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Search for the Optimal Spin Decoherence Effect in a QFS Lattice

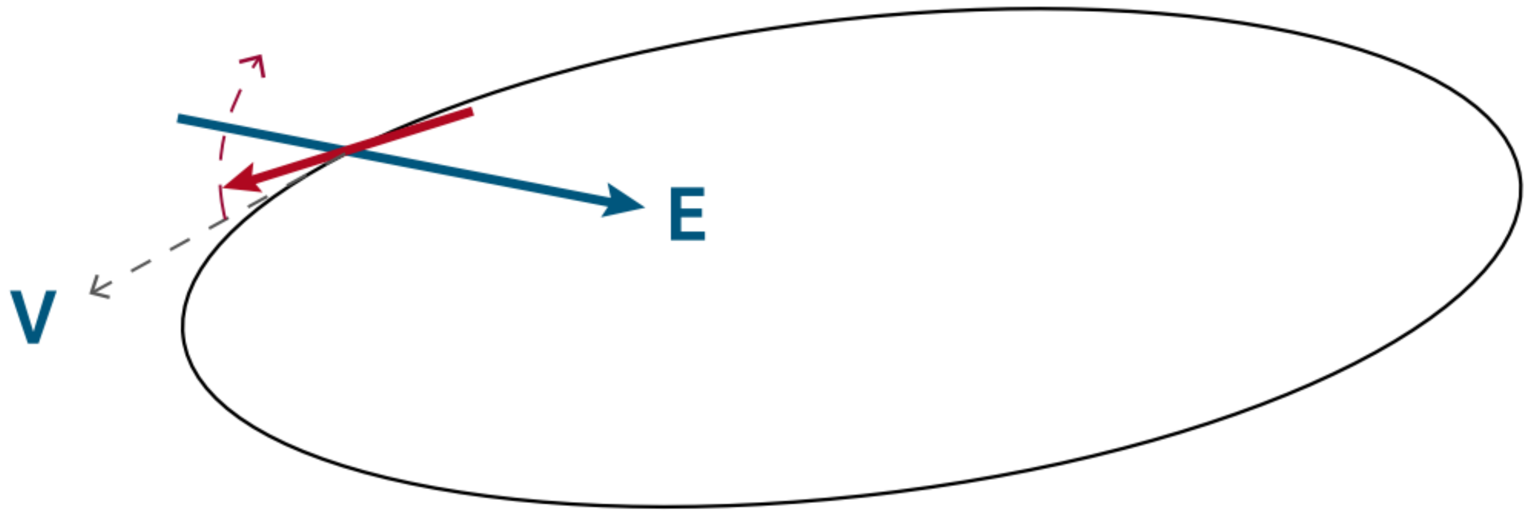
E. Valetov (MSU, USA), Yu. Senichev (FZJ, Germany), M. Berz (MSU, USA)

On behalf of the JEDI Collaboration

05-Oct-2015

Introduction
Principle of EDM Search [1]

Particle spin alignment along momentum (*frozen spin*)

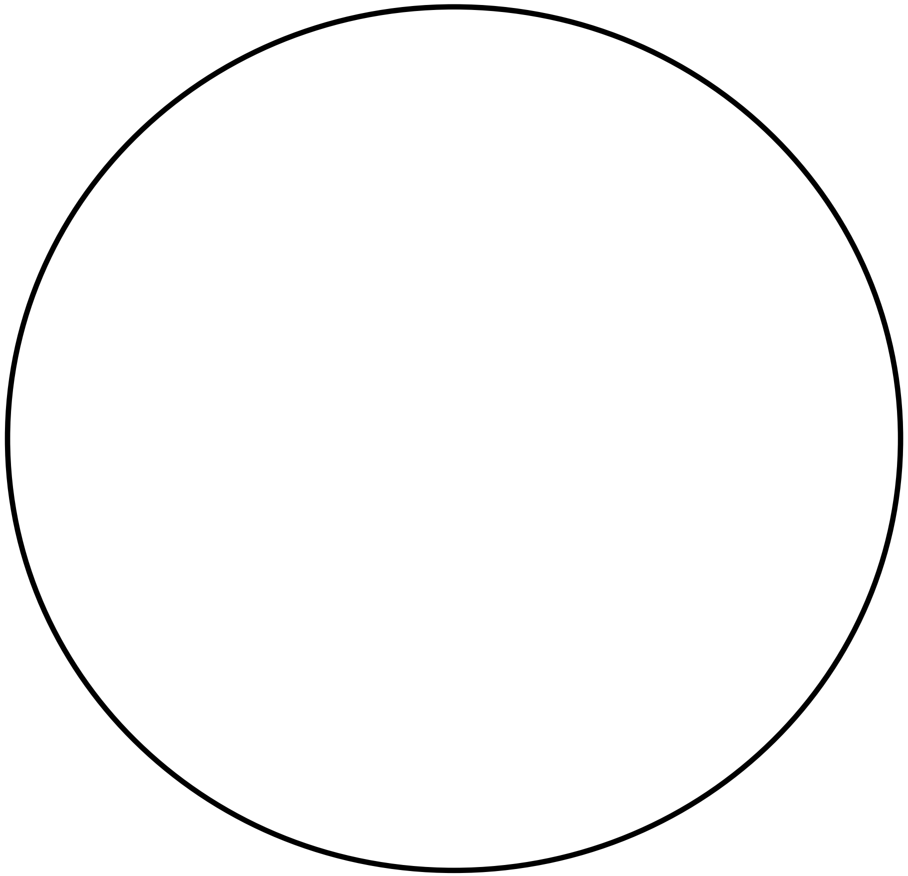


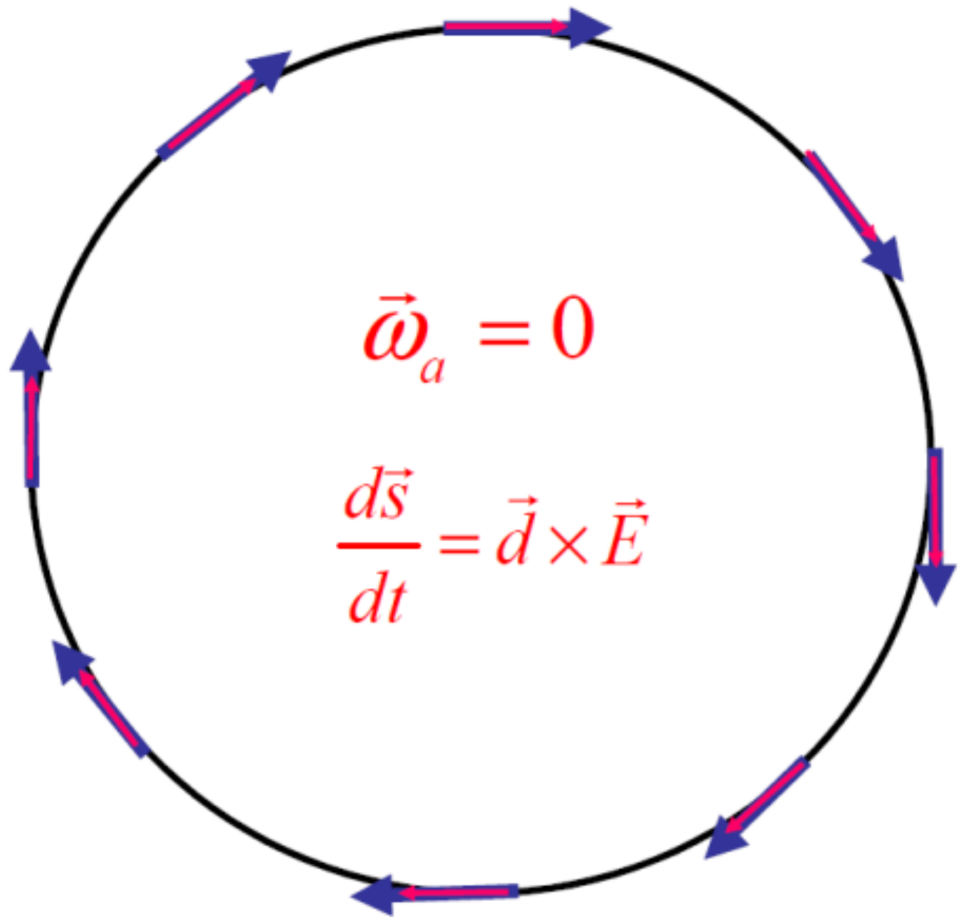
Radial E-field: torque on spin – rotation out of ring plane

