# SIMULATION AND OBSERVATION OF THE SPACE CHARGE INDUCED MULTI-STREAM INSTABILITY OF LINAC MICRO BUNCHES IN THE SIS18 SYNCHROTRON

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#### Abstract

For the future operation as an injector for the FAIR project the SIS18 synchrotron has to deliver intense and high quality ion bunches with high repetition rate. One requirement is that the initial momentum spread of the injected coasting beam should not exceed the limit set by the SIS18 rf bucket area. Also the Schottky spectrum should be used to routinely measure the momentum spread and revolution frequency directly after injection. During the transverse multi-turn injection the SIS18 is filled with micro bunches from the UNILAC linac at 36 MHz. For low beam intensities the micro bunches debunch within a few turns and form a coasting beam with a Gaussian-like momentum distribution. With increasing intensity we observe persistent current fluctuations and an accompanying pseudo-Schottky spectrum. We will explain that the multi-stream instability of the micro bunch filaments is responsible for the turbulent current spectrum that can be observed a few 100 turns after injection. The current spectrum observed in the SIS18 and the results from a longitudinal simulation code will compared to an analytical model of the multistream instability induced by the space charge impedance.

## **INTRODUCTION**

In the SIS18 intense heavy-ion micro bunches (36 MHz) are injected from the UNILAC at 11.4 MeV/u for up to 15 turns. For the SIS18 intensity upgrade the resulting rms momentum spread of the debunched beam has to remain below  $\sigma_p = 5 \cdot 10^{-4}$  because of the available rf bucket area for fast ramping. Furthermore the momentum spread before rf capture should be measured within a few ms using the Schottky spectrum from the coasting beam. Longitudinal space charge can be responsible for a sustained linac micro bunch structure after injection into a synchrotron or storage ring with no rf (see e.g. [1]). Persistent coherent structures on the debunched beam can modify the Schottky spectrum ('pseudo-Schottky'). In combination with broadband impedance sources the structures could also cause an undesired beam energy loss. In [2] it was pointed out that during the debunching of linac micro-bunches a fast multi-stream instability can develop. The saturated instability shows a broad coherent fluctuation spectrum. In the present study we compare the analytic and simulation models of the multi-stream instability with experimental results obtained in the SIS18.

#### **MULTI-STREAM INSTABILITY**

If the injected micro bunches are allowed to debunch freely this leads to the formation of filaments in longitudinal phase space. After sufficiently many revolutions the beam distribution consists of many almost parallel filaments. The width of a filament and the distance to the next neighbor is shrinking with time. The beam distribution for M filaments can be approximated as

$$f_0(v) = \frac{\lambda_0}{M} \sum_{j=1}^M \delta(v - v_j), \qquad (1)$$

where  $v = -\eta_0 \beta_0 c \Delta p/p$  is the velocity deviation,  $v_j$  is the velocity of the *j*th filament,  $\eta_0 = 1/\gamma_t^2 - 1/\gamma_0^2$  is the slip factor,  $\beta_0$  and  $\gamma_0$  are the relativistic parameters,  $\lambda_0$  is the total line density. For a perturbed *M*-filament distribution  $f_0 + \delta f$  the dispersion relation for frequencies below the cut-off ( $\omega \ll \omega_{cut} = 2\pi b$ , *b* beam radius) was obtained in [2] as

$$-i\frac{qI_0\eta_0}{2\pi\gamma_0\beta_0^2mc^2}\frac{Z_{sc}}{n}\frac{n^2\omega_0^2}{M}\sum_{j=1}^M\frac{1}{(\omega-nv_j)} = 1, \quad (2)$$

where  $\omega_0$  is the revolution frequency, n the harmonic number, q the ion charge, m the ion mass,  $I_0$  the beam current. The space charge impedance is

$$\frac{Z_n^{sc}}{n} = -i\frac{gZ_0}{2\beta_0\gamma_0^2},$$
(3)

where  $Z_0 = 377 \ \Omega$  is the vacuum impedance. The geometry factor with the beam radius *a* is

$$g = 1 + 2\ln\frac{b}{a}.\tag{4}$$

From the dispersion relation Eq. 2 one can obtain the threshold for the onset of a multi-stream instability in terms of the critical number of filaments

$$M_{thr} = \frac{32}{\pi^2 U_{sc}},\tag{5}$$

where

$$U_{sc} = \frac{2I_0 qg Z_0}{\pi mc^2 \beta_0^3 \gamma_0^3 |\eta_0| \sigma_p^2} \tag{6}$$

is the space charge parameter for a costing beam. After debunching the time needed to reach  $M_{thr}$ -filaments is

$$t_{M_{thr}} = \frac{1}{2} \frac{CM_{thr}}{\sigma_p \beta c N_{micro}},\tag{7}$$

**05 Beam Dynamics and Electromagnetic Fields** 

D03 High Intensity in Circular Machines - Incoherent Instabilities, Space Charge



Figure 1: Measured frequency spectrum after injection into the SIS18. The space charge parameter is  $U_{sc} = 0.1$ . The dashed line indicates the time needed for the formation of the critical number of filaments  $t_{M_{thr}}$ .

where C is the ring circumference and  $N_{micro}$  is the number of micro bunches per turn. Simulation results indicate that after  $t = t_{M_{thr}}$  a fast transition to a broad band coherent fluctuation spectrum occurs. Such a fast transition to a broad frequency spectrum was observed in the SIS18 shortly after injection of  $N = 2 \cdot 10^9$  Ar<sup>18+</sup> ions in one turn. The measured momentum spread was  $\sigma_p = 0.7 \cdot 10^{-3}$  and the space charge parameter  $U_{sc} = 0.1$ . Fig. 1 shows the obtained frequency spectrum during the first 5 ms. The observed transition occurs after  $M_{thr}$  filaments have been generated (dashed line, representing Eq. 7). Below this line one can observe the 36 MHz structure of the UNILAC micro bunches and their harmonics.

