LOW EMITTANCE LATTICE OPTIMIZATION USING MULTIOBJECTIVE GENETIC ALGORITHM*

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Abstract

Low emittance is a desirable performance for high brightness synchrotron light source and damping ring. The work presented in this paper demonstrates that the lattice of a given electron storage ring, which has fixed circumference and magnet layout, can be optimized to obtain low emittance by using MOGA (Multi-objective Genetic Algorithm). Both dispersion-free and non-dispersion-free lattices of HLS (Hefei Light Source) upgrade project are computed as an illustration. Simulation result shows that this method is fast and straightforward.

INTRODUCTION

Presently, there are many codes for storage ring lattice design, such as MAD, OPA, ELEGANT, etc. The critical problem of designing lattice by these codes is that they will consume the designers lots of time to adjust it repeatedly. Though the designers could get a satisfactory structure, but they couldn't judge whether the ultimate performance is reached.

MOGA is a highly effective randomly searching algorithm, which can globally search a set of solutions over a domain. These solutions are superior to the rest of solutions when considering all of the objectives, which are called Pareto-optimal solutions.

This algorithm was introduced into damping ring [1] and storage ring lattice [2] optimization in recent years. It overcomes the above difficulties of the existing codes and converges to a set of Pareto optimal solutions. Also this algorithm settles the high computational complexity. In this paper we will use a nondominated sorting-based multi-objective evolutionary algorithm (NSGA-II) [3] to optimize linear optics of low emittance lattice.

NSGA-II AND CONSTRAINT HANDLING APPROACH

The Non-dominated Sorting Genetic Algorithm (NSGA) was proposed by Srinivas and Deb in 1994. Later the upgraded algorithm NSGA-II was developed. It overcomes the following difficulties of NSGA: high computational complexity, non-elitist and the need for specifying sharing parameters. The optimization steps of this algorithm are listed as below:

First of all, rank and crowding distance of the initial population p_0 will be computed. And then sort these individuals based on their non domination. Generate a child population $p_0^{'}$ from the initial population, and keep the Pareto optimal solutions q_0.

From the first generation to the maximum generation the flowing loop are repeated.

1. Merge parent population $p_{t\text{-}1}$ and child population $p_{t\text{-}1}^{'}$.
2. Sort the individuals of the merged population based on their rank and crowding distance. In this step the individual of lower rank is better than the higher one. If two individuals have the same rank the one with larger crowding distance is better. The best N individuals of the merged population are chosen to make up parent population p_t.
3. Use tournament selection, crossing, and mutating to generate a child population $p_t^{'}$ from parent population P_t.
4. For the sake of keeping elastic individuals of every generation, an external population q_t is used to memorize the Pareto-optimal front, which is preferred from composite set $q_{t\text{-}1}$ and $p_t^{'}$.
5. Increase the generation counter t=t+1.

In this algorithm constraints are handled without any penalty functions. We define all feasible solutions have a better rank than infeasible solutions, two feasible solutions are sorted based on their objective functions, for both infeasible solutions the better is chosen according to constraint violation (the lower the better), which is calculated from the sum of the every equality constraint and inequality constraint.

LOW EMITTANCE LATTICE OPTIMIZATION

HLSII (Hefei Light Source II) storage ring is composed of DBA structures, which is designed to achieve some desirable beam qualities such as low beam emittance.

The brightness of a light source is defined as equation (1), which is the photon flux per unit solid angle and unit area emitted in a relative bandwidth. In this equation, $d\Omega dS$ is proportional to transverse beam emittance $\varepsilon_x \varepsilon_y$, thus lower beam emittance is preferred to produce higher beam brilliance [4]. In addition, the smaller beam size, the higher brightness will be, thus small betatron function at the insertion device (ID) is expected.

$$B = \frac{d^4 N_{\rm ph}}{dt d\Omega dS (d\lambda / \lambda)} \tag{1}$$

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The horizontal equilibrium emittance for an isomagnetic ring is given by expression (2):

$$\varepsilon_x = \frac{C_q \gamma^2 \langle H \rangle_{dipole}}{J_x \rho} \tag{2}$$

where ρ is the bending radius, γ the total energy in mc^2 units, J_x the horizontal damping partition number, $C_q = 3.83 \times 10^{-13} m$ and $\langle H \rangle = \frac{1}{2\pi\rho} \oint_{dipole} \left(\gamma_x \eta^2 + 2\alpha_x \eta\eta' + \beta_x \eta'^2 \right) ds$.