Introduction

Frozen Spin Technique [1]

Left: a polarized charged particle (beam) in a storage ring





Right: fixing the horizontal spin along the momentum direction

Introduction
Quasi-Frozen Spin (QFS) Technique

Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{S}\times(\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM})$$

where

$$\vec{\Omega}_{MDM} = \frac{e}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{E}\times\beta}{c}\right]$$

$$\vec{\Omega}_{EDM} = \frac{e}{m}\frac{\eta}{2}\left[\frac{\vec{E}}{c} + \beta\times\vec{B}\right]$$

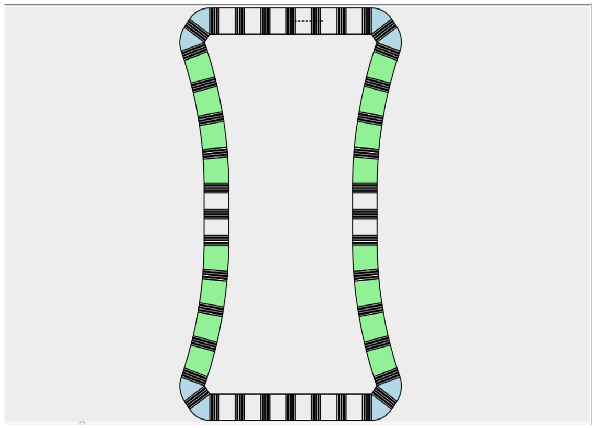
Quasi-Frozen Spin condition

$$\gamma G\,\Phi_B = \left[\frac{1}{\gamma}(1 - G) + \gamma G\right]\Phi_E$$

where Φ_B and Φ_E are the angles of momentum rotation in magnetic and electric bend parts of the ring correspondingly.

QFS Lattices

Codename "Senichev 6" Lattice



Lattice parameters

Length: 16667 cm

Particles: deuterons

Kinetic energy: 270 MeV

Lattice reference: Yu. Senichev et al., "Quasi-Frozen Spin Method for EDM Deuteron Search", Proceedings of IPAC'2015, Richmond, VA (2015).

Lattice structure

- 4 straight sections (light grey)
- 4 magnetic sections (blue)
- 4 electrostatic sections (green)

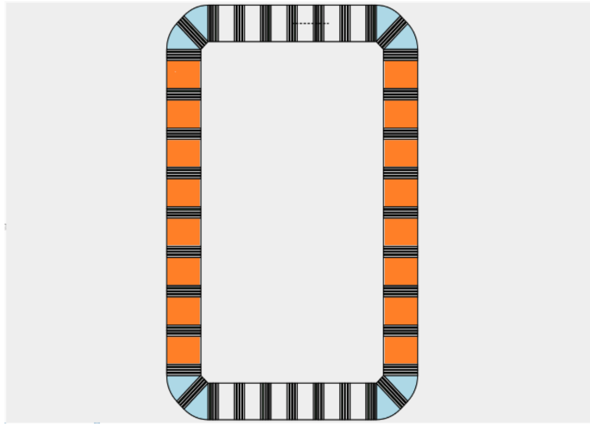
Decoherence order suppression

- RF cavity: 1st and partially 2nd order
  - ❖ by mixing the particles relatively to the average field strength, and therefore, averaging out $\Delta\gamma G$ for each particle

- Sextupoles: remaining 2nd order component
  - ❖ which is due to average of $\Delta\gamma G$ being different for each particle

QFS Lattices

Codename "Senichev E+B" Lattice



Lattice parameters
Length: 14921 cm
Particles: deuterons
Kinetic energy: 270 MeV

Lattice structure

- 4 straight sections (light grey)
- 4 magnetic sections (blue)
- 4 E+B sections (orange)

Decoherence order suppression

- RF cavity: 1st and partially 2nd order
- Sextupoles: remaining 2nd order component

E+B Wien Filter elements are used instead of the electrostatic deflector.

Purpose:

- remove corresponding nonlinear components
- simplify from engineering perspective

QFS Lattices

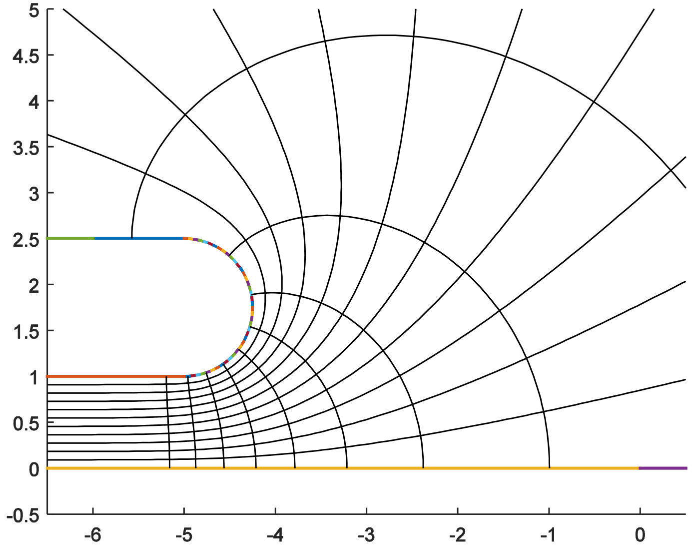
System analysis

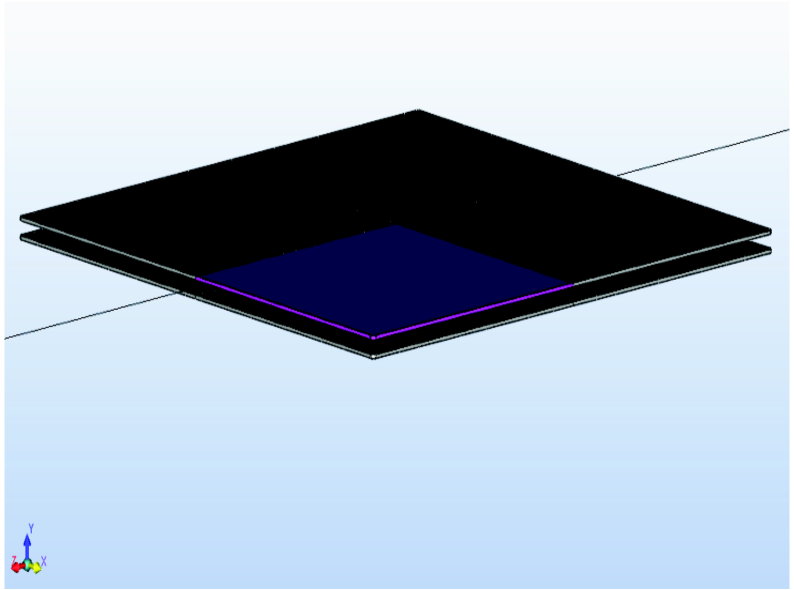
- Analytic relations (quadratic, etc.): see the general character of the system
- Numerical methods (system tracking in *COSY Infinity*): have final understanding of which orders are needed for spin decoherence less than 1 rad in 1000 s / 1 billion turns

Developed solution

- *COSY Infinity* programs
    - Code for manual and automatic optimization of lattice
        - Choice of three objective functions
        - One differential algebra (DA) objective function
    - Spin tracking code
    - Output data to files for storage and further processing
- *Mathematica* programs
    - Store certain results of *COSY* runs in an organized way
    - Process and QA check that data
    - Generate reports that aggregate processed data in plot and table format

