NEGATIVE ION AND ELECTRON PLASMA SHEATH AND BEAM EXTRACTION*

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Abstract

In the development of powerful negative ion sources for neutral beam injectors, the modelling of extraction of negative ions is fundamentally complicated by the existence of coextracted electrons, and the practically necessary transverse magnetic field. After recalling that for positive ion extraction the transition from presheath to sheath happens near Bohm speed, as definitely proved by kinetic models in agreement with simpler fluid models, a kinetic model for electron transport is solved for in a large class of potentials, including examples of interest. Relations to usual mobility coefficient and equilibrium density are found. Kinetic concepts are also included in a new fluid model, showing some results for electron extraction from an e - H^+ plasma, with a magnetic field B_x and with an electron collision frequency dependent from electron speed. Extraction voltage increases with $|B_x|$ at constant extracted current j_z.

INTRODUCTION

The Negative Ion Sources (NIS) used, for example, for multiturn synchrotron injection[1] or in Neutral Beam Injectors for fusion application[2], are based on a two stage plasma: gas (H_2 or D_2) is dissociated or ionized in a driver region (electron temperature $T_e \geq 4$ eV), while negative ions propagates in a cooler plasma region (temperature $T_0 \cong 1$ eV) near extraction. A transverse magnetic field (called filter, in x or y direction when z is the beam axis) is necessary in the latter region to reduce electron flow toward extraction; another transverse magnetic field system in the 1st acceleration gap is useful to deflect and dump the coextracted electrons, before they are accelerated over 10 keV; D^- extraction voltage ranges from 60 kV sources to the 1 MV planned for NBI system. From electron orbits, it is evident that plasma collisions are responsible for coextracted electrons, and this paper attempts a step towards the clarification of the transition from a collisional plasma to a collisionless ray tracing, by discussing some newly found solutions to the integrodifferential transport equations.

In singly charged positive ion sources, the study of beam extraction was greatly simplified by the absence of magnetic field; consider only two charge species, say e^- and H^+; so both fluid and kinetic models have success[3]. The vast majority of models is one dimensional (1D), that is any variation in space coordinates x and y is ignored. As an example of kinetic model without collision, the integrodifferential plasma equation [4] for the adimensional variable $u = -e\phi/T_0$ with ϕ the electric potential was solved numerically[5], explaining the transition between two regions: a quasineutral plasma (named presheath) and a positively charged sheath, where the ion velocity $v_z(z)$ becomes approximately equal for all ions and greater than $c_s = \sqrt{T_0/m_H}$ and the ion density N_{H^+} becomes

$$N_{H^+} = -j_{H^+}/(ev_z) \qquad v_z \cong c_s\sqrt{2u - 2u_p} \qquad (1)$$

here j is the extracted current density and u_p a plasma potential. Electron density N_e exponentially decreases in the sheath region; to fix ideas, sheath ends and ion beam begins when $N_e < 0.01 N_{H^+}$, that is, fully negligible; sheath thickness is very small, of the order of ten Debye lengths λ_D. In fluid models (much simpler to solve), eq. 1 is assumed for the sheath and beam region, while $N_H^+ = N_e = N_0 \exp(-u)$ is usually assumed in the presheath; in fact, we have two different fluid models. Presheath model breaks when the ion fluid velocity $< v_z >$ reaches the speed of a sonic wave, known as Bohm speed $\sqrt{(T_e + T_H)/m_H} \cong c_s$. As regards to the beam fluid model, it requires $u > u_p + \frac{1}{2}$ for stability, so that the two models do not overlap and do not contradict, as discussed in a vast literature[3]. Most of the ion extraction simulation codes were implicitly based on the concept of quasi neutrality in the plasma region and of ion Bohm speed.

In negative ion extraction, we have to consider electron speed, magnetic fields and collisions and additional charged species (H^-, and in most experiment Cs^+). A kinetic model starting from Vlasov equation with a thermalized scatterer collision term was reduced to a 1D transport equation[6] and fully solved for H^- ions. Selfconsistent solutions for u and a fluid transport model were also found[7]. Monte Carlo simulations are also widely used to investigate plasma behavior, and resolution down to λ_D is becoming possible with parallelized computing and variance reduction techniques[8]; some regularized sampling techniques seems also promising.

