

DISPERSION ENGINEERING AND DISORDER IN PHOTONIC CRYSTALS

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Abstract

The possibility of achieving higher accelerating gradients at higher frequencies with the reduction of the effect of HOMs, compared to conventional accelerating structures, is increasing interest in the possible use of Photonic Crystals (PC) for accelerator applications. In this paper we analyze how the properties of the lattice of a PC resonator can be engineered to give a specific band structure, and how by tailoring the properties of the lattice specific EM modes can either be confined or moved into the propagation band of the PC. We further go on to discuss the role of disorder in achieving mode confinement and how this can be used to optimize both the Q and the accelerating gradient of a PC based accelerating structure. We also examine the use of high disorder to give rise to Anderson Localization, which gives rise to exponential localization of an EM mode. Discussing the difference between the extended Bloch wave, which extends over the entire PC, and the Anderson localized mode.

INTRODUCTION

Conventional accelerating technologies suffer from a number of issues which limit performance, such as wakefields, which has created interest in the possible use of Photonic Crystals (PC) for accelerator applications [1]. PC can be described as a periodic array of varying permittivity (ϵ). Propagation of EM waves through this lattice is described by Bloch-Floquet theorem, where for specific lattice configurations band-gaps in the frequencies of EM waves able to propagate appear. PC resonators can be formed by creating a defect in the lattice, excitation of EM fields in the region will confine frequencies that are inside the band-gap of the lattice whilst the rest propagate freely away. Defects can be realized in PC structures by either increasing or reducing the effective refractive index in a localized region of the structure. In this paper, we describe how PC structures can be engineered to confine specific frequencies whilst forcing others to propagate away. Previous studies on designing PC structures have focused on maximising the band gap, where the effects of refractive index contrast, filling factors and different lattice geometries on the size of the band gap have been considered [2].

DISORDER

The most commonly used type of PC in high power applications is the 2-Dimensional (2D) triangular lattice of rods with separation a and radius r , where the central rod is removed to create the defect, forming the resonator. Figure 1 shows the ideal PC structure considered.

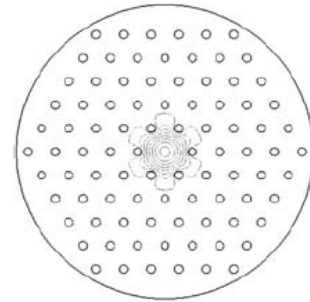


Figure 1: Ideal PC Structure, consisting of Perfectly Electrically Conducting (PEC) rods. Removal of the central rod localises a single EM field at $f_0 = 9.4072$ GHz. rods have radius of 0.00186 m and separation of 0.0124 m. The contours show the extent of the electric field of the mode.

Any physically realizable structure will have some degree of disorder. Previous papers have extensively studied the effect that disorder in PC structures has on transmission and reflection properties (for example [3]). Previous work has focused on disorder in bulk crystals. In this paper we focus on the effect of disorder on the fundamental frequency and the peak electric field confined in a photonic resonator, the investigations are carried out using the methodology of reference [3].

Disorder was introduced to the structure by adding a random number between $\pm 0-15\%$, $0-10\%$, $0-5\%$, or $0-1\%$, of the initial parameters a and r , to each individual rod. Disorder is applied to the position, radius and both a and r of each rod. The random number used was taken from a uniform distribution pseudo-random number generator. This effectively introduces a white noise error to the dimensions of the PBG structure. 300 different configurations of the structure were generated for each level of disorder. Each ensemble was processed to find the resonant frequency and the peak electric field at the centre of the structure, and then averaged to give a mean value of the resonant frequency and peak electric field.

The resonant frequency of each structure was calculated using the commercially available finite-element package COMSOL, to find the eigenmodes of each structure. The peak electric field simulations were performed using a finite-difference time-domain method, with subpixel smoothing for increased accuracy [4]. Each structure was excited with a point source at the defect generating gaussian pulse.

The effects of disorder on the resonant frequency are shown in figure 2. Although the mean resonant frequency remains fairly constant, we note that for some structures increasing disorder causes the resonant frequency to significantly deviate from the base value. This behaviour, as explained in [3], is due to the dominate effect of the scatters closest to the defect region.

