SIMPLIFIED APPROACH TO EVALUATION OF BEAM-BEAM TUNE SPREAD COMPRESSION BY ELECTRON LENS *

A.L. Romanov, BINP SB RAS, Novosibirsk, Russia
A.A. Valishev, V. Shiltsev, FNAL, Batavia, IL 60510

Abstract

One of the possible ways to increase luminosity of hadron colliders is the compensation of beam-beam tune-spread with an electron lens (EL). At the same time, EL as an additional nonlinear element in the lattice can increase strength of nonlinear resonances so that its overall effect on the beam lifetime will be negative. Time-consuming numerical simulations are often used to study the effects of the EL. In this report we present a simplified model, which uses analytical formulae derived for certain electron beam profiles. Based on these equations the idealized shapes of the compressed tune spread can be rapidly calculated. Obtained footprints were benchmarked against several reference numerical simulations for the Tevatron in order to evaluate the selected configurations. One of the tested criteria was the so-called "folding" of the compensated footprint, which occurs when particles with different betatron amplitudes have the same tune shift. Also studied were the effects of imperfections, including misalignment of the electron and proton beams, and mismatch of their shapes.

ANALYTICAL CALCULATION OF FOOTPRINT

Introduction

Beam-beam effects, space charge and nonlinear elements cause betatron tunes of particles in a circulating beam to be different. The variety of these tunes form the tune spread (footprint).

To avoid a loss of particle due to chaotic drift, its tunes should be located away from harmful resonances. Beam intensities are often limited by the maximum size of the footprint that can be fitted between resonances.

The full footprint of all particles in the ring is formed by a superposition of individual footprints of bunches. Thus, bunch by bunch tune shift variations will lead to an effective increase of the tune spread.

There are two ways in which electron lenses can be used to compress the footprint. First, they allow to eliminate the bunch by bunch tune shift variations using the fast electron current modulation. Second, they allow to compress the individual footprints of each bunch.

Let us consider two types of the electron beam profile (Fig. 1). One is the Gaussian profile $\rho(r) = \rho_0 \exp(-r^2/2r_0^2)$, that is best for footprint compression of the individual bunch. The other is the distribution with the smooth edges and flat top (SEFT) $\rho(r) = \rho_0/(1+(r/r_0)^4)$, that produces mostly tune shift.

* Work supported by the Fermi Research Alliance, LLC under Contract DE-AC02-07CH11359 with the United States Department of Energy, and by the DOE through the US LHC Accelerator Research Program (LARP).

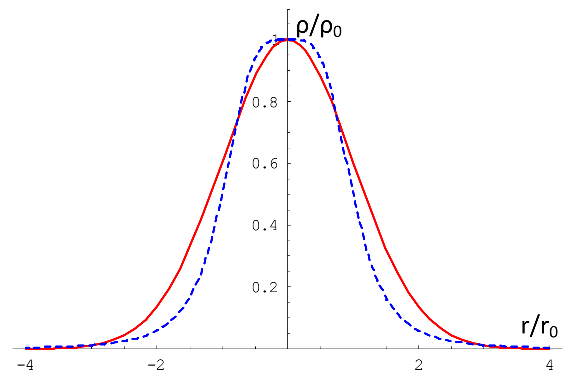

Figure 1: Gaussian profile (red, solid) and SEFT profile (blue, dashed)

Tune shifts from various beam profiles

For simplicity, we will not take into account the variation of the electrons' velocity with transverse coordinate. Let us consider an electron lens with length L_{EL}, charge density in the center ρ_0, electron velocity $c\beta_e$, placed in accelerator at the point with beta functions $\beta_{x,y}$. In the case of short EL the tune shift of the particle will be:

$$\Delta\nu_z(x,y) = \frac{\beta_e+1}{4\pi}\frac{\beta_z L_{EL} G_0}{\gamma H_p r_p}\frac{G(x,y)}{G_0}, \qquad (1)$$

here $H_p = e/r_p^2 = 2.036{\cdot}10^{22} Gs$, $r_p = 1.535{\cdot}10^{-16} cm$, z represents either x or y and $\tilde{z}$ represents y or x respectively.