In the next section the transport of electrons is discussed. A sheath model is also discussed in the last section.

ELECTRON TRANSPORT

Let T_0 be a fixed reference plasma temperature (that is $T_0 = 1$ eV) and N_0 be a reference density and assume $\mathbf{B} = \hat{x}B_x(z)$, so that $\mathbf{A} = \hat{y}A_y(z)$. To discuss transport of any particle a we find convenient to use scaled variables: velocities in units of $c_a = \sqrt{T_0/m_a}$, mechanical momenta

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p_z in unit of $m_a c_a$, density in units of N_0, currents in units of $q_a N_0 c_a$ with q_a the particle charge, potential $v = q_a\phi/T_0$ and vector potential $a = q_a A_y/\sqrt{m_a T_0}$. Let $\partial_z a = a_{,z}$ be the z-derivative of a. For electrons, $c_e = \sqrt{T_0/m_e}$ and $v = u$.

It is convenient to separate the current j^+ of electrons moving in the forward direction from the absolute value j^- of the current in the backward direction, and define

$$j_a(z) = \frac{j^+(z)+j^-(z)}{2} \quad ; \quad j_h = \frac{j^+(z)-j^-(z)}{2} = \tfrac{1}{2} j \quad (2)$$

Note that $2j_a$ is the total current impinging on a scatterer (analogous to a neutron flux), while j is the net particle current. With λ the mean free path and M the average current propagation from a collision at z' to observation at z without any other collision, by integrating Vlasov equation we got a closed equation[6] for j_a

$$\frac{\partial j_a}{\partial z} + \frac{j_h}{\lambda} = j^B_{,z} + \int_{z_\alpha}^{z_\beta} \frac{dz'}{\lambda} e^{-\frac{|z'-z|}{\lambda}} j_a(z') \frac{\partial M(z,z')}{\partial z} \quad (3)$$

with

$$M = \tfrac{1}{2}\mathrm{erfc}\frac{\bar{a}^2+2\bar{v}}{|\bar{a}|\sqrt{8}} + \tfrac{1}{2}e^{-\bar{v}}\mathrm{erfc}\frac{\bar{a}^2-2\bar{v}}{|\bar{a}|\sqrt{8}} \quad (4)$$

for $\bar{a} \neq 0$, where $\bar{a} = a(z) - a(z')$ and $\bar{v} = v(z) - v(z')$ are the potential differences between z and z'; moreover $M(0,\bar{v}) = \min(1, e^{-\bar{v}})$ and j^B, due to particles injected at boundaries z_α and z_β with speed not sufficient to climb $v(z)$ and $a(z)$, is known and it is negligible when $-z_\alpha = z_\beta \to \infty$. In a pure scatter problem (no absorption), j_h does not depends from z. Density n^+ of the forwardly directed particles is

$$n^+(z) = n_\alpha(z) + \int_{z_\alpha}^{z} ds' e^{-|z'-z|/\lambda} N(\bar{a},\bar{v}) j_a(z') \quad (5)$$

where n_α is the known boundary term; similarly for n^-; here $N(0,\bar{v}) = c_2 e^{-\bar{v}}\mathrm{erfc}\,\Re[(-\bar{v})^{1/2}]$ with $c_2 = \sqrt{\pi/2}$ and

$$N(\bar{a},\bar{v}) = \int_0^\infty dp_z e^{-\bar{v}-(p_z^2/2)}\mathrm{erfc}\frac{\bar{a}^2-p_z^2-2\bar{v}}{|\bar{a}|\sqrt{8}} \quad (6)$$

For a constant magnetic field, we get $\bar{a} = (z-z')/L$ where L is the Larmor radius $\sqrt{m_e T_0}/e|B_x|$. In the source, $\bar{a} \gg \bar{v}$ typically holds for electrons, so that M may be approximated as

$$M(z,z') \cong \exp\{-\tfrac{1}{2}[v(z)-v(z')]\}\,\mathrm{erfc}\frac{|z'-z|}{\sqrt{8}L} \quad (7)$$