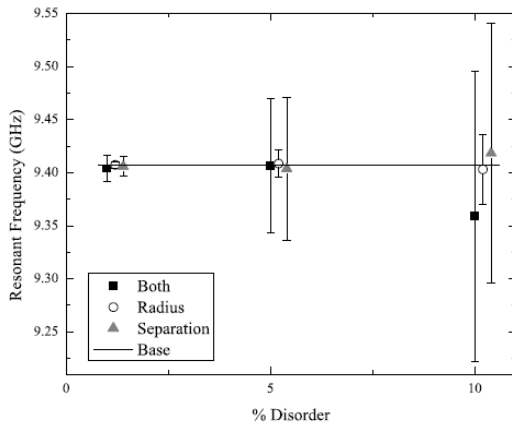


Figure 2: The effect of disorder in radius, separation and both applied to the whole structure. The vertical error bars show one standard deviation from the mean. The horizontal line indicates the base frequency

The effects of disorder on the peak E_z component of the electric field is shown in figure 3. As expected, increasing disorder of the structure causes the mean peak field to decrease. In terms of percentage disorder, separation has a larger effect than radius. While the effect on the peak field of applying disorder to both, can be seen as approximately equal to the sum of the separate variations in position and radius. In terms of absolute variation, we can see that the effect on the peak field is approximately equal in both cases. There are some specific ensembles that produce results showing an increase in the maximum peak electric field. As discussed in [3] this can be understood in terms of the defect volume over which EM energy is distributed. The energy stored in the EM field is given by $(E \cdot D + |H|)/2$, where E and H are the electric and magnetic field components and D the electric displacement field. To maintain a constant energy, a reduction in the volume results in an increase in EM field magnitude.

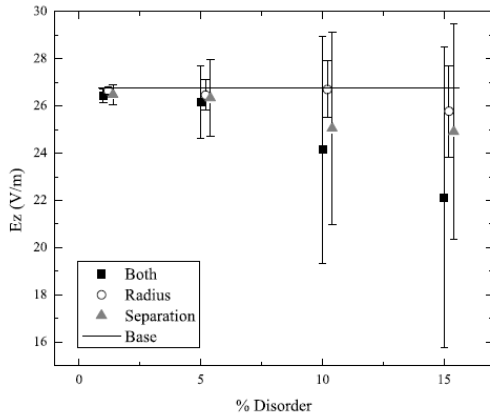


Figure 3: The effect of disorder in radius, separation and both applied to the whole structure, on the peak electric field.

In a perfect lattice waves exist as extended states over the structure (Bloch waves). By introducing disorder, we break coherence, requiring 2nd quantisation to study propagation, treating transport as discrete particles. We

consider the regime where the wavelength (λ_{EM}) is less than the mean free path of a photon in the lattice. In this regime specific ensembles can exponentially localise a specific frequency. As the wave is not an extended state the losses are minimised and the Q maximised by several orders of magnitude. Figure 4 shows experimental and numerical results for an Anderson localisation structure with a $Q > 10^9$. Although the high Q presents difficulty in coupling a field in and out of the region. Conventional techniques either cannot couple or they perturb the system altering the spectrum.

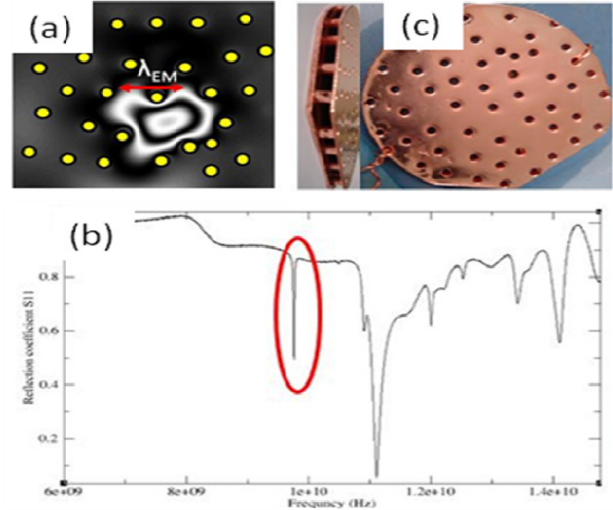


Figure 4: (a) The numerical results showing localisation of a field at 9.412 GHz in a highly disordered structure, (b) Experimental results for the reflection over the structure, the red circle shows the measured resonance predicted in (a) but is shifted due to the perturbation of the probe by 300 MHz (c) physical structure of (a).

DISPERSION ENGINEERING

We now consider how structures can be engineered to confine specific modes whilst other modes propagate away. The dispersion was computed using a Plane Wave Expansion (PWE) technique. The confined field position and frequency within the band gap was computed by the FDTD technique used in the previous section. The PWE was used to perform a multi-parameter scan by varying the dielectric constant of the scatterers while sweeping the ratio from $r/a = 0.1 - 0.3$. As shown in [2] the size of any particular band gap increases with refractive index contrast and decreases with higher values of the ratio r/a . Also, as the ratio r/a increases, higher order band gaps are introduced. For a given r/a , the number of band gaps increases with increasing permittivity.

In Figure 5 ($r/a = 0.2$) five distinct band gaps were found. A single gap was found for $\epsilon = 3$ to 11. Two gaps were found for $\epsilon = 11$ to 18, three gaps were found for $\epsilon = 18$ to 52, four gaps for $\epsilon = 52$ to 63, and five for $\epsilon = 63$ to 65. We found that TM₀₁₀ like modes were confined not just in the first band gap, but also in higher order band gaps.