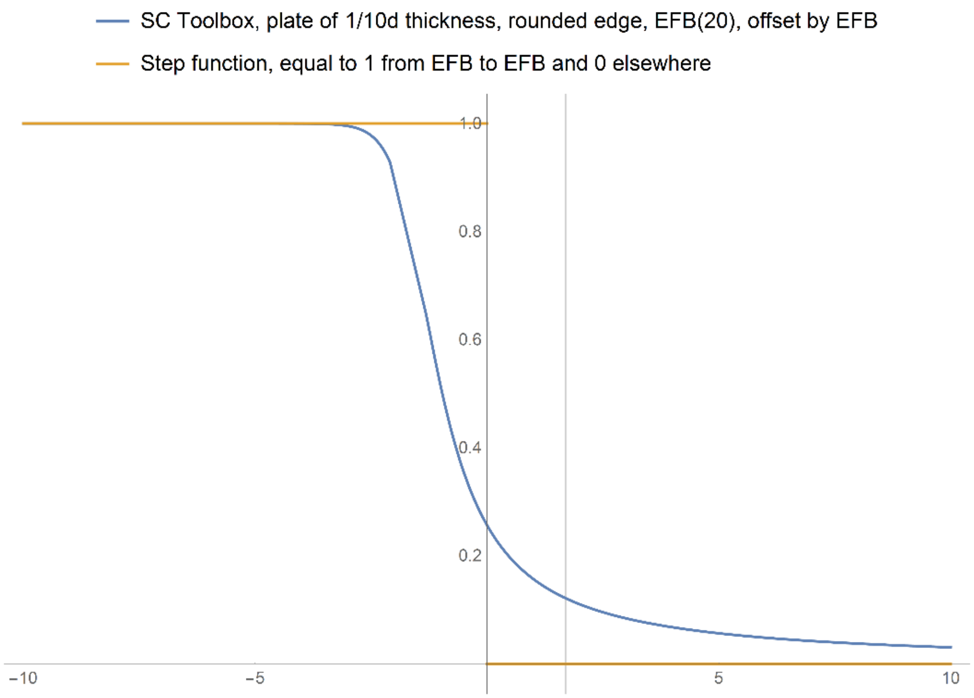
Fringe Field of the Electrostatic Deflector [4]





- Fringe fields of semi-infinite capacitors with solid metal plates were modeled in *MATLAB* using *Schwarz-Christoffel Toolbox* v.2.3 **[5]** and analyzed in *Mathematica*.
- Results were compared with those obtained for finite rectangular solid metal capacitors in *Coulomb* by H. Soltner (FZJ, Germany).

Fringe Field of the Electrostatic Deflector [4]

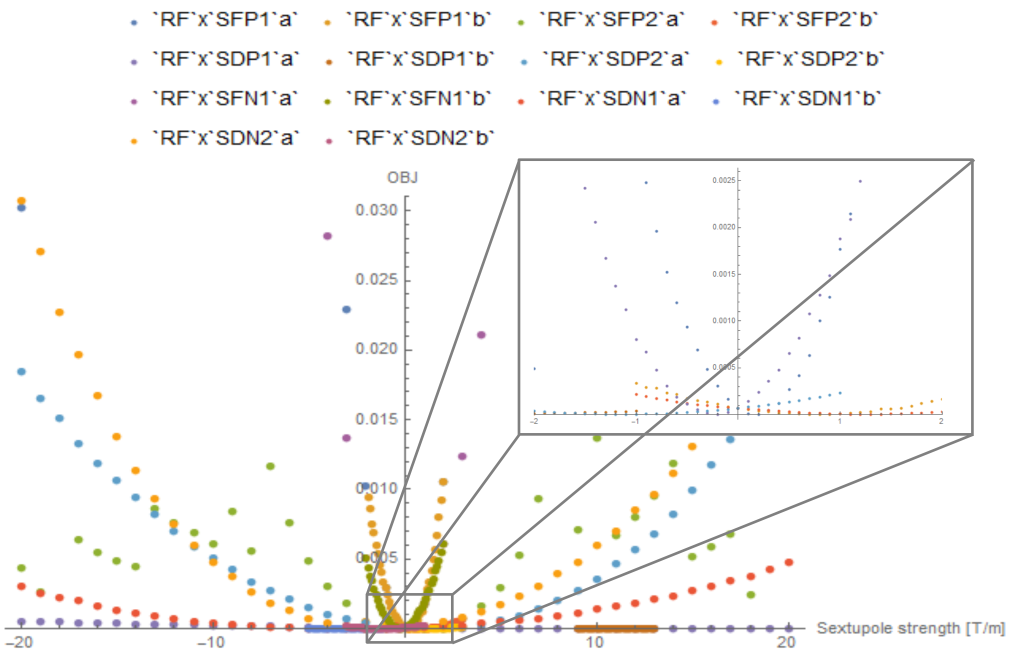


Enge Coefficient	Value
h_0	1.0614024399605924
h_1	1.6135741290714967
h_2	-0.9401447081042862
h_3	0.4781500036872176
h_4	-0.14379986967718494
h_5	0.017831089071215347

"Senichev 6" lattice electrostatic deflector:

- Semi-infinite capacitor
- Plate thickness 10% of distance between plate and midplane
- Rounded edges

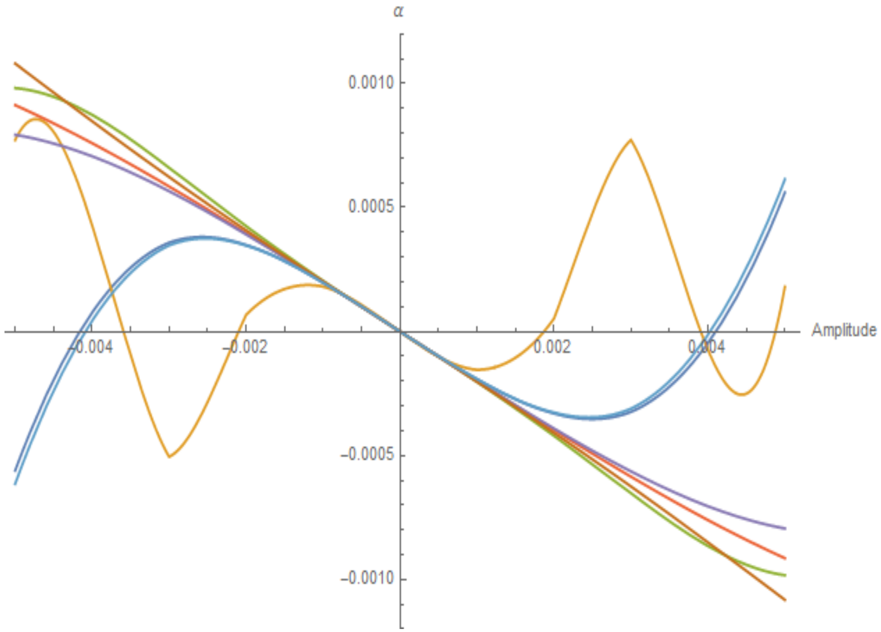
Manually optimized sextupole strengths

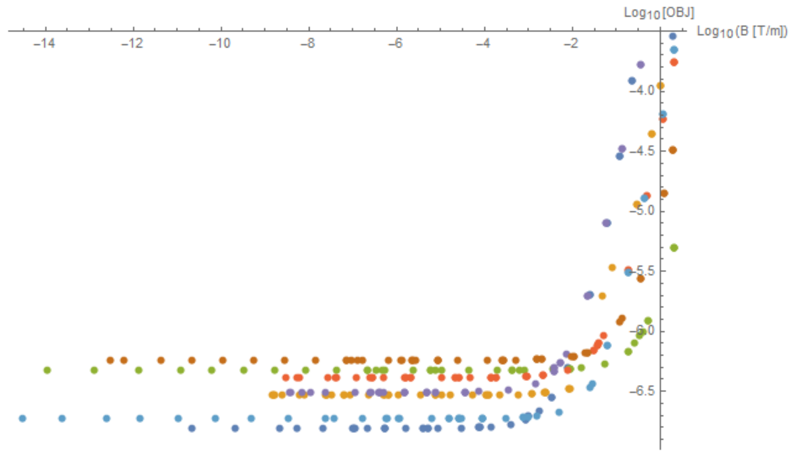


Optimization Context	Optimal Point	OBJ Value
\`RF\`x\`SFP1\`a\`	0.	0.0000662735
\`RF\`x\`SFP1\`b\`	0.2	3.01296×10^{-6}
\`RF\`x\`SFP2\`a\`	1.	9.43392×10^{-6}
\`RF\`x\`SFP2\`b\`	0.8	3.2609×10^{-7}
\`RF\`x\`SDP1\`a\`	11.	8.8197×10^{-7}
\`RF\`x\`SDP1\`b\`	10.6	5.23617×10^{-7}
\`RF\`x\`SDP2\`a\`	1.	4.79843×10^{-6}
\`RF\`x\`SDP2\`b\`	1.2	8.71622×10^{-7}
\`RF\`x\`SFN1\`a\`	0.	0.0000662735
\`RF\`x\`SFN1\`b\`	-0.2	1.5856×10^{-6}
\`RF\`x\`SDN1\`a\`	-3.	1.03277×10^{-6}
\`RF\`x\`SDN1\`b\`	-2.9	5.88324×10^{-7}
\`RF\`x\`SDN2\`a\`	-1.	2.22602×10^{-6}
\`RF\`x\`SDN2\`b\`	-1.2	3.48314×10^{-7}