It is clear that the horizontal emittance is determined by $\langle H \rangle$ function, and designing a low emittance lattice means optimizing dispersion function and β-function in dipole magnets.

HLSII double bend lattice with four families of quadrupoles (Q1, Q2, Q3, Q4) in standard focusing cell is optimized by NSGA-II. The four quadrupole strength parameters are varied to minimize emittance, and their variation range is $-5 < Q_i[k_1] < 5$ respectively. Two objectives are minimized in the non-dispersion-free mode, they are emittance and β_y (in the ID section). There are three objectives to be minimized in the dispersion-free mode, who are emittance, β_y (in the ID section), and dispersion function (in the ID section). Some interested optical parameters should meet the designing constraints. They are listed as follows:

1. $\beta_{x,y}(\max) < 35\text{m}$.
2. $\eta_x(\max) < 1.5\text{m}$.
3. Horizontal tune $Q_x = 4.4$, and permissible error is ± 0.05.
4. Vertical tune $Q_y = 3.2$, and permissible error is ± 0.05.

During the optimization by NSGA-II, all the evolving individuals have to satisfy stable conditions of both planes from initial population to the last generation, because unstable point is no use in storage ring lattice design. This means that the initial population is equivalent to stable points of the lattice. Fig.1 shows these stable points of non-dispersion-free mode onto Q1/Q2 and Q3/Q4 plane.

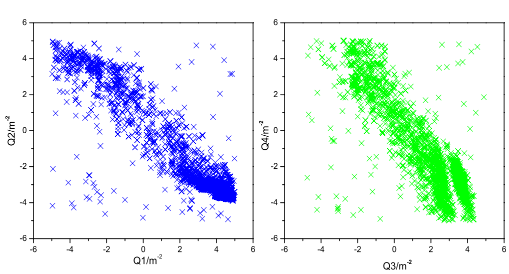

Figure 1: Initial population.

This treatment of stable points will speed up the convergence. Fig.2 gives the convergency (non-dispersion-free mode) of a population with 1500 individuals and a maximum generation 50.

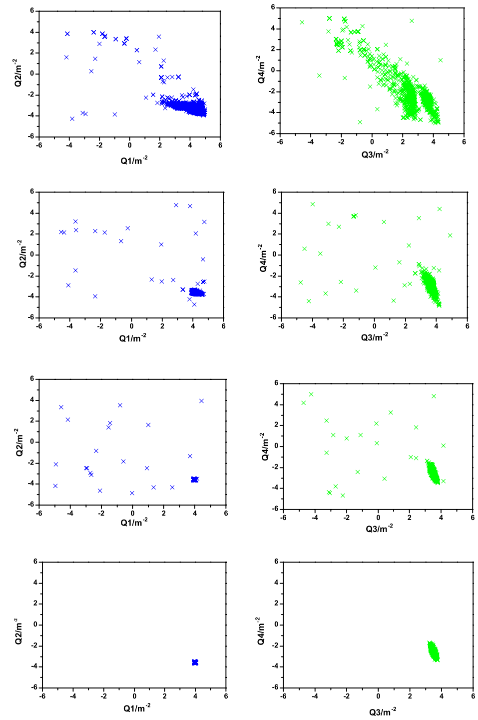

Figure 2: The 5th, 10th , 20th and 50th generation

From the above figure, it is easy to see that the populations converge to the optimal solutions with a quick speed. An appropriate crossover rate p_c and mutational rate p_m is critical for convergence. A large p_c will speed up the convergence but probably a local optimal, while too large p_m will lead to non convergence. In this problem we use a crossover rate 0.60, and mutational rate 0.01. The Pareto-optimal solutions of the last generation are drawn in the objectives space (see Fig.3 and Fig.4).

