Computation of Nonlinear Fields and Orbit and Spin Transfer Maps of Electrostatic Elements using Differential Algebras

Kyoko Makino, Martin Berz

Department of Physics and Astronomy, Michigan State University

The work is supported by the U.S. Department of Energy.

The Particle Optical Equations of Motion

$$
\begin{aligned}
x' &= a \cdot (1+hx)\frac{p_0}{p_z}, \\
y' &= b \cdot (1+hx)\frac{p_0}{p_z}, \\
a' &= \left((1+\delta_m)\frac{1+\eta}{1+\eta_0}\frac{p_0}{p_z}\frac{E_x}{\chi_{e0}} - \frac{B_y}{\chi_{m0}} + b \cdot \frac{p_0}{p_z}\frac{B_z}{\chi_{m0}} \right)(1+hx)(1+\delta_z) + h\frac{p_z}{p_0}, \\
b' &= \left((1+\delta_m)\frac{1+\eta}{1+\eta_0}\frac{p_0}{p_z}\frac{E_y}{\chi_{e0}} + \frac{B_x}{\chi_{m0}} - a \cdot \frac{p_0}{p_z}\frac{B_z}{\chi_{m0}} \right)(1+hx)(1+\delta_z), \\
l' &= (1+\delta_m)(1+hx)\frac{1+\eta}{1+\eta_0}\frac{p_0}{p_z},
\end{aligned}
$$

where h is the curvature of the reference orbit, and

$$
\begin{aligned}
\chi_{e0} &= \frac{p_0 v_0}{z_0 e}, \quad \chi_{m0} = \frac{p_0}{z_0 e}, \quad \frac{p_z}{p_0} = \left((1+\delta_m)^2 \frac{\eta(2+\eta)}{\eta_0(2+\eta_0)} - a^2 - b^2 \right)^{1/2}, \\
\eta &= \frac{K_0(1+\delta_k) - z_0 e(1+\delta_z)V(x,y,s)}{m_0 c^2 (1+\delta_m)}.
\end{aligned}
$$

The Lorentz Equations of Motion

The motion of a charged particle in the electromagnetic fields is described by the **Lorentz force law**, which in SI units is

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right).$$

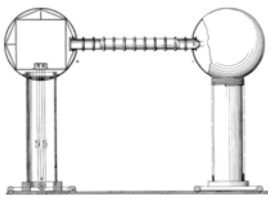
Observe

- $\vec{E}$ contributes to an acceleration, but not $\vec{B}$.
- Both $\vec{E}$ and $\vec{B}$ can be used for guiding/bending.
- In the relativistic limit, the bending power of $\vec{E}$ is $\sim 1/c$ of $\vec{B}$'s.

Achievable High Fields

Electrostatic Fields

The Cockcroft-Walton generator – 750 kV over ~10m at Fermilab



The Van de Graaff generator – up to ~10 MV over ~10m

Lightning – ~100 MV over ~100m

Magnetic Fields

The LHC, superconducting bending magnets – ~8 T

The Lorentz Equations of Motion

The motion of a charged particle in the electromagnetic fields is described by the **Lorentz force law**, which in SI units is

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right).$$

Observe

- $\vec{E}$ contributes to an acceleration, but not $\vec{B}$.
- Both $\vec{E}$ and $\vec{B}$ can be used for guiding/bending.
- In the relativistic limit, the bending power of $\vec{E}$ is $\sim 1/c$ of $\vec{B}$'s.
- We saw that the achievable high fields: $|\vec{E}| \sim MV/m$, $|\vec{B}| \sim 10T$.
- To match the bending power of $|\vec{B}| \sim 10T$, $|\vec{E}|$ would need $\sim GV/m$.

Hence, high energy rings use $\vec{B}$ for the beam guidance,
except for special cases.

