $N_e^+ = 3.3\,10^{10}$, $E_{kin} = 3.5$ GeV) represented by 1 million macro-particles with an e-cloud of a homogeneous density $\rho_e = 10^{12}$ $[1/m^3]$. The interaction was simulated for e-clouds with different transversal sizes as shown in Figure 2. The e-cloud was represented by 0.5 million macro-particles and its thickness was taken to be 1 cm. The bunch was set to enter the e-cloud with an offset $\triangle y = \sigma_y = 0.06$ mm. The interaction simulation was for $t = 2$ ns with time discretization steps of $dt = 1$ ps. For the beam pipe was assumed a cylindrical cross-section with PEC boundary.

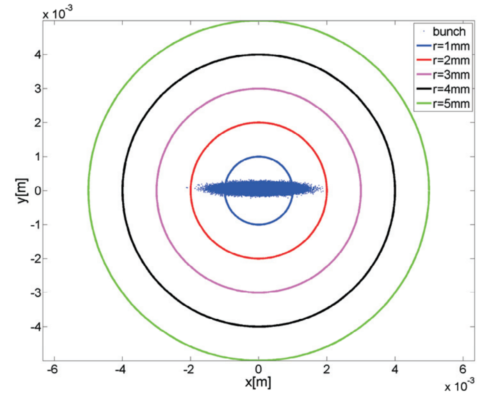

Figure 2: Different transverse e-cloud size and the offset beam.

We started the interaction simulation for a cloud with $r = 1$ mm which in horizontal direction is even smaller then the bunch itself. The kick in Figure 3 shows that the center of mass of the electrons was shifted in the positive y-direction towards the one of the bunch. The positive momentum of the electrons and the fact that their number is not that big to start repelling, pull the second part of the positron bunch in the positive y-direction. The kick from the second e-cloud ($r = 2$ mm) starts lower simply because the forces of the more distant electrons sum up and give a stronger base force that pulls the offset bunch down. After the middle of the bunch passed through the cloud, the center of mass of the e-cloud moved up and it starts also to push the kick in the positive y-direction but this doesn't work because the electrons start disintegrating because of their own space charge forces.

The same explanation could be applied to the kick curve from the cloud with $r = 3$ mm with the notice that because of the bigger number of electrons the forces from the distant electrons are stronger and those forces are reinforced with the slowly approaching electrons from higher radii. Again the electrons start repelling as their concentration on the beam axis starts to grow.

For the e-clouds with $r = 4$ and 5 mm the kick on the bunch particles is nearly constant over the whole bunch length and a saturation of the kick amplitude can be observed. The last conclusions coincide with those from [1] except that the computed kicks with the PIC simulation are of one order of magnitude lower then the one in [1]. This

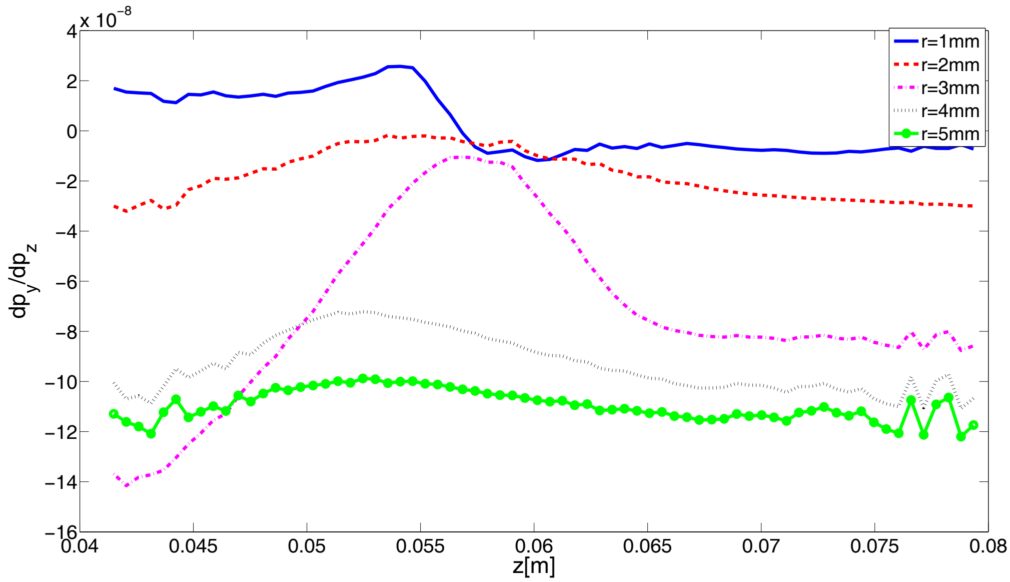

Figure 3: Vertical kick on the bunch particles along it's length (the head of the bunch is at larger z values).

may be due to the fact that in the PIC simulation the own space charge forces of the electrons are taken into account so that the electrons are not just gathering on the beam axis but as their concentration grows they will also repel and lower the number of electrons on the beam axis. The Figures 4 and 5 show the impulse of the electrons during the bunch passage. Figure 4 visualize that as almost the half of the bunch went through the e-cloud the electrons on the beam axis start repealing although they are still in the strong beam potential. However there are still electrons from the higher radii heading towards the bunch axis and the concentration of electrons on the beam axis oscillates. Figure 5 shows the electrons disintegrating in the transversal direction after the bunch passed through. Another conclusion we derive from the results is that the tune shift is not linearly depend on the cloud density.

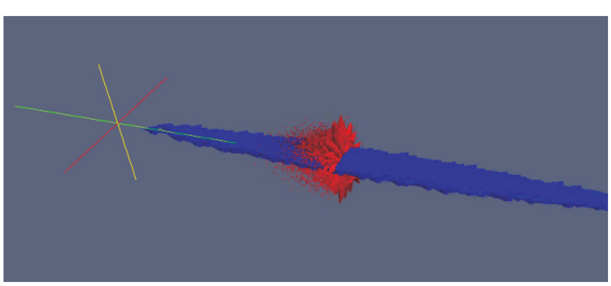

Figure 4: Impulse of the electrons as only the first half of the bunch progressed through.

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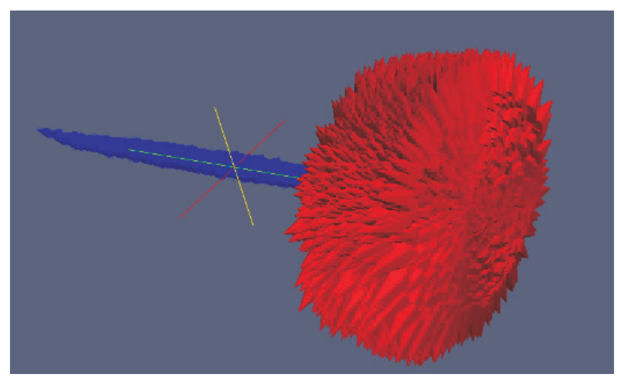

Figure 5: Impulse of the electrons as the whole bunch progressed through.

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