Defining $j_b(z) = e^{v(z)/2} j_a(z)$ and $m_b = e^{\bar{v}/2} M_{,z}$ we have

$$\frac{\partial j_b}{\partial z} + \frac{j_h e^{v(z)/2}}{\lambda} = \int_{z_\alpha}^{z_\beta} \frac{dz'}{\lambda} e^{-\frac{|z'-z|}{\lambda}} m_b(z,z') j_b(z') \quad (8)$$

$$m_b = -\tfrac{1}{2} v_{,z}\,\mathrm{erfc}\frac{|z''|}{L\sqrt{8}} + e^{-(z''/L)^2/8}\frac{\mathrm{sign}(z'')}{\sqrt{2\pi}L} \quad (9)$$

with $z'' = z' - z$. Most remarkably, when $v_{,z}$ is a rational function of z, thanks to the m_b form, the integrodifferential eq. 8 can be converted to an ordinary differential equation for $j_F(k) = \mathcal{F}\, j_b(z)$ with $\mathcal{F}$ the Fourier transform. For example, with a classical barrier $v = -p_2 z^2$ with $p_2 > 0$, we get

$$\frac{j_h}{\lambda}\frac{e^{-k^2/2p_2}}{p_2^{1/2}} = (ik + W_2^F) j_F + p_2 i\partial_k (1 - W_1^F) j_F \quad (10)$$

which is exactly solvable; here

$$W_1^F(k) = \int_{-\infty}^{\infty} \frac{dz''}{\lambda} e^{-\frac{|z''|}{\lambda}}\mathrm{erfc}\frac{|z''|}{L\sqrt{8}} \quad (11)$$

$$W_2^F(k) = \int_{-\infty}^{\infty} \frac{dz''}{\lambda} e^{-\frac{|z''|}{\lambda}} e^{-(z''/L)^2/8}\frac{\mathrm{sign}(z'')}{\sqrt{2\pi}L} \quad (12)$$

Details of solution are omitted for brevity. In the example $v(z) = v_1 z$ we also have the translational symmetry since E_z is uniform, and Fourier transformed equation is

$$j_h\sqrt{2\pi}\,\delta(k - \tfrac{i}{2}v_1) = D_F(k) j_F(k) \quad (13)$$

with $D_F = \lambda\{ik + W_2^F + \tfrac{1}{2}v_1(1-W_1^F)\}$. Its general solution and inversion of $\mathcal{F}$ give $j_b(z) = \sum_{n=0}^{3} c_n \exp(-ik_n z)$ where $n = 0$ is the inhomogeneous term, that is $k_0 = \frac{i}{2}v_1$ and $c_0 = j_h/D_F(\frac{i}{2}v_1)$. The other k_n are the roots of $D_F(k) = 0$; we are able to prove that no solution is nonzero and real and we found 3 solutions on the imaginary axis. In detail, $k_2, k_3 \cong \pm i/L$, so they correspond to very sharply decaying modes, which require huge input currents to be maintained, so we drop them here. The solution k_1 happens to be of v_1 order, and can be computed from Taylor expansions $W_1 = W_{10} + O(k^2)$ and $W_2 = W_{21}k + O(k^3)$; we get

$$\begin{aligned} k_1 &= \tfrac{i}{2}v_1(1-W_{10})/(1-iW_{21}) \\ W_{21} &= -ix\sqrt{32/\pi} + 8ix^2 e^{2x^2}\mathrm{erfc}(\sqrt{2}x) \\ W_{10} &= 2 - 2x^2 e^{2x^2}\mathrm{erfc}(\sqrt{2}x) \end{aligned} \quad (14)$$

with $x = L/\lambda$. Final solution is

$$j_a(z) = \frac{2j_h/\lambda}{v_1(iW_{21} - W_{10})} + c_1 e^{s_1 z} \quad (15)$$

with c_1 an integration constant and $s_1 = -\frac{1}{2}v_1 - ik_1 = v_1 R_1$ with the ratio $R_1 = \frac{1}{2}(W_{10} - iW_{21})/(1 - iW_{21})$. This ratio is clearly the modification to Maxwell density distribution in this transport problem. Moreover note that when $c_1 = 0$ we have an uniform plasma subjected to a constant electric field, so that results can be compared to Fick law (usual diffusion theory) $j = -q_e N_e \mu E_z$ with μ the mobility coefficient (positive, non scaled, in m2/(Vs) units). Now $j = 2j_h$ and, thanks to eq. 5, $n = n^+ + n^- = R_{n/j} j_a$ (in scaled variables), where $R_{n/j} \cong \sqrt{2\pi}$ is a fixed number. We get $\mu = e\lambda[(W_{10} - iW_{21})/R_{n/j}]/(m_e c_e)$ where the square bracket factor is due to magnetic field.