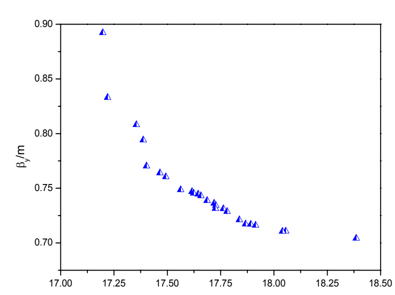

Figure 3: Pareto-optimal front of non-dispersion-free mode.

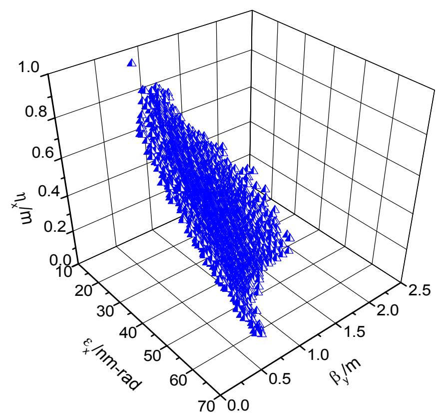

Figure 4: Pareto-optimal front of dispersion-free mode.

Optics of dispersion-free mode and non-dispersion-free mode obtained from MOGA are drawn in Fig.5 and Fig.6 respectively. The emittances of these two modes are about 17nm-rad and 35nm-rad, which is 1.7 and 1.18 times of the theoretical minimum emittance of each structure.

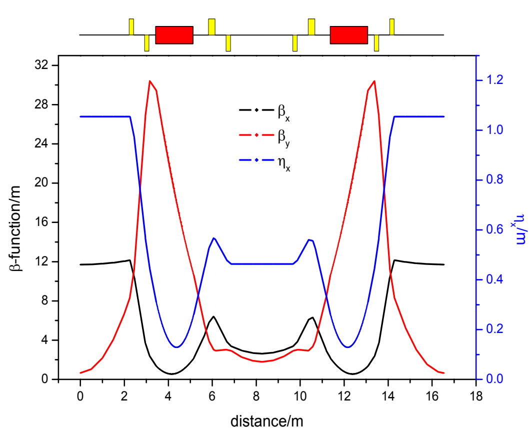

Figure 5: Lattice optics of non-dispersion-free mode.

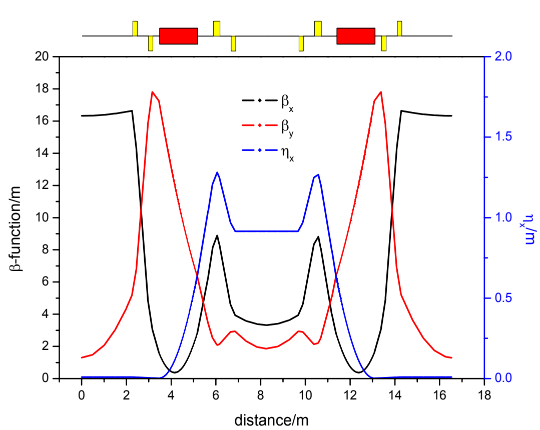

Figure 6: Lattice optics of dispersion-free mode.

The quadrupole strength, which obtained from the last generation of these two modes, are compared. From the variable space Q1/Q2 and Q3/Q4, we can see that the quadrupoles strength of non-dispersion-free mode is a subset of dispersion-free mode. This is easy to understand that dispersion-free mode has three objectives, and two of them are the same as non-dispersion-free mode. If the dispersion function in the long straight section is not cared, it becomes the non-dispersion-free mode. This suggests that we can transform these two modes by changing quadrupole Q3 and Q4 smoothly.

CONCLUSION

This paper uses a multi-objective genetic algorithm (NSGA-II) to optimize lattice optics of storage ring. Two examples with four variables, four constraints and some objectives (two objectives in non-dispersion-free mode, three objectives in dispersion-free mode) are optimized. The result given by this algorithm is a set of solutions, which satisfy all of the constraints and trade off between each of the optimizing objectives. Therefore we get a lattice which satisfies all of the constraints and has a best beam performance. Since the excellence property of MOGA, such as high computational capability, no need for understanding the solving problem, robust and inherently parallel, we can extend its application to further nonlinear optimization problem in accelerator.

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