- 20000 turn spin tracking in $x-a$, $y-b$, and $l-\delta$ planes, RF cavity on/off, various RF cavity frequencies and voltages
- Objective function represents spin decoherence
- Each curve shows manual optimization by a sextupole strength
- Compared with $x-a$ plane, in $y-b$ plane
  - curves are more parabolic
  - objective function values tend to be lower

Automatically optimized sextupole strengths





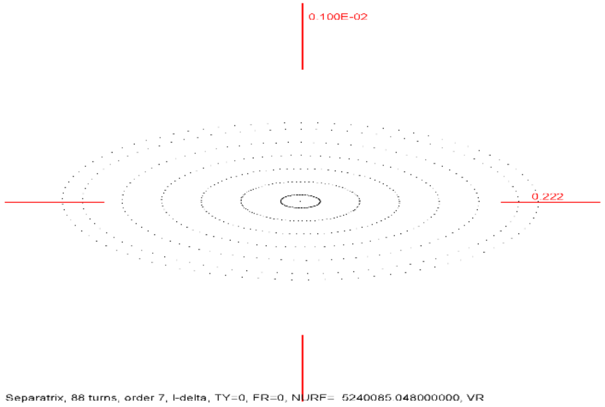
- ❖ Start from manually optimized sextupole strengths values
- ❖ Further optimize them using *LMDIF* optimizer
- ➢ When RF on, curves are typically bounded by symmetric slanted lines
- ❖ On the right plot, the thickness of the optimums is shown on $log - log$ scale
- ➢ Considering the accuracy, with which the physical sextupole strengths can be set, the thickness is acceptable

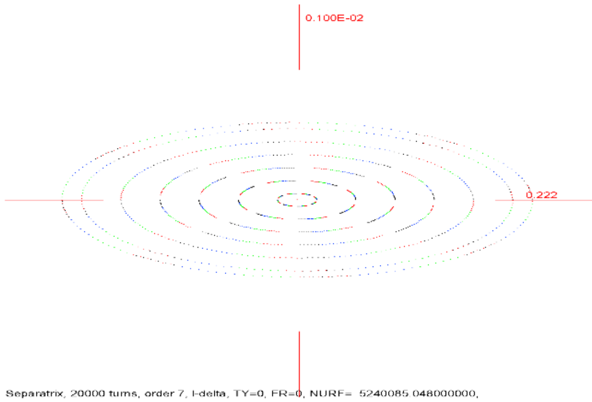
MICHIGAN STATE UNIVERSITY

Period of spin precession and longitudinal oscillation

(http://accelconf.web.cern.ch/AccelConf/IPAC2013/papers/wepea036.pdf)

JÜLICH FORSCHUNGSZENTRUM

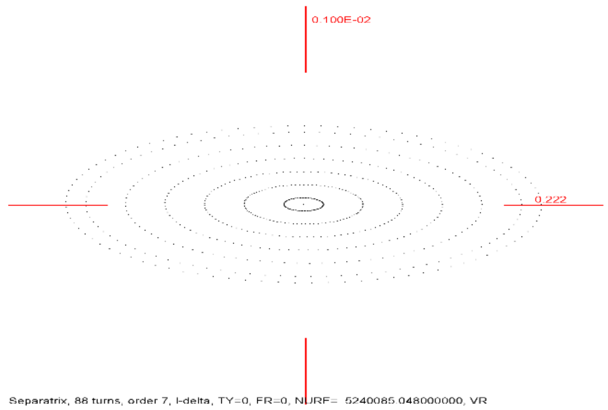


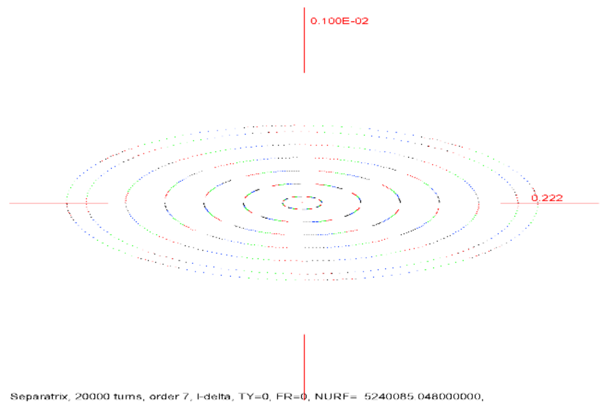


- ❖ Spin decoherence $\Delta\gamma G$ is order-wise proportional to orbit lengthening $\Delta l/l$
- ❖ In the example above, RF cavity is set to 5.2 MHz 100kV
- ➢ Period particle orbit in $l-\delta$ plane is approximately 88 turns near ref. particle
- ❖ Therefore, the period of energy averaging $\Delta\gamma G$ is 88 turns
- ❖ The larger the period of this averaging, the larger the amplitude of oscillations caused by the RF cavity (*cf. infra*)

Period of spin precession and longitudinal oscillation

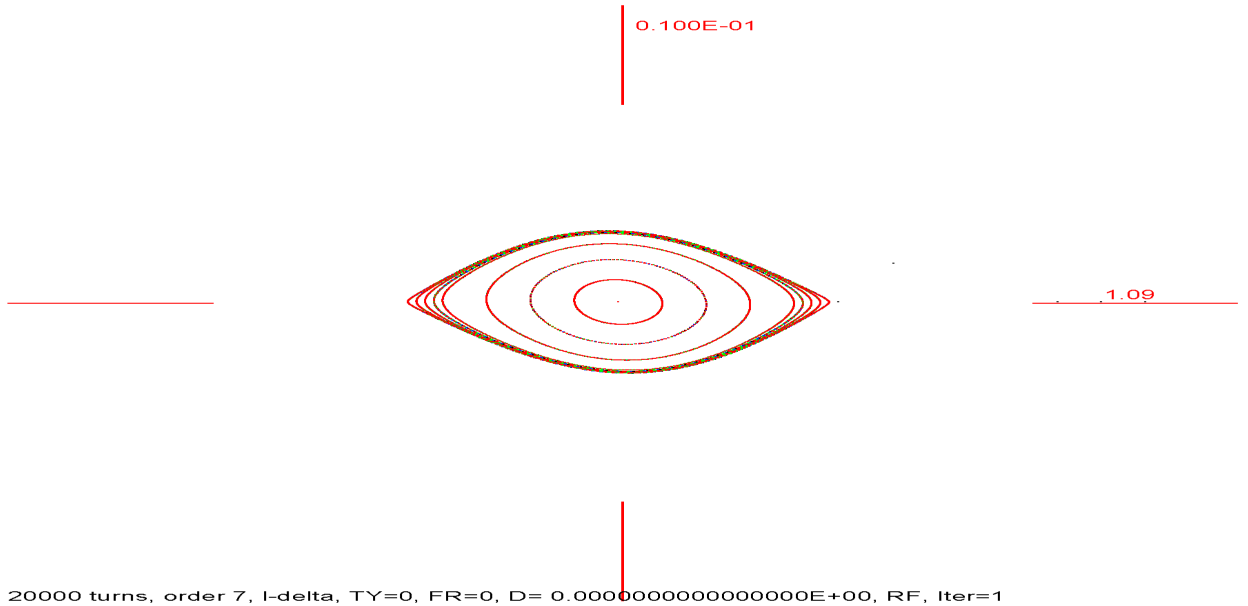
(http://accelconf.web.cern.ch/AccelConf/IPAC2013/papers/wepea036.pdf)