A PRESHEATH-SHEATH MODEL

To begin with, we here consider electron extraction from a e-H^+ plasma. The Poisson equation becomes

$$\lambda_D^2 u_{,zz} = n_{H^+} - n_e\,, \quad \lambda_D = (\varepsilon_0 T_0/e^2 N_0)^{1/2} \quad (16)$$

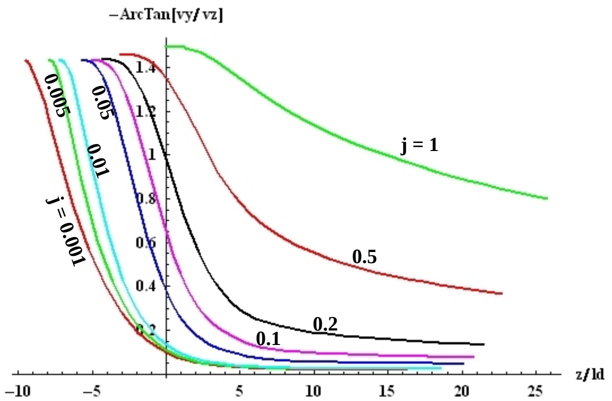

Figure 1: The angle $-\tan^{-1}(v_y/v_z)$ vs z/λ_D, for $j = 0.001$ to 1; for each curve, $z = 0$ is where $v_z = 1$ (in scaled units), that is $v_z = c_e$. Here $\lambda_D = 14\mu$m

where $n = N/N_0$ are scaled densities. Even if a transport model is appropriate also for the confined protons[7], we approximate it with $n_{H^+} = n_0 e^u$ as usual, where $n_0 \cong 1$, and we set up a fluid model for electrons, still approximately retaining the effect of backward and forward currents.

In scaled units for velocity v, the motion fluid equations for $\mathbf{v}^F = <\mathbf{v}>$ are

$$v_z v_{z,z} + (n_{e,z}/n_e) = -u_{,z} - (v_y/L) - (\nu_e^m/c_e)v_z$$
$$v_z v_{y,z} = +(v_z/L) - (\nu_e^m/c_e)v_y \qquad (17)$$

where the F superscript of v is dropped as usual and the collision frequency ν^m may depend on electron speed $v = |\mathbf{v}|$.

Let us assume that collision cross sections are $\sigma = \sigma_n v_R^{-n}$ where v_R is the relative velocity (and here n is an index); $n = n^c \cong 3.9$ for Coulomb collision with H^+; for collisions with H_2 molecules, $n = 1$ is a fair fit of experimental data from 0.5 to 10 eV[9]. The collision frequency is $< v_R\sigma > N_s$ where N_s is the scatterer density, so that

$$\nu_e^m \equiv k_g g(v) + k_c h(v) = k_g + \frac{k_c n_{H^+}}{(c_s^2 + c_e^2 + v^2)^{\alpha_c}} \qquad (18)$$

where the gas term is $k_g = N_g\sigma_1$ and in the Coulomb collision term $k_c = N_0\sigma_{n^c}$ with $\alpha_c = (n_c - 1)/2 = 1.45$. From collision data, $\sigma_1 = 1.01 \times 10^{-13}$ m^3/s and $\sigma_{n^c} c_e^{1-n_c} = 1.25 \times 10^{-10}$ m^3/s. We take $N_0 = 3.3 \times 10^{17}$ m^{-3} and $N_g = 7.7 \times 10^{19}$ m^{-3} for a typical NIS.

