SYNCHRO-BETA RESONANCE SIMULATION USING MEASURED CHROMATIC ABERRATIONS

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Abstract

Synchro-beta enhance resonances beam sizes dynamically. For accelerators aimed for high luminosity, the effect can be more serious since a difference between vertical emittance and longitudinal emittance tends to be larger. Therefore, it is necessary to estimate a tune spread of the synchro-beta resonances properly. Synchro-beta effect is caused by chromatic aberrations, which characterize how linear optics parameters, including tune, Twiss parameter, x-y coupling parameter, and other parameters, depend on the momentum deviation. The chromatic aberrations are defined by coefficients of an optics parameter expanded in terms of momentum deviation. The synchro-beta resonances caused by chromatic aberrations are discussed in this proceeding. We use 6-dimensional symplectic map, which is obtained from measured linear optics parameters, in order to simulate realistic beam motion.

INTRODUCTION

Lattice design codes like SAD [1] and MAD [2] calculate tune, twiss parameters and chromaticities. However the design values sometimes differ from measurements. This is because machine errors change chromatic aberrations. The discrepancies are corrected by introducing fudge factors in the magnet strength for example. However introduction of the factors do not ensure realistic simulation. In such case, it is better to construct an accelerator model with measured chromatic aberrations [3,4].

Chromatic aberrations sometimes play an important role in the beam-beam, space charge, electron cloud, and impedance phenomena. Symplectic expression for chromatic aberrations, which is implemented in computer program codes for studying the above-mentioned phenomena, makes it possible to study their effects directly. The synchro-beta resonance was studied to demonstrate the utility of the symplectic expression.

PARAMETRIZATION

Momentum deviation is $\delta = (p - p_0)/p_0$ normalized by reference momentum p_0 . As is well known, δ varieties slowly compare to betatron variables whose form is $\mathbf{x}_{\beta} = (x_{\beta}, p_{x,\beta}, y_{\beta}, p_{y,\beta})$, and it changes only in RF cavity. The transverse coordinate of beam particle, $\mathbf{x} = (x, p_x, y, p_y)$, and the longitudinal coordinate *z* are expressed by

$$\mathbf{x} = \mathbf{x}_{\beta} + \vec{\eta}(\delta)\delta \tag{1}$$

$$z = z_{\beta} - \eta'_{x}(\delta)x_{\beta} + \eta_{x}(\delta)p_{x\beta} - \eta'_{y}(\delta)y_{\beta} + \eta_{y}(\delta)p_{y\beta}$$
(2)

where the orbit distortion is characterized by the dispersion $\vec{\eta}(\delta) = (\eta_x(\delta), \eta'_x(\delta), \eta_y(\delta), \eta'_y(\delta))$, which is one of linear optics parameters. This transformation is represented by a 6 by 6 matrix $R_n(\delta)$ as follows:

$$(x, p_x, y, p_y, z, \delta) = R_{\eta}(\delta)(x, p_x, y, p_y, z, \delta)_{\beta}$$
(3)

 $R_{\eta}(\delta)$ partially diagonalizes $M_{6}(\delta)$, which is transfer matrix in a revolution, into 4 by 4 and 2 by 2 matrixes $M_{4}(\delta)$ and $M_{z}(\delta)$, respectively.

$$M_6(\delta) = R_\eta(\delta) M_{4\times 2}(\delta) R_\eta^{-1}(\delta) \tag{4}$$

where

$$M_{4\times 2}(\delta) = \begin{pmatrix} M_4(\delta) & 0\\ 0 & M_z \end{pmatrix}$$
(5)

 $M_4(\delta)$ and M_z represent the betatron and synchrotron motions, respectively. Definitions of the linear optics parameters $v_{xy}, \alpha_{xy}, \beta_{yy}, r_i (i = 1 \sim 4)$ are as follows [5,6]

$$M_{4}(\delta) = R(\delta)M_{2\times 2}(\delta)R^{-1}(\delta)$$
(6)

where the 4 by 4 matrix $M_{2\times 2}(\delta)$ is partially diagonalized as follows:

$$M_{2\times2}(\delta) = \begin{pmatrix} M_x(\delta) & 0\\ 0 & M_y(\delta) \end{pmatrix}$$
(7)

$$M_{i}(\delta) = \begin{pmatrix} \cos\mu_{i}(\delta) + \alpha_{i}(\delta)\sin\mu_{i}(\delta) & \beta_{i}(\delta)\sin\mu_{i}(\delta) \\ -\gamma_{i}(\delta)\sin\mu_{i}(\delta) & \cos\mu_{i}(\delta) - \alpha_{i}(\delta)\sin\mu_{i}(\delta) \end{pmatrix}$$
(8)