The equation for electric field in the axially symmetric case is:

$$E_z(x,y) = \frac{2z}{x^2+y^2}\int_0^{\sqrt{x^2+y^2}} 2\pi R\rho(R)\,dR \qquad (2)$$

To get the gradients one should take respective derivatives:

$$G_z(x,y) = \frac{2(\tilde{z}^2 - z^2)}{(x^2+y^2)^2}\int_0^{\sqrt{x^2+y^2}} 2\pi R\rho(R)\,dR + \frac{4\pi z^2}{x^2+y^2}\rho(\sqrt{x^2+y^2}) \qquad (3)$$

In the case of the Gaussian electron beam profile the electric field gradient is:

$$G_z(x,y) = 4\pi\rho_0 \left[\frac{\tilde{z}^2 - z^2}{x^2+y^2} \frac{1-\exp(-(x^2+y^2)/2r_0^2)}{(x^2+y^2)/r_0^2} + \right. \\ \left. + \frac{z^2}{x^2+y^2} \exp(-(x^2+y^2)/2r_0^2) \right] \tag{4}$$

In the case of the SEFT electron beam profile the electric field gradient is:

$$G_z(x,y) = 4\pi\rho_0 \left[\frac{\tilde{z}^2 - z^2}{x^2+y^2} \frac{\arctan((x^2+y^2)/r_0^2)}{(x^2+y^2)/r_0^2} + \right. \\ \left. + \frac{z^2}{x^2+y^2} \frac{1}{1+(x^2+y^2)^2/r_0^4} \right] \tag{5}$$

It is also necessary to take into account the magnetic field. This adds the coefficient $(1 \pm \beta_e)$ depending on the direction of the particle velocities.

To obtain the tune spread generated by the EL, one should convolve the equations for G with the particle distribution in the treated bunch. Fig. 2 illustrates footprints from Gaussian and SEFT profiles.

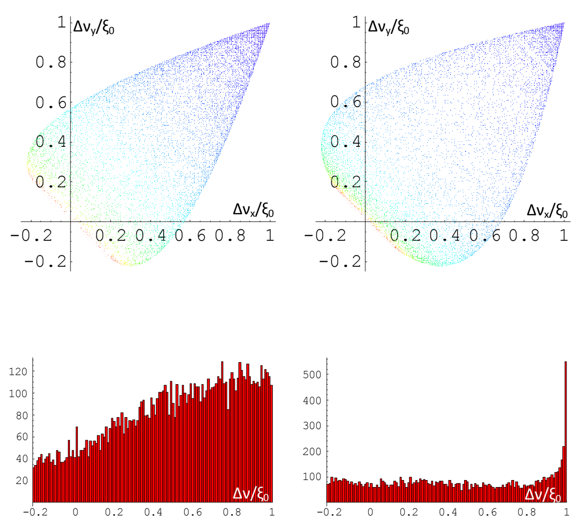

Figure 2: Footprints and histograms for Gaussian (left) and SEFT (right) profiles. Colors represent particle amplitudes, blue to red - from 0 to $3r_0$

Imperfections of EL beam alignment

Since the length of the interaction region in the Tevatron electron lenses (TEL) is much smaller than the beta-function at their locations, we can treat TELs as short elements. In this case to model the imperfect alignment one can split the simulated beam into several longitudinal slices and sum the effects.