The third band gap was found to confine both TM010 like and TM011 like modes.

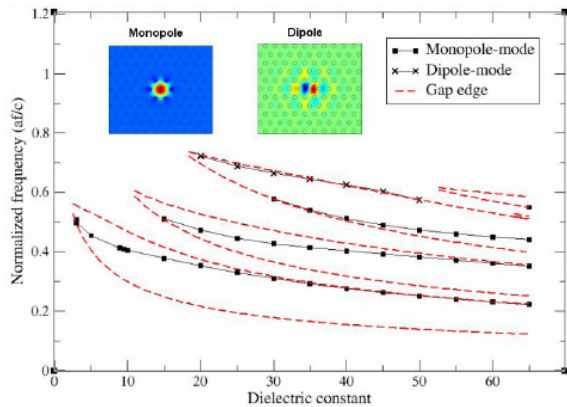


Figure 5: Confined fields ($r/a=0.2$) within the band gaps with varying dielectric constants. The field profiles of the Confined fields are inset.

Considering a structure where a larger defect region was created by removing multiple rods, as shown in figure 6 ($r/a=0.1$), the monopole mode drifts down towards the lower edge of the band gap, and a dipole enters into the gap. The frequency of both modes increases with higher values of ϵ . The two higher order band gaps did not confine any mode.

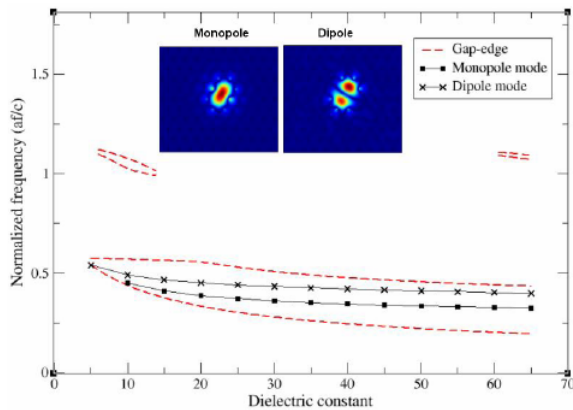


Figure 6: confined fields ($r/a=0.1$) within the band gaps for varying dielectric constants. The field profiles of the Confined fields are inset.

In order to remove the monopole mode from the bandgap, additional perturbations must be introduced to the lattice. Therefore, we investigated the effect of varying the radius of the rods around the double diagonal defect. Focusing on a dielectric constant of 9.5 (Dynalox100) for the scatters. The results are presented in figure 7. We found that the frequency of the both monopole and dipole modes decreased with increased r/a of the inner rods. At $R/a = 0.11$, the monopole mode slipped below the lower edge of the band gap. As the frequency of the modes continue to drop, HOMs were introduced into the gap at $r/a = 0.14$. The region between the two dashed lines in figure 7 is the operational region for a structure where only the dipole is

confined within the band gap and all other modes are extended and are able to propagate away.

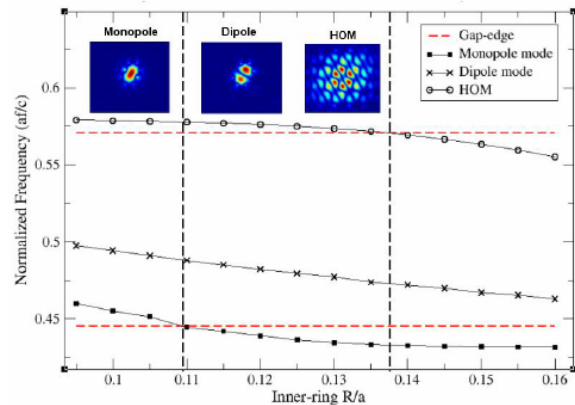


Figure 7: The effect of varying the radius of the innermost rods around the defect $r/a = 0.1$ and $\epsilon = 9.5$. The field profile is shown in the inset

CONCLUSIONS

We have found that disorder and systematical movement of individual rods result in the ability to ‘tune’ the PBG structure, and it is possible to increase peak field by approximately. In terms of structure fabrication, a maximum error in the innermost ring of rods of 1% in separation, 5% in radius, and less than 10% in disorder in all outer rods, leads to an average resonant frequency equal to the ideal structure (9.4072 GHz) with a maximum variation of 0.2% (20 MHz), and maximum variation of 0.5 V/m in the peak field. To achieve this for the structure considered in this paper requires fabrication of the rods with a radius variation of 150 μm , and a separation variation of 100 μm for the innermost ring of rods. This level of accuracy in fabrication, although difficult, is within the capability of modern fabrication.

We have demonstrated a systematic approach to mode engineering within the band gap of a photonic lattice. We have designed a monomodal structure that confines only the TM011-like dipole mode while allow all other modes to propagate away. A similar approach can be employed to design monomodal structures that confines only monopole, quadruple or higher order modes..

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