$$M_{z} = \begin{pmatrix} \cos \mu_{z} + \alpha_{z} \sin \mu_{z} & \beta_{z} \sin \mu_{z} \\ -\gamma_{z} \sin \mu_{z} & \cos \mu_{z} - \alpha_{z} \sin \mu_{z} \end{pmatrix}$$
(9)

where i = x, y. α_z, β_z, v_z are determined from momentum compaction factor, length of ring, accelerated gradient, and energy. $R(\delta)$, which characterises x-y coupling, is parameterized by,

$$R(\delta) = \begin{pmatrix} r_0(\delta)I_2 & -S_2R_2^t(\delta)S_2 \\ -R_2(\delta) & r_0(\delta)I_2 \end{pmatrix}$$
(10)

$$R_2(\delta) = \begin{pmatrix} r_1(\delta) & r_2(\delta) \\ r_3(\delta) & r_4(\delta) \end{pmatrix} \quad S_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
(11)

where $r_0(\delta) = \sqrt{1 - \det(R_2(\delta))}$ and I_2 is 2 by 2 unit matrix. These linear optics parameters is expanded in terms of δ as follows:

$$\nu(\delta) = \sum_{n=0}^{\infty} \nu_n \delta^n, \beta(\delta) = \sum_{n=0}^{\infty} \beta_n \delta^n$$

$$\alpha(\delta) = \sum_{n=0}^{\infty} \alpha_n \delta^n, r_i(\delta) = \sum_{n=0}^{\infty} r_{i,n} \delta^n$$
(12)

SYMPLECTIC EXPRESSION

Notice that 6-dimensional matrix $M_{4\times 2}(\delta)$ satisfies symplectic condition only when δ equal zero. In order to make 6-dimensional symplectic map for non-zero δ ,

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following Hamiltonian (Generating function [7]) is supposed.

$$\begin{aligned} H_{I}(x,\bar{p}_{x},y,\bar{p}_{y},\bar{\delta}) &= \sum_{n=1}^{n} (a_{n}x^{2}+2b_{n}x\bar{p}_{x}+c_{n}\bar{p}_{n}^{2}+2d_{n}xy \\ &+2e_{n}x\bar{p}_{y}+2f_{n}y\bar{p}_{x}+2g_{n}\bar{p}_{x}\bar{p}_{y}+u_{n}y^{2}+2v_{n}y\bar{p}_{y}+w_{n}\bar{p}_{y}^{2})\bar{\delta}^{n}/2 \\ &= (Ax^{2}+2Bx\bar{p}_{x}+C\bar{p}_{n}^{2}+2Dxy+2Ex\bar{p}_{y} \\ &+2Fy\bar{p}_{x}+2G\bar{p}_{x}\bar{p}_{y}+Uy^{2}+2Vy\bar{p}_{y}+W\bar{p}_{y}^{2})/2 \end{aligned}$$
(13)

The upper bar of x, p_x and others means those after transformation. The Hamiltonian gives particle coordinates following transformation,

 $\overline{x} = x + Bx + C\overline{p}_x + Fy + G\overline{p}_y$

$$p_{x} = \overline{p}_{x} + Ax + B\overline{p}_{x} + Dy + E\overline{p}_{y}$$

$$\overline{y} = y + Vy + W\overline{p}_{y} + Ex + G\overline{p}_{x}$$

$$p_{y} = \overline{p}_{y} + Uy + V\overline{p}_{y} + Dx + F\overline{p}_{x}$$

$$2H$$

$$(14)$$

$$\overline{z} = z + \frac{\partial H_I}{\partial \overline{\delta}}$$
(15)

Then, the transformation of transverse coordinates and momentums is represented by $M_H(\delta)$,

$$\begin{pmatrix} \bar{x} \\ \bar{p}_{x} \\ \bar{y} \\ \bar{y}_{p_{y}} \end{pmatrix} = M_{H}(\delta) \begin{pmatrix} x \\ p_{x} \\ y \\ p_{y} \end{pmatrix}$$
(16)

Relations of $a_n, ..., w_n$ and α_{xn} , $\beta_{x,n}$, $\mu_{x,n}, \alpha_{y,n}$, $\beta_{y,n}$, $\mu_{y,n}$, $r_{i,n}$, $i = 1 \sim 4$ are obtained by comparison of the transfer matrices written by the chromaticities and by $M_H(\delta)$,

$$M_4(\delta) = M_4(0)M_H(\delta) \tag{17}$$

Then, the coefficients $a_n,...,w_n$, which are described by linear optics parameters, are derived from

$$M_H(\delta) = M^{-1}(0)M(\delta)$$
(18)

Since linear optics parameters are measureable, $a_n,...,w_n$ are numerically determined. Therefore, a one-turn map is obtained by

$$\mathcal{M}_{4\times 2}(0) \circ e^{-:H_I:} \circ \mathbf{x} \tag{19}$$

where $\mathbf{x} = (x, p_x, y, p_y, z, \delta)$.