To determine the sufficient amount of slices for different levels of distortion, one can compare gradients obtained from the sliced approximation with gradients obtained by exact numerical integration along the electron beam. Fig. 3 shows relative gradient differences for various numbers of slices for inclined electron beam with Gaussian and SEFT profiles:

$$\Delta(x,y) = \frac{1}{G(0,0)} \left[\frac{1}{N_s} \sum_i G(x - x_{s,i}, y - y_{s,i}) - \right. \\ \left. \frac{1}{L_{EL}} \int_0^{L_{EL}} G(x - x_e(s), y - y_e(s)) ds \right] \tag{6}$$

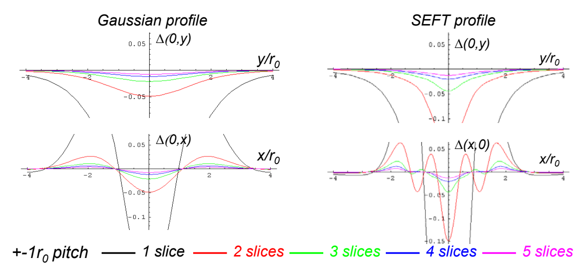

Figure 3: Comparison of the gradients from smooth and sliced beams

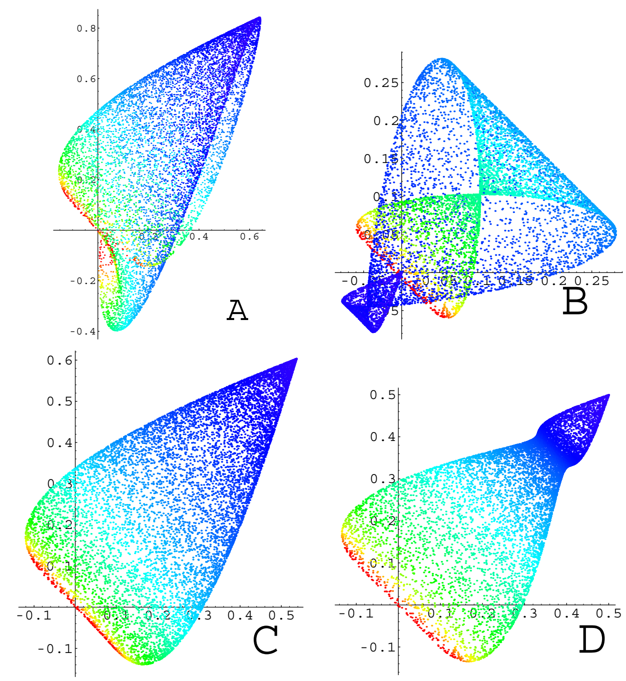

Figure 4: Footprint distortions, initial tunespread generated by a Gaussian beam of equal size. A - Gaussian EL, $\xi_e = 0.5\xi_p$ with $1r_e$ shift; B - SEFT, $\xi_e = \xi_p$, no shift; C - Gaussian EL, $\xi_e = 0.5\xi_p$ with $\pm 1r_e$ tilt; D - SEFT, $\xi_e = 0.5\xi_p$, no shift.

The first approximation for the footprint in the case of combined action of beam-beam interaction and EL can be obtained by the convolution of the tunespread generated by beam-beam with one from the EL. Even such a simple

Table 1: Parameters of the first set of simulations

Tunes in the 1st simulation	$\nu_x = .578 \quad \nu_y = .575$
ξ from protons	0.012
ξ from TEL	0.0, 0.003, 0.006
TEL radius	0.52 mm
antiproton beam size	$\sigma_x = 0.45, \sigma_y = 0.6$
Turns in simulation	10^6

Table 2: Parameters of the second set of simulations

Tunes	$\nu_x = .581 \quad \nu_y = .576$
ξ from protons	0.012
ξ from TEL	0.0, 0.003
TEL radius	0.52 mm
antiproton beam size	$\sigma_x = 0.45, \sigma_y = 0.6$
Turns in simulation	10^6