PROJECT OVERVIEW AND COMPUTATIONAL NEEDS TO MEASURE ELECTRIC DIPOLE MOMENTS AT STORAGE RINGS

A. Lehrach# on behalf of the JEDI collaboration*, Forschungszentrum Jülich, Germany

Abstract

Different approaches to measure Electric Dipole Moments (EDMs) of proton, deuteron and light nuclei are pursued at Brookhaven National Laboratory (BNL) and Forschungszentrum Jülich (FZJ) with an ultimate goal to reach a sensitivity of 10^{-29} $e{\cdot}$cm in a dedicated storage ring. As an intermediate step, a first direct EDM measurement of protons and deuterons at 10^{-24} $e{\cdot}$cm sensitivity level will be carried out in a conventional storage ring, the Cooler Synchrotron COSY at FZJ [1].

Full spin-tracking simulations of the entire experiment are absolutely crucial to explore the feasibility of the planned storage ring EDM experiments and to investigate systematic limitations. For a detailed study of particle and spin dynamics during the storage and buildup of the EDM signal, one needs to track a large sample of particles for billion of turns.

INTRODUCTION

Permanent EDMs of fundamental particles violate both time invariance $\mathcal{T}$ and parity $\mathcal{P}$. Assuming the $\mathcal{CPT}$ theorem this implies $\mathcal{CP}$ violation. The Standard Model (SM) predicts non-vanishing EDMs, their magnitudes, however, are expected to be unobservably small in the near future. Hence, the discovery of a non-zero EDM would be a signal for "new physics" beyond the SM. It is mandatory to measure EDMs on different species of particles in order to disentangle various sources of $\mathcal{CP}$ violation. While neutron EDM experiments are pursued at many different places worldwide, no such direct measurements have been conducted yet for protons and other light nuclei due to special difficulties of applying performed with COSY Infinity must be benchmarked with other simulation programs and experiments performed at the Cooler Synchrotron COSY, to ensure the required accuracy of the obtained simulation results.

EDM MEASUREMENTS AT STORAGE RINGS

Principle

The principle of every EDM measurement (e.g., neutral and charged particles, atom, molecule) is the interaction of an electric field with the dipole moment of the particle. In the center-of-mass system of a particle electric dipole moments $\vec{d}$ couple to electric fields, whereas magnetic dipole moments $\vec{\mu}$ couple to magnetic fields. The spin precession in the presence of both electric and magnetic fields is given by:

$$\frac{d\vec{S}}{dt} = \vec{d}\times\vec{E}^* + \vec{\mu}\times\vec{B}^*. \quad (1)$$

Here, $\vec{E}^*$ and $\vec{B}^*$ denote the electric and magnetic fields in the particle rest frame. In case of moving particles in a circular accelerator or storage ring, the spin motion is covered by the Thomas-BMT equation and its extension for EDM:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}\times\vec{S}$$

$$\vec{\Omega} = \frac{e\hbar}{mc}\left\{G\vec{B} + \left(G - \frac{1}{\gamma^2-1}\right)\left(\vec{v}\times\vec{E}\right) + \frac{\eta}{2}\left(\vec{E}+\vec{v}\times\vec{B}\right)\right\}. \quad (2)$$

The BMT Equation

Classical equation of Motion for Spin:

$$\frac{d\vec{s}}{dt} = \vec{w} \times \bar{s}, \text{where}$$

$$\vec{w} = k \left(-(1+G\gamma)\vec{B} + \frac{G}{1+\gamma}(\vec{P}\cdot\vec{B})\ \vec{P} + (G + \frac{1}{1+\gamma})\ \vec{P} \times \frac{\vec{E}}{c} \right),$$

and $k = e/\gamma m_o c$, $G = (g-2)/g$, $\vec{P} = \vec{p}/m_0 c$.