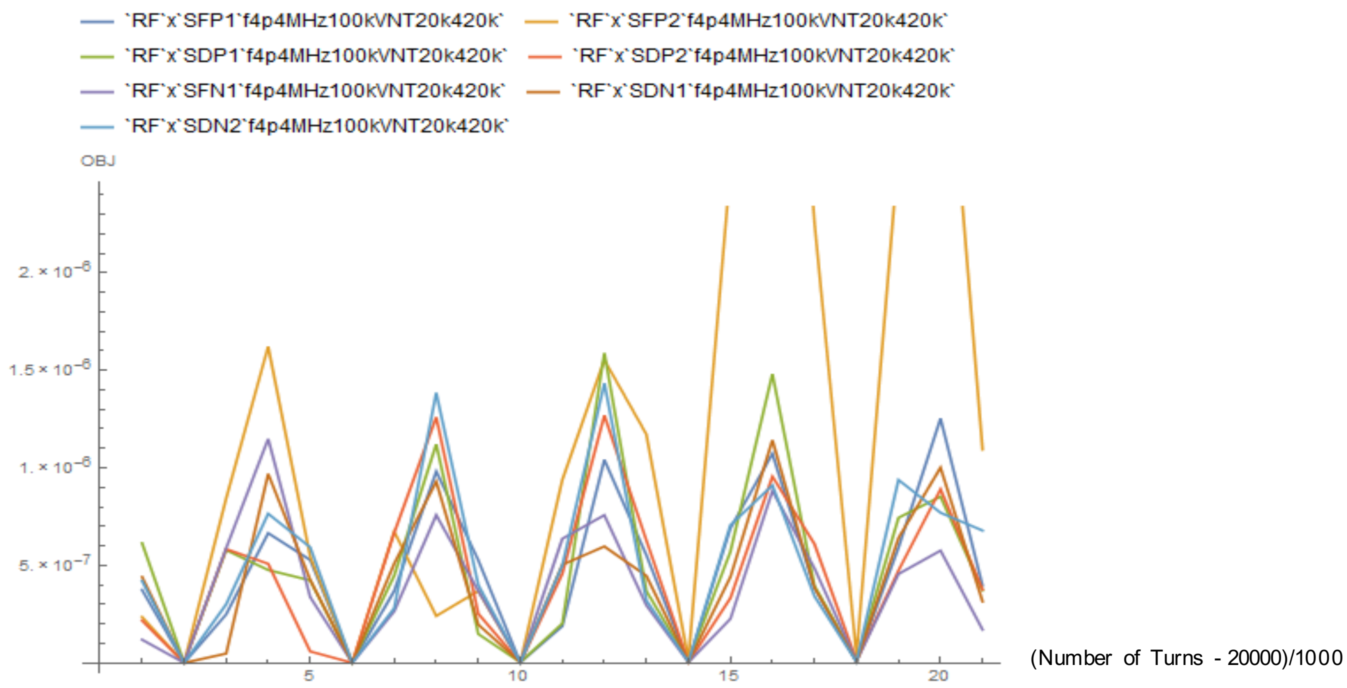
- W/o RF cavity, spin motion would have a period proportional to $\Delta\gamma G \sim 10^{-4}$, where $\Delta\gamma \sim 10^{-3}$ and $G = 0.14$
- For good energy averaging, the order of RF frequency must be 1-2 times higher than $\Delta\gamma G$
- RF cavity frequency in the lattice is ~2 orders faster $\Longrightarrow$ good energy averaging

Separatrix in the $l - \delta$ plane



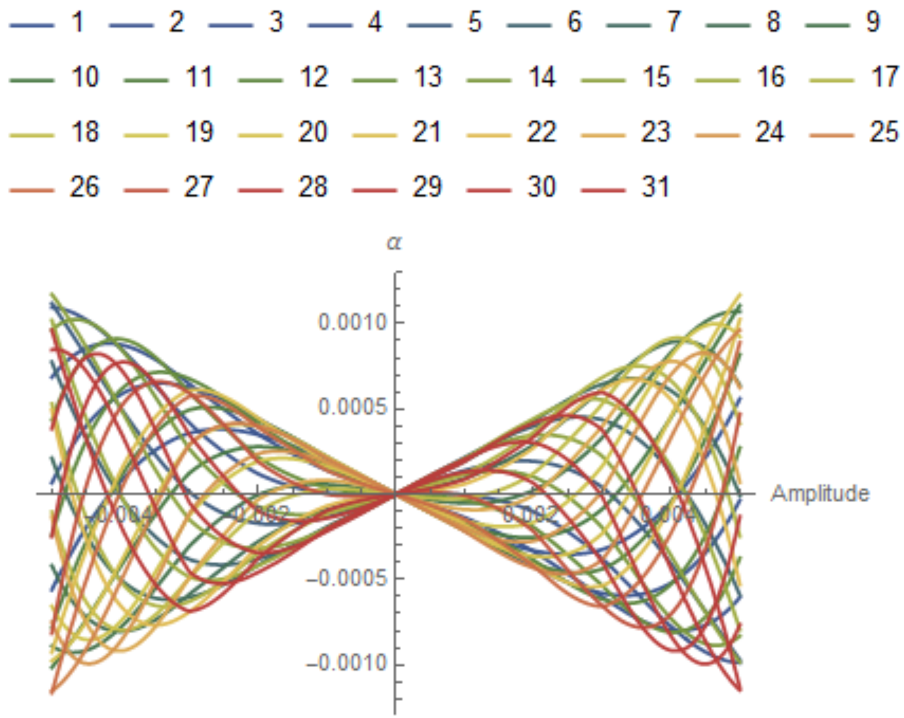
- 20000 turn spin tracking in $l - \delta$ plane
- Separatrix for the motion of particles
- In the example above, RF cavity is set to 1MHz 50kV
  - Order of δ-variance: 10^{-2}
  - Size of the separatrix: 5×10^{-3}
  - Period of the energy variance averaging: 20 turns
- Spin precession frequency: 10^{-4}
- Therefore, energy averaging due to RF cavity is ~ 2 orders faster

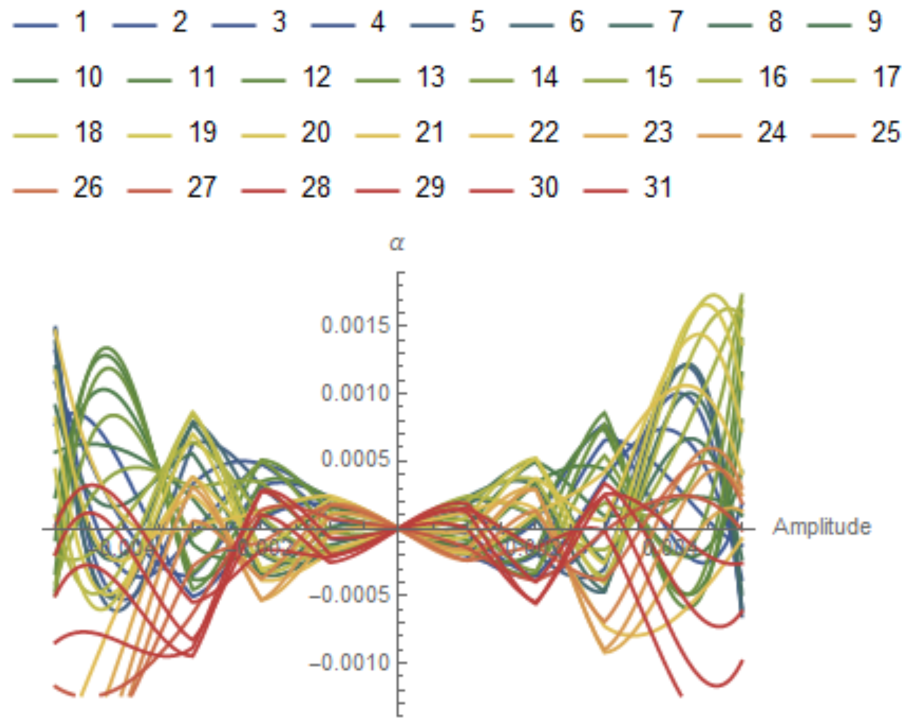
Time evolution of spin decoherence measure: “Senichev 6.2” $x - a$ 420k turns (with RF cavity)



- At 20000 turns, the order of spin decoherence was not sufficiently low to be satisfactory at that point of analysis
- So we have observed what happens to the objective function as a function of number of turns
- We plot the objective function against number of turns from 20000 to 420000 turns with the step of 20000
  - We note that significant minima occur periodically
  - In $x - a$ and $y - b$ planes for most sextupole strengths: indication that objective function **remains in the same range**