Since no absorption of electrons (or ionization) is considered (as in the previous section), we do not need the particle balance equation discussed elsewhere[7] and simply take j_z as a parameter j. Note that using $n_e = j_z/v_z$ to close eqs. (16-17) is incorrect, since backward and forward directed particles sums in n_e , but subtracts in j_z.

To close eqs. (16-17), we need a robust relation between n_e, j and v_z^F: consider a distribution $f(v_z)$ of v_z with variance 1 and mean $v_f = v_z^F$, that is

$$f(v_z) = n_e \exp[-(v_z - v_f)^2/(2c_e^2)]/c_e\sqrt{2\pi} \qquad (19)$$

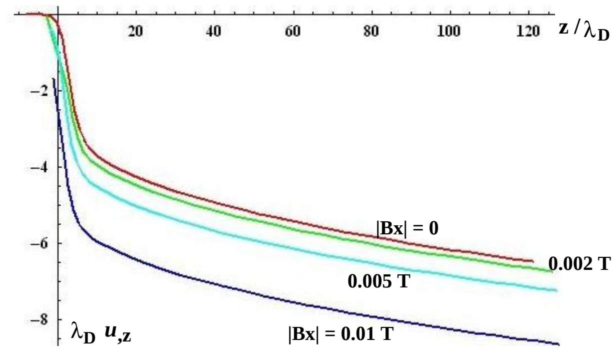

Figure 2: The (scaled) electric field $\lambda_D u_{,z}$ for $j = 0.5$ and several B_x; others conditions as in fig 1, but $N_g = 0$.

computing n^+ and j^+ by integration on $v_z > 0$, we get

$$n^\pm/j^\pm \cong -\tfrac{1}{2}\left\{\pm v_f - \sqrt{v_f^2 + 4}\right\} \equiv 1/v^\pm(v_f) \qquad (20)$$

which we take as the definition of $v^\pm$. Rearranging, and remembering that $j^+ = j^- + j$ we get

$$n_e = \frac{j^-}{v^-} + \frac{j^+}{v^+} = j^-\sqrt{v_f^2+4} + \tfrac{1}{2}\left\{\sqrt{v_f^2+4} - v_f\right\}j \qquad (21)$$

Since backscattering decrease with v_f, we also estimate $j^- = k_j \exp(-v_f^2/8)$.

Starting conditions at $z = z_{st}$ must be suited to represent a point in the quasineutral plasma. Some conditions are obvious: for example $n_0 = 1$ and $u(z_{st}) = 0$, so that n_{H^+}=1; and k_j is adjusted so that $n_e(z_{st}) = 1$; we also set $v_z(z_{st}) = j$. Other conditions are chosen to avoid oscillations: $v_y(z_{st}) = c_e v_z/L\nu_e^m$ and $u_{,z} = -v_z[(\nu_e^m/c_e) + (c_e/\nu_e^m L^2)]$; in other words, RHS of eq. 17 be zero at start.

Some result from simulation is shown in fig. 1, for $B_x = -20$ G, $T_0 = 1$ eV and several values of (scaled) j, from 0.001 to 1; a value $j = 0.4835$ will correspond to a positive ion unmagnetized sheath. The angle between electron beam and extraction field is large (as an effect of magnetic field), especially at extraction. An additional amount of extraction field, proportional to the applied magnetic field seems also necessary from fig 2.

REFERENCES

[1] M. P. Stockli et al., Rev. Sci. Instrum., **81**, 02A729 (2010)

[2] P. Sonato et al., Fusion Eng. Des. **84**, 269 (2009).

[3] M. S. Benilov, Plasma Sources Sci. Technol. **18**, 014005 (2009), and references herewithin

[4] L. Tonks, I. Langmuir, *Phys. Rev.*, **34**, 826, (1929).

[5] S. A. Self, Phys. Fluids, **6** 1762 (1963)

[6] M. Cavenago, Rev. Sci. Instrum., **79** 02B709 (2008)

[7] M. Cavenago, Rev. Sci. Instrum., **81**, 02B501 (2010), and references herewithin

[8] D. Wunderlich et al., Plasma Sources Sci. Technol. **18** 045031 (2009)

[9] http://www-amdis.iaea.org/ALADDIN/