In particle optical relative coordinates:

$$\frac{d\vec{s}}{ds} = t' \cdot \vec{w} \times \bar{s} + \vec{h} \times \vec{s}, \text{where } s\text{: arclength, } \vec{h}\text{: curvature}$$

Solution is a *linear orthogonal transformation* depending on orbital variables. Thus

$$\vec{s}_f = \hat{A}(\vec{z}) \cdot \vec{s}_i, \text{ where}$$

$$\hat{A}(\vec{z}) \in SO(3)$$

Motion Of Spin Matrix

Nine dimensional motion of particle with spin, neglecting spin-orbit coupling

$$\begin{pmatrix} \vec{z} \\ \vec{s} \end{pmatrix}' = \vec{F}(\vec{z}, \vec{s}, s) = \begin{pmatrix} \vec{f}(\vec{z}, s) \\ \hat{W}(\vec{z}, s) \cdot \vec{s} \end{pmatrix}$$

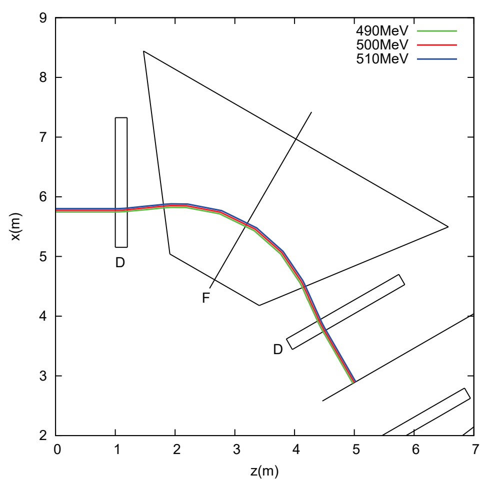
$$\begin{pmatrix} \vec{z}_f \\ \vec{s}_f \end{pmatrix} = \vec{M}(\vec{z}_i, \vec{s}_i, s) = \begin{pmatrix} \mathcal{M}(\vec{z}_i, s) \\ \hat{A}(\vec{z}_i, s) \cdot \vec{s} \end{pmatrix}$$

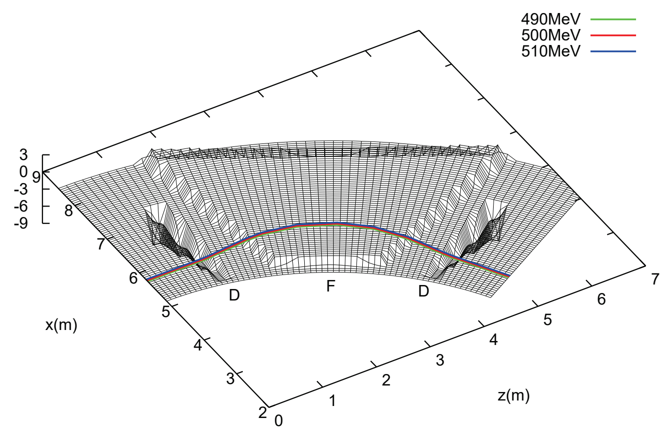
To *reduce dimensionality* and *utilize linearity*, it is advantageous to set up EOM for $\hat{A}$. Insertion yields EOM for 3×3 spin matrix depending on only the 6 orbital variables:

$$\hat{A}'(\vec{z}, s) = \hat{W}(\vec{z}, s) \cdot \hat{A}(\vec{z}, s)$$

Particle Tracking in a Conventional Style

- Computations of the electromagnetic fields – FEM, BEM,...
  Field values at grid points. → Interpolate anywhere else.
- Numerical integrations of the trajectories through the fields.





A coarse grid (d_R=5cm, d_theta=1deg) is used for the field data, only for the easy demonstration.

Transfer Map Method and Differential Algebras

- The transfer map $\mathcal{M}$ is the flow of the system ODE.

$$\vec{z}_f = \mathcal{M}(\vec{z}_i, \vec{\delta}),$$

where $\vec{z}_i$ and $\vec{z}_f$ are the initial and the final condition, $\vec{\delta}$ is system parameters.