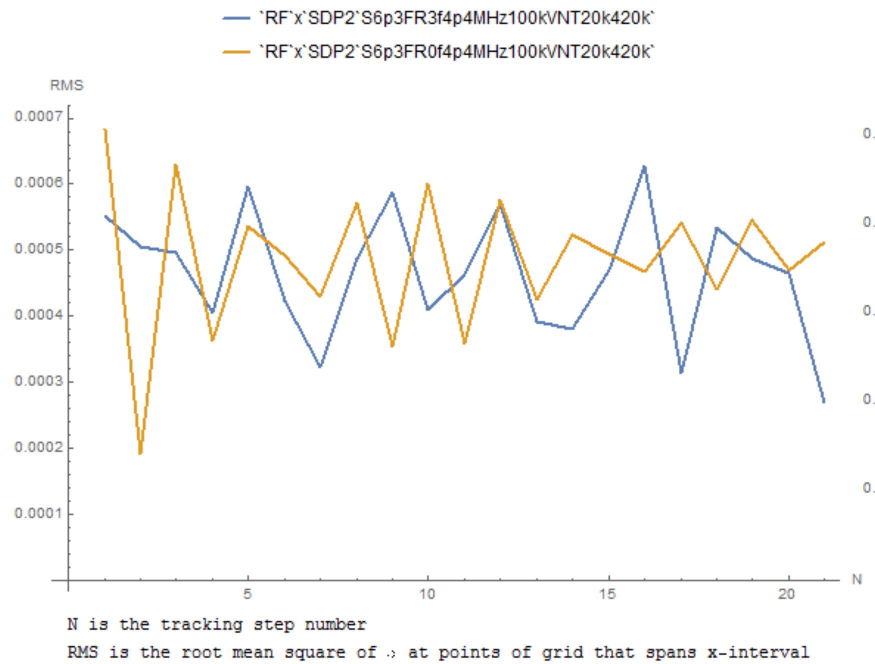
Time evolution of spin decoherence measure: “Senichev 6.2” $x-\alpha$ 420k turns (with RF cavity)

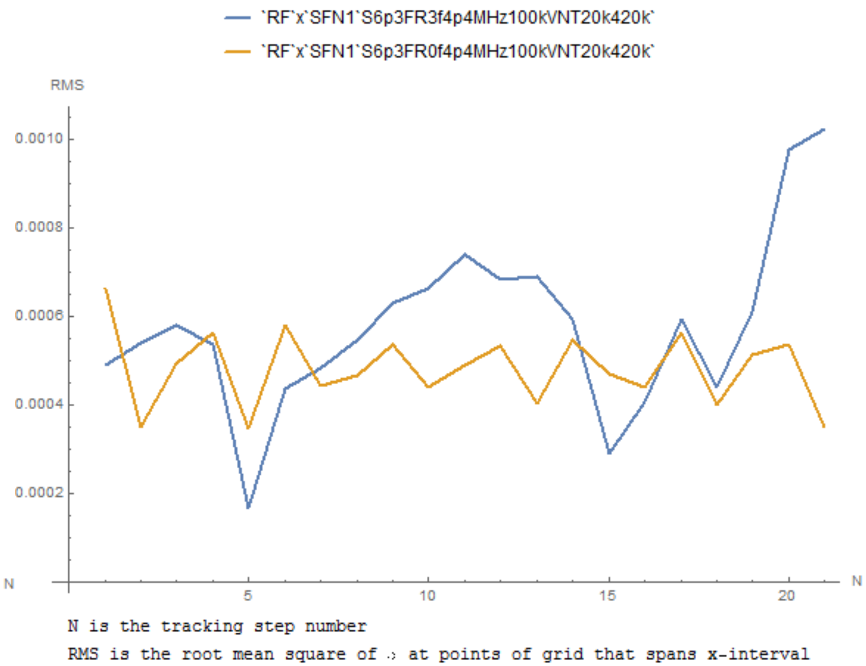




- ❖ We plot evolution of α-amplitude plots against number of turns from 20000 to 420000 turns
- ➢ For most sextupoles, oscillations seem to be bounded by two slanted lines as seen in the first picture
- ➢ Plot for SFP2 was exceptional due to low dispersion at the sextupole
- ❖ This is because RF cavity introduces forced oscillation into the system
- ❖ We could attempt to reduce its amplitude by decreasing RF cavity’s voltage and decreasing its frequency by about an order, but we simply deleted SFP2

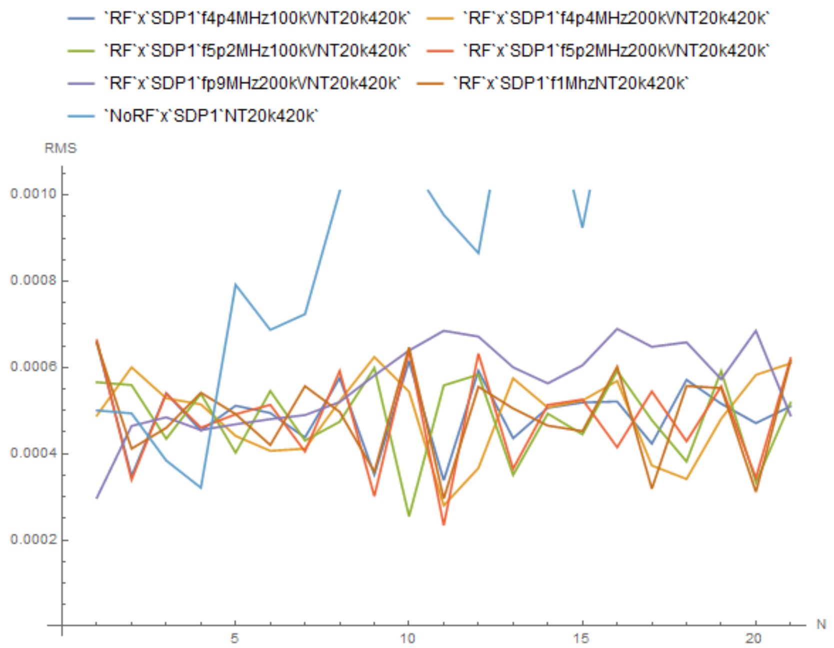
Time evolution of RMS of spin decoherence: fringe fields vs. no fringe fields



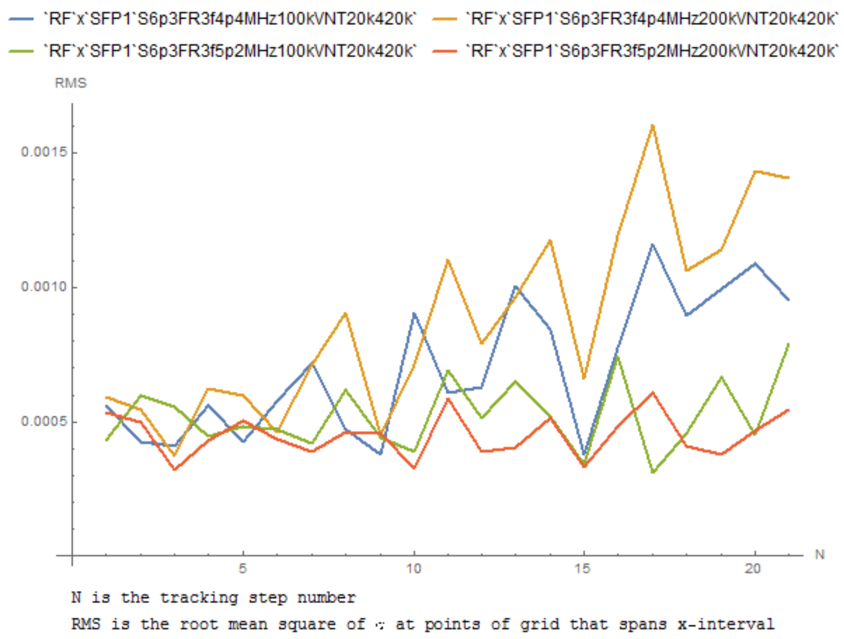


- Spin decoherence as a function of number of turns is often similar in fringe field modes 0 (no fringe fields) and 3 (most accurate).
- Sometimes, there is spin decoherence growth in FR 3 but no growth in FR 0.
- Previous plots were FR 0; starting from this slide we specify FR mode.