# SIMULATION MODEL AND RESULTS

A longitudinal simulation code has been developed in order to study the evolution of micro bunches after injection. The code solves the longitudinal equation of motion together with the self-consistent space charge field. The evolution of the multi-stream instability is shown in Fig. 2. As an example case we use  $Ar^{18+}$  ions with  $U_{sc} = 1$ . In the simulation we only treat a subsection of the ring which includes three micro-bunches of the total 168 per turn. In order to verify the instability threshold in the simulations we performed a parameter scan by varying the micro-bunch intensity. All other parameters was kept constant. The waterfall plot in Fig. 3 shows the evolution of the total electric field energy W as a function of time and of the number of ions in one micro bunch. During a fraction of the first millisecond the electric field energy decreases during the debunching process. During the subsequent filamentation the field energy is very low until the critical number of filaments is reached (Eq. 7, red line). Afterwards the electric field energy increases very fast due to the coherent fluctuations generated by the instability. The instability results in a 'turbulent' frequency spectrum with a corresponding



Figure 2: Snapshots of the time evolution of the multistream instability for  $U_{sc} = 1$ . The onset of the instability between the inner filaments (middle) and the fluctuations after saturation (bottom) are clearly visible.

persistent electric field energy. For very small  $U_{sc} \ll 1$  we do not observe the onset of the multi-stream instability.

## LONGITUDINAL SCHOTTKY SIGNALS

In SIS18 Schottky bands from coasting beams are routinely used to measure the revolution frequency  $f_0$  and the momentum spread  $\sigma_p$ . The rms width of a Schottky band  $\sigma_f$  is linked via

$$\sigma_f = n f_0 \left| \eta \right| \sigma_p \tag{8}$$

to the rms momentum spread of the beam (see e.g. [3]). For low beam intensities the shape of the Schottky band reproduces the shape of the momentum distribution. For intense





Figure 3: Simulation scan of the total electric field energy W divided by the initial field energy  $W_0$  for different micro bunch intensities versus time. The red line indicates the time needed to reach the critical number of filaments given through Eq. 7.

beams the shape of the Schottky band will be deformed by space charge effects and a characteristic double-peak structure develops (see e.g. [4]). However, the Schottky noise theory applies only to the incoherent, natural fluctuation spectrum in stable beams. The current fluctuation or longlived coherent structures generated as a result of the injection process can cause a pseudo-Schottky spectrum that is difficult to interpret. For intense beams in the SIS18 pronounced, long-lived peaks have been observed on top of the Schottky bands [5] after injection. Here we present example results of noise bands obtained from our macro-particle simulation code. The simulation noise bands are obtained from a FFT of the beam current over a time interval of 1 ms right after injection. For weak space charge  $U_{sc} \ll 1$ the shape of the obtained simulation noise band at a given harmonic n agrees very well with the underlying momentum distribution. The obtained noise band for  $U_{sc} \approx 1$  is shown in Fig. 4. The expected shape of the Schottky band for a coasting beams with a Gaussian momentum distribution and the same space charge parameter is also shown. For  $U_{sc} \approx 1$  the expected shape is still close to the underlying momentum distribution. However, the simulated noise band in Fig. 4 already shows large discrepancies in the form of a 'burst'.

### CONCLUSIONS

In this contribution we study the space charge induced multi-stream instability during the debunching of intense linac micro bunches in SIS18. The instability can be clearly identified in the measured fluctuation spectrum obtained in SIS18 right after injection. The observed 'turbulent' broadband fluctuation spectrum might cause a slowing down of the unbunched beam and affect the efficiency of the subsequent rf capture. The threshold for the multi-stream in-

Figure 4: Simulated noise band obtained for  $U_{sc} \approx 1$ . The dashed green curve represents the underlying momentum distribution. The dashed-dotted blue curve represents the expected shape of the Schottky spectrum from a coasting beam with  $U_{sc} = 1$ .

stability can be reproduced very well with a longitudinal simulation code. With the same code we also studied the shape of the Schottky bands that are expected directly after injection. For space charge parameters expected in the SIS18 the noise bands show large deviation from the expected shape. Here we only present experimental and simulation results for single turn injection. In the SIS18 the frequency ratio between the UNILAC injection frequency (36 MHz) and the revolution frequency ( $\approx 215$  kHz) is not an integer. Therefore the filling pattern can be rather complex. Still the onset of the multi-stream instability can be identified after the injection of up to  $\approx 5$  turns. The detailed analysis of simulation and experimental results for multi-turn injection is still ongoing. For the injection of many turns and for a matched frequency ratio one would expect the formation of large phase space hole [1] instead of the multi-stream instability. Future work will also address the applicability of these two limiting cases for the situation in the SIS18.

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