MACHINE ERROR

There are many kinds of machine error. Some of them affect chromatic aberrations, for example, edge effect error or gradient errors of quadrupoles or position errors of sextupoles. It is known that the gradient errors of quadrupoles are very small. Here, position errors of sextupoles are treated. By assuming random position errors of sextupoles, effect of the machine errors on chromatic aberrations can be estimated. Amplitude and seed of errors are chosen so that emittance coupling ($\varepsilon_v / \varepsilon_x$) is 1% after optics correction.

While, chromatic aberrations of the linear optics parameters at the interaction point are measured using a turn-by-turn monitor by changing the energy of the beam

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[8]. The linear optics parameters have been measured in a single bunch operation, and not in collision in KEKB.

In Figure 1, red points show measured linear optics parameters r_3, r_4 , which depend on momentum deviation. While, green dashed line describes average of the simulation results used 1000 kinds of random position errors. Blue dashed line represents standard deviation of them.

Since measurement of absolute value (value at $\delta = 0$) is difficult, difference of it between the measurement and the simulation is not important. However difference of their shape is not negligible. Especially, higher order of δ is not seen in simulation results. It is very difficult to match measurement data to simulation results.



 r_3 and r_4 .

SYNCHRO-BETA RESONANCE

The chromatic aberrations cause synchro-beta resonances, because the Hamiltonian contains quadratic terms of transverse coordinate and power series of momentum deviation. The synchro-beta resonances are studied using the symplectic map expressed by the Hamiltonian. A multi-particle tracking code is developed to simulate the synchro-beta resonance.

1000 macro-particles are initialized with a Gaussian distribution in which the initial size is given by the emittance and linear optics parameters, where the horizontal, vertical, and longitudinal emittances are $\varepsilon_x = 1.8 \times 10^{-8} m$, $\varepsilon_y = 1.8 \times 10^{-10} m$, and $\varepsilon_z = 4.9 \times 10^{-6} m$, respectively. The macro-particles are tracked with

radiation damping (4000/2000 turns in transverse/ longitudinal) and excitation. The equilibrium beam sizes in the horizontal and vertical planes are obtained by an average of the particle coordinates after 30,000 turns.

The simulation is performed for scanning the horizontal tune with a fixed vertical tune $v_y = 0.588$ and longitudinal tune $v_z = 0.021$, where the coefficients of Hamiltonian are kept constant in the tune scan. We also assume that 0-th order coupling parameters are zero because they are optimized by a day-by-day operation in KEKB.

Figure 2 shows the tune scan of the normalized beam sizes with a step of $\Delta v_x = 1.4 \times 10^{-4}$. The normalized beam sizes mean beam sizes divided by $\sigma_0 = \sqrt{\epsilon\beta(0)}$. Dispersion is not taken into account in this simulation. This simulation is treated up to the 3rd order. Several peaks in the beam sizes are observed in the figure.



Figure 2: Horizontal (blue) and vertical (red) beam sizes in the tune space on the simulation. ($v_y = 0.588$)

The beam sizes are measured with a synchrotron light monitor using an interferometer [9]. Figure 3 shows the beam size measurement performed in KEKB on May 16, 2008. Several peaks in the beam sizes observed in the figure are similar to those found in the simulation. The measurement was done for various v_y . The peaks shifted with a change in v_y . The peaks correspond to x-y



Figure 3: Horizontal (blue) and vertical (red) beam sizes in the tune space on the measurement. ($v_v = 0.6$)

coupling and its synchrotron side bands. The behaviour of the horizontal and vertical sizes at the synchrotron side band resonance of x-y coupling agrees with the simulation; that is, the vertical size increases while the horizontal size decreases in the measurement.

SUMMARY

The synchro-beta resonance was studied to demonstrate the utility of the symplectic expression. A symplectic expression is obtained for chromatic aberrations of linear optics parameters up to 3rd order in momentum deviation, which is given by performing measurements using turnby-turn position monitors. Multi-particle tracking simulation using the symplectic expression was compared with a beam size measurement in the tune space. The simulation results qualitatively agree with the measurement in the resonance behavior, while some discrepancies are seen in stop band widths quantitatively.

This symplectic map can be used for six-dimensional particle tracking simulations to study the synchro-beta resonance, beam-beam, space charge, impedance effects, and so on, which are influenced by chromatic aberrations.

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