Time evolution of RMS of spin decoherence: "Senichev 6.2" $x - a$ 420k turns



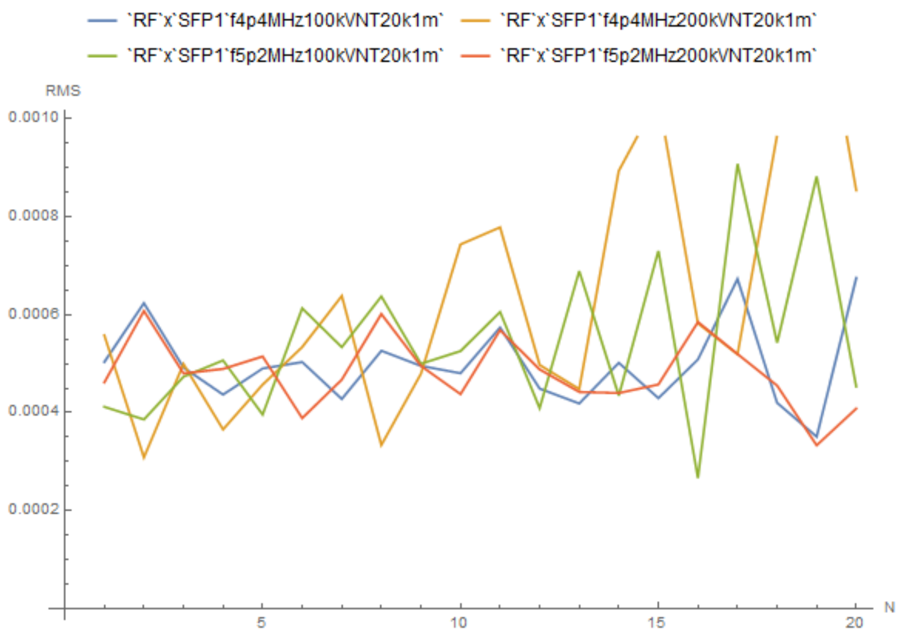
FR 0



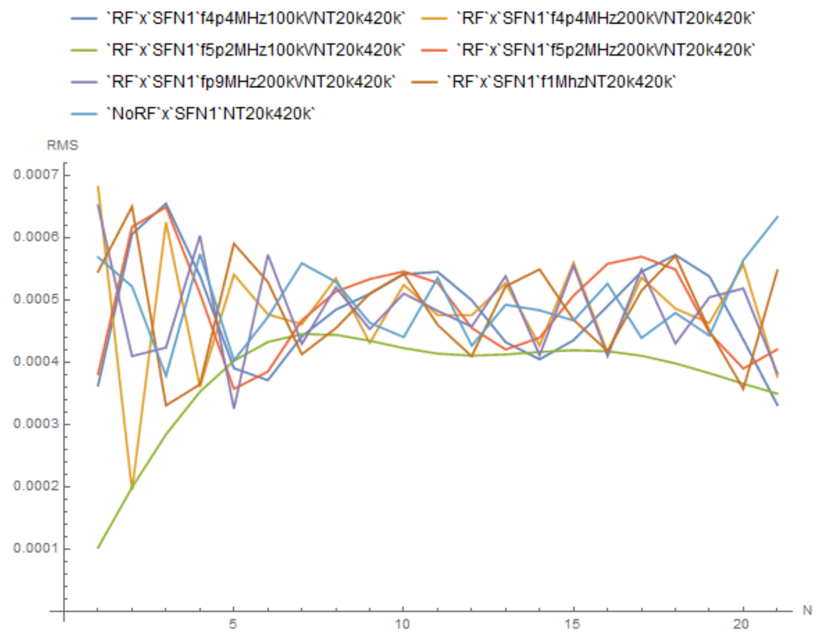
FR 3

- The RF cavity often limits spin decoherence to a range, at least for the number of turns of the order of 5×10^5.

Time evolution of RMS of spin decoherence: "Senichev 6.2" $x - a$ 1M turns



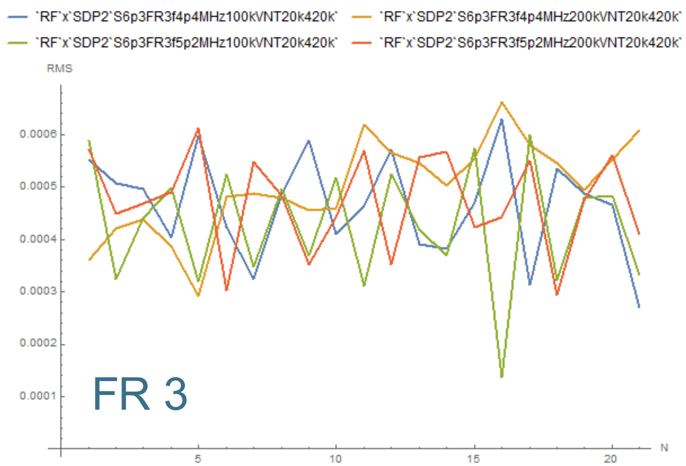
FR 0



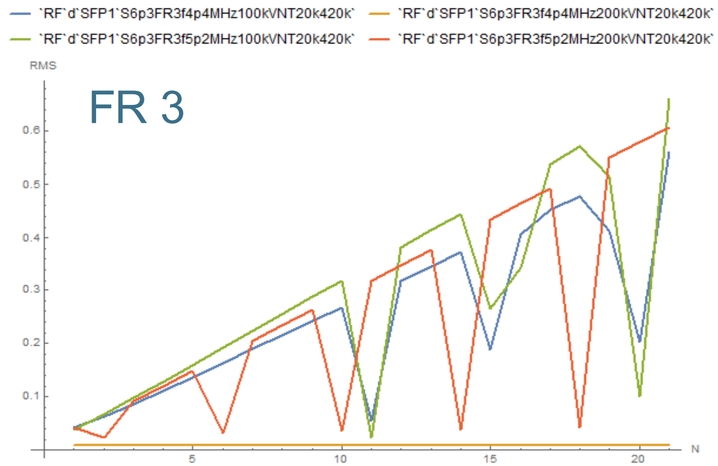
FR 0

- For some sextupoles and RF cavity settings, spin decoherence goes out of range and starts to increase after $\sim 10^6$ turns
- We have to find a set of RF cavity settings, sextupoles, and sextupole settings such that spin decoherence is sufficiently small in all planes, even if there is such an increase

Time evolution of RMS of spin decoherence: "Senichev 6.2" $l - d$ 420k turns



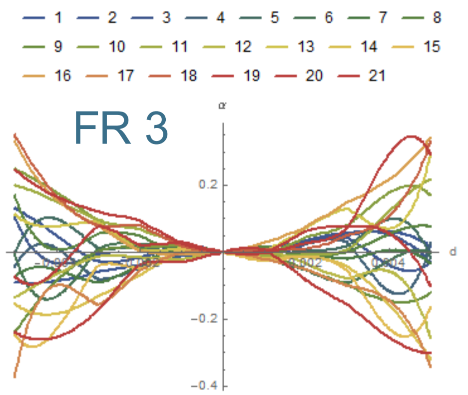
N is the tracking step number
RMS is the root mean square of α at points of grid that spans x-interval

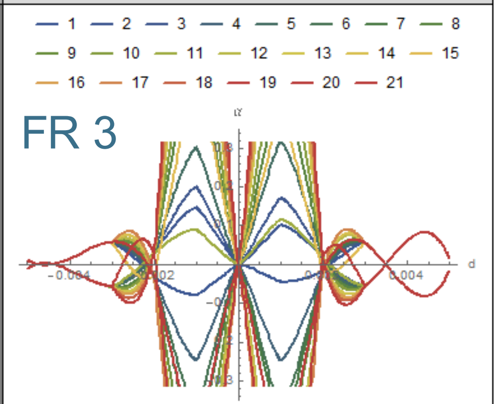


N is the tracking step number
RMS is the root mean square of α at points of grid that spans d interval

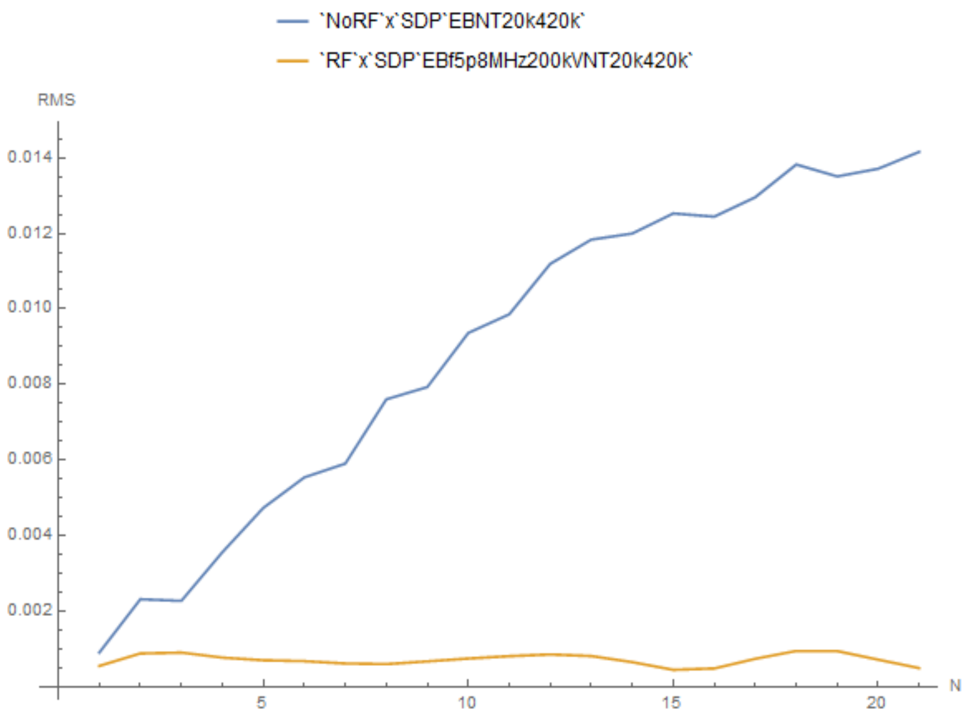
High nonlinearities introduces by a sextupole family can cause decoherence growth for some RF cavity settings.