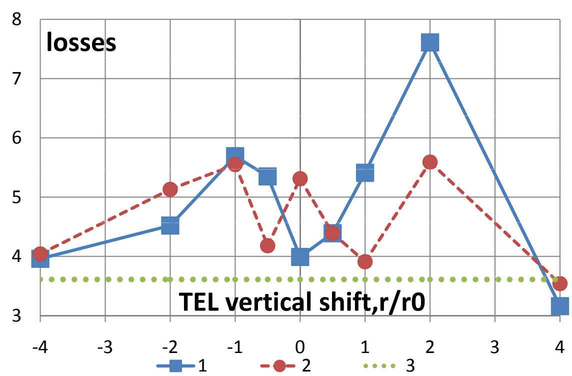

Figure 5: Results of the first set of simulations. Vertical scale is increased by a factor of 10^4, losses normalized by initial beam intensity. "1" corresponds to $\xi_{TEL} = 0.006$, "2" corresponds to $\xi_{TEL} = 0.003$, "3" corresponds to TEL turned off.

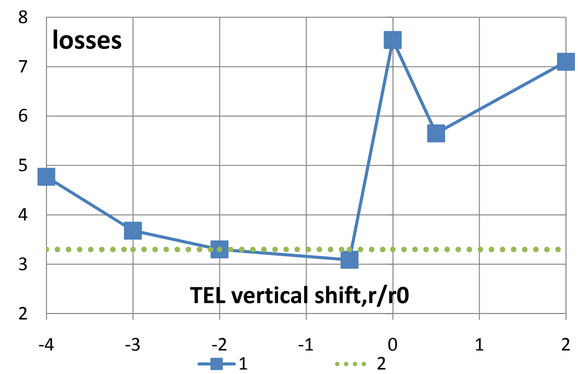

Figure 6: Results of the second set of simulations. Vertical scale is increased by a factor of 10^4, losses normalized to initial beam intensity. "1" corresponds to $\xi_{TEL} = 0.003$, "2" corresponds to TEL turned off.

model can demonstrate some crucial moments of the imperfect alignment of the electron beam along the treated one, such as the "folding" of the footprint, which may cause the lifetime degradation [3]. Figure 4 shows footprint distortions for several conditions.

NUMERICAL SIMULATIONS

Tune spread compensation without folding is very sensitive to the alignment of antiproton and electron beams. If the tune shift of antiprotons from EL is $1/2$ of that from protons, then maintaining electron beam shift of less than $0.3r_{e-beam}$ is critical.

One method for testing the electron beam alignment is based on monitoring the antiproton losses while scanning the vertical and horizontal displacement of the electron beam. If the electron beam is parallel to the antiproton beam, then losses should decrease if beams are aligned perfectly or separated far away. Recent results of such TEL beam studies are presented in [2]

Numerical simulations of the misaligned TEL were performed with the Lifetrac code [4], in which the sliced TEL model was included. The vertical displacement of the TEL was scanned for two Tevatron tune working points.

Figure 5 shows losses during the first set of simulations. Note the double hump shape of curve no. 1, which looks similar to Fig. 2 in [2]. The main details of the first set of simulations are listed in Table 1. Figure 6 gives losses during the second set of simulations. The main details of the second set of simulations are listed in Table 2.

SUMMARY

A semi-analytical method was developed for fast estimation of the footprint distortion under the influence of the misaligned EL with two different types of electron beam profiles. A numerical model of EL beam misalignment was included in the Lifetrac beam-beam simulation code. The numerical simulation of particle losses as a function of vertical TEL beam displacement shows good qualitative agreement with experimental data.

REFERENCES

[1] *V. Shiltsev, V. Danilov, D. Finley, and A. Sery*, Phys. Rev. ST Accel. Beams 2, 071001 (1999).

[2] *A. Valishev et al.*, "Progress with Tevatron Electron Lens Head-on Beam-Beam Compensation", these Proceedings, TUPD070

[3] *Yu. Alexahin et al.*, "Feasibility of the Nonlinear Beam-Beam Compensation at Tevatron", PAC'01, Chicago, 2001, TPPH147

[4] *D. Shatilov et al.*, "Lifetrac Code for the Weak-Strong Simulation of the Beam-Beam Effects in Tevatron", PAC'05, Knoxville, 2005, TPAT084