Context	d–α plot for each iteration	Context	d α plot for each iteration
`RF`d`SFN1`S6p3FR3f4p4MHz100kVNT20k420k`		`RF`d`SFP1`S6p3FR3f4p4MHz100kVNT20k420k`	

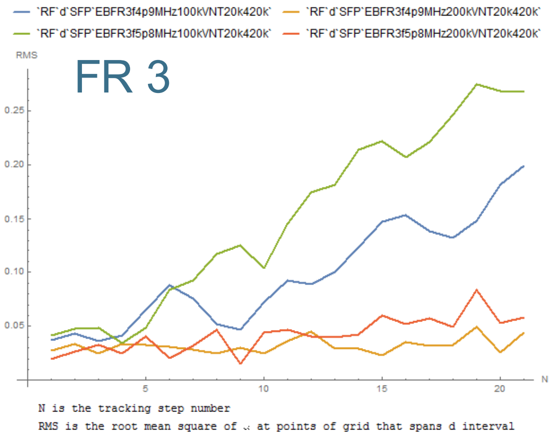


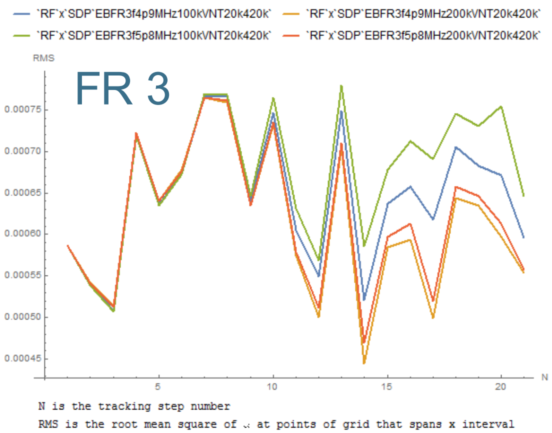


Time evolution of RMS of spin decoherence: “Senichev E+B” $x-a$ 420k turns



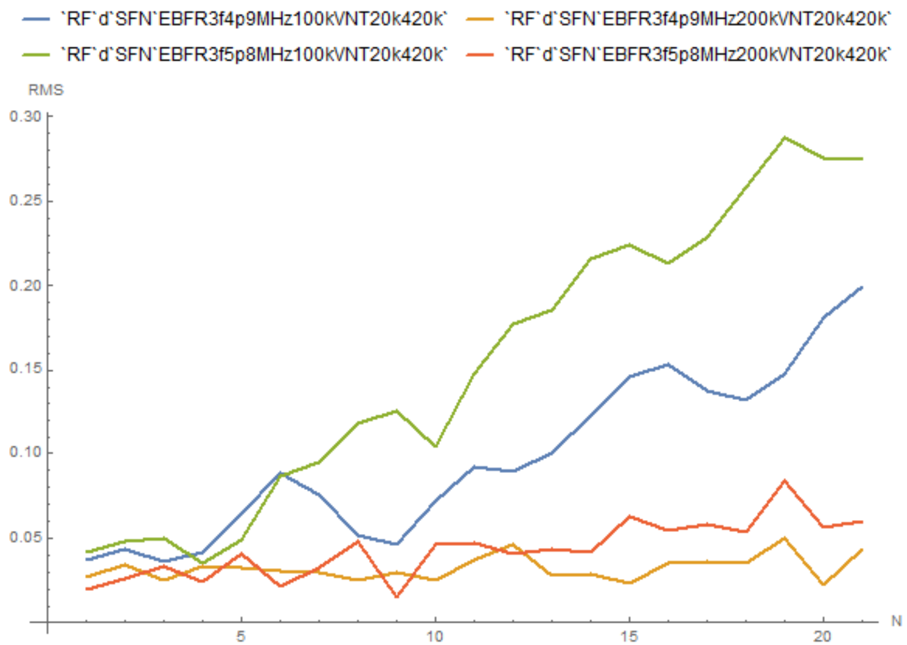
FR 3



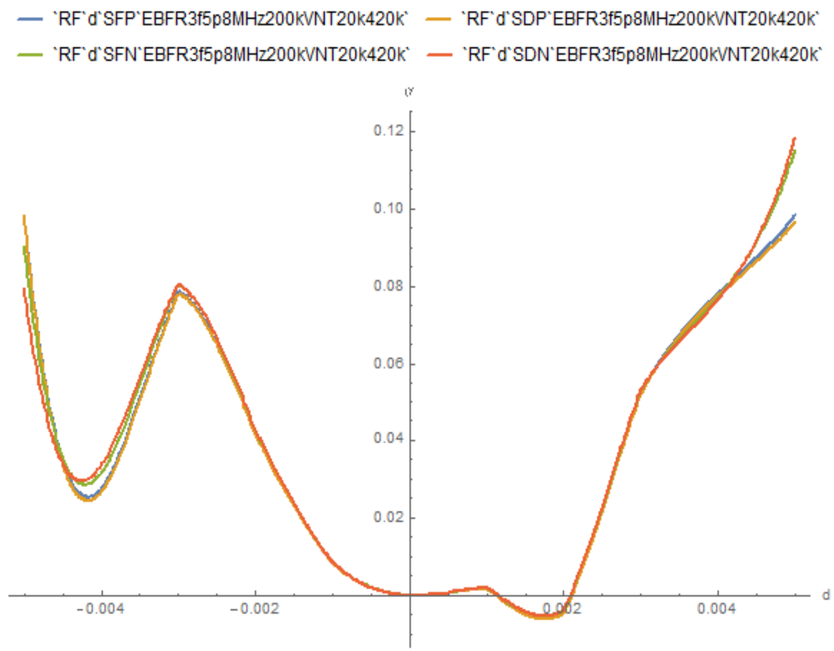


- Again, the RF limits the spin decoherence to range, at least for the number of turns of the order of 5×10^5.
- High nonlinearities introduced by a sextupole family can cause decoherence growth for some RF cavity settings.

Time evolution of RMS of spin decoherence: "Senichev E+B" $l - d$ 420k turns



N is the tracking step number
RMS is the root mean square of ω at points of grid that spans d interval



d is a spacial or relative energy difference amplitude of the particle
ω is the angle offset of the particle's spin, relative to the reference particle

FR 3

FR 3

In this case, sextupole family behavior was practically the same.

Conclusion

- At present, calculations indicate that the use of RF cavity and sextupoles alone in the considered QFS lattices may be sufficient to optimize the spin decoherence to less than 1 rad in 1 billion turns
- We will continue to work on optimization spin decoherence using the RF cavity and the sextupole strengths in all planes simultaneously
- We will track the obtained solution for a larger number of turns
- We will try to obtain an improved objective function based on tracking of differential algebra (DA) vector-valued particle rays.
- We will try suppressing decoherence using octupoles in addition to sextupoles

References

[1] H. Ströher, *Design Study* EDM, Forschungszentrum Jülich, presentation at NuPECC Meeting Edinburgh, October 10, 2014

[2] Yu. Senichev et al., *Quasi-Frozen Spin Method for EDM Deuteron Search*, Proceedings of IPAC'2015, Richmond, VA, 2015

[3] Yu. Senichev et al., *Spin Tune Decoherence Effects in Electro- and Magnetostatic Structures*, Proceedings of IPAC'2013, Shanghai, China, 2013.

[4] E. Valetov and M. Berz, *Calculation of Fringe Fields of Semi-Infinite Electrostatic Deflectors*, preprint of report, Michigan State University, East Lansing, MI, 2015.

[5] T. Driscoll, *A MATLAB toolbox for Schwarz-Christoffel mapping*, ACM Trans. Math. Softw., 22(2):168-186, June 1996.


Thank you!