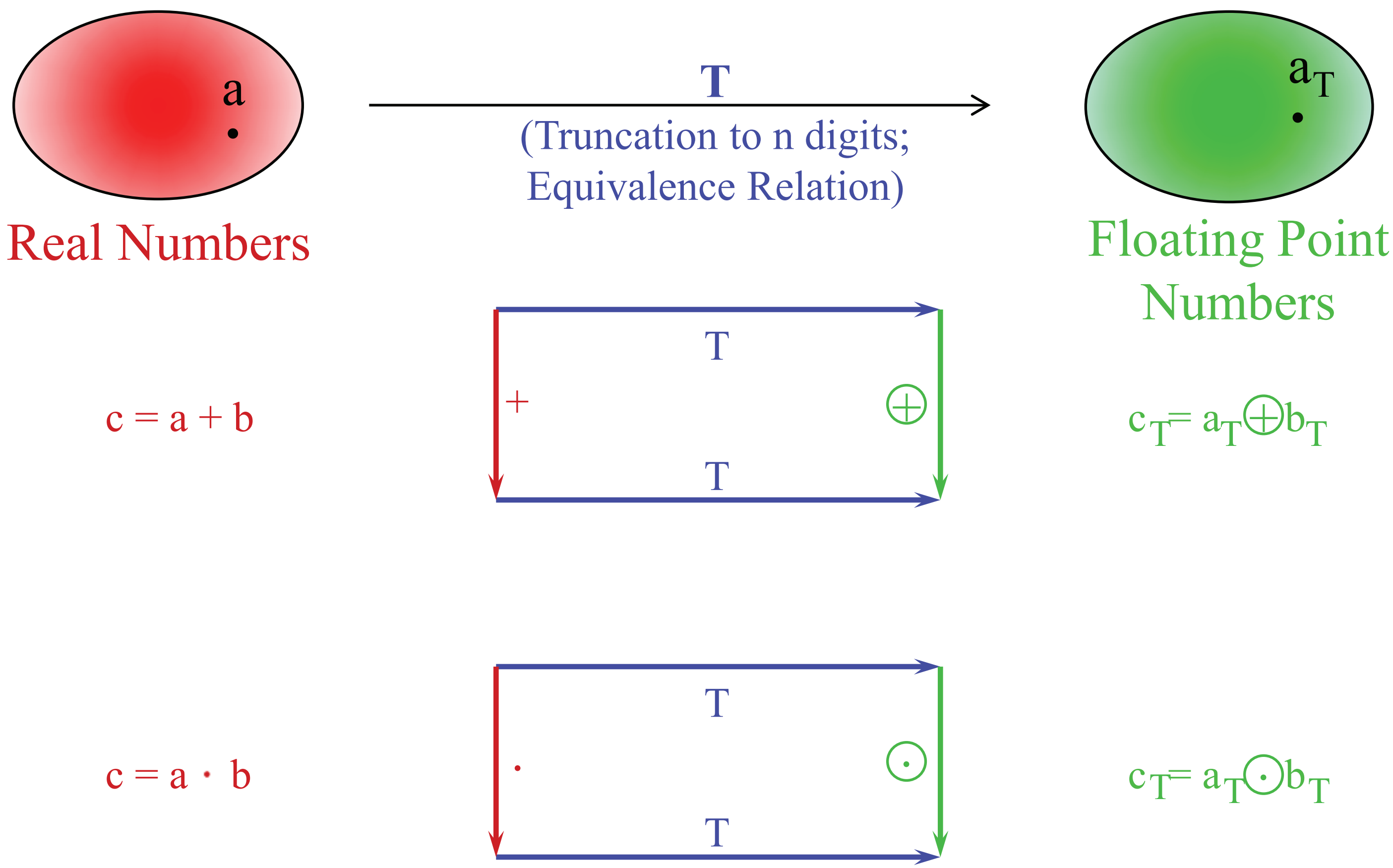
- For a repetitive system, only one cell transfer map has to be computed. Thus, it is much faster than ray tracing codes (i.e. tracing each individual particle through the system).
- The Differential Algebraic method allows a very efficient computation of high order Taylor transfer maps.
- The Normal Form method can be used for analysis of nonlinear behavior.

Differential Algebras (DA)

- it works to arbitrary order, and can keep system parameters in maps.
- very transparent algorithms; effort independent of computation order.

The code **COSY Infinity** has many tools and algorithms necessary.

Number Fields and Floating Point Numbers



Field

(Also want "exp", "sin" etc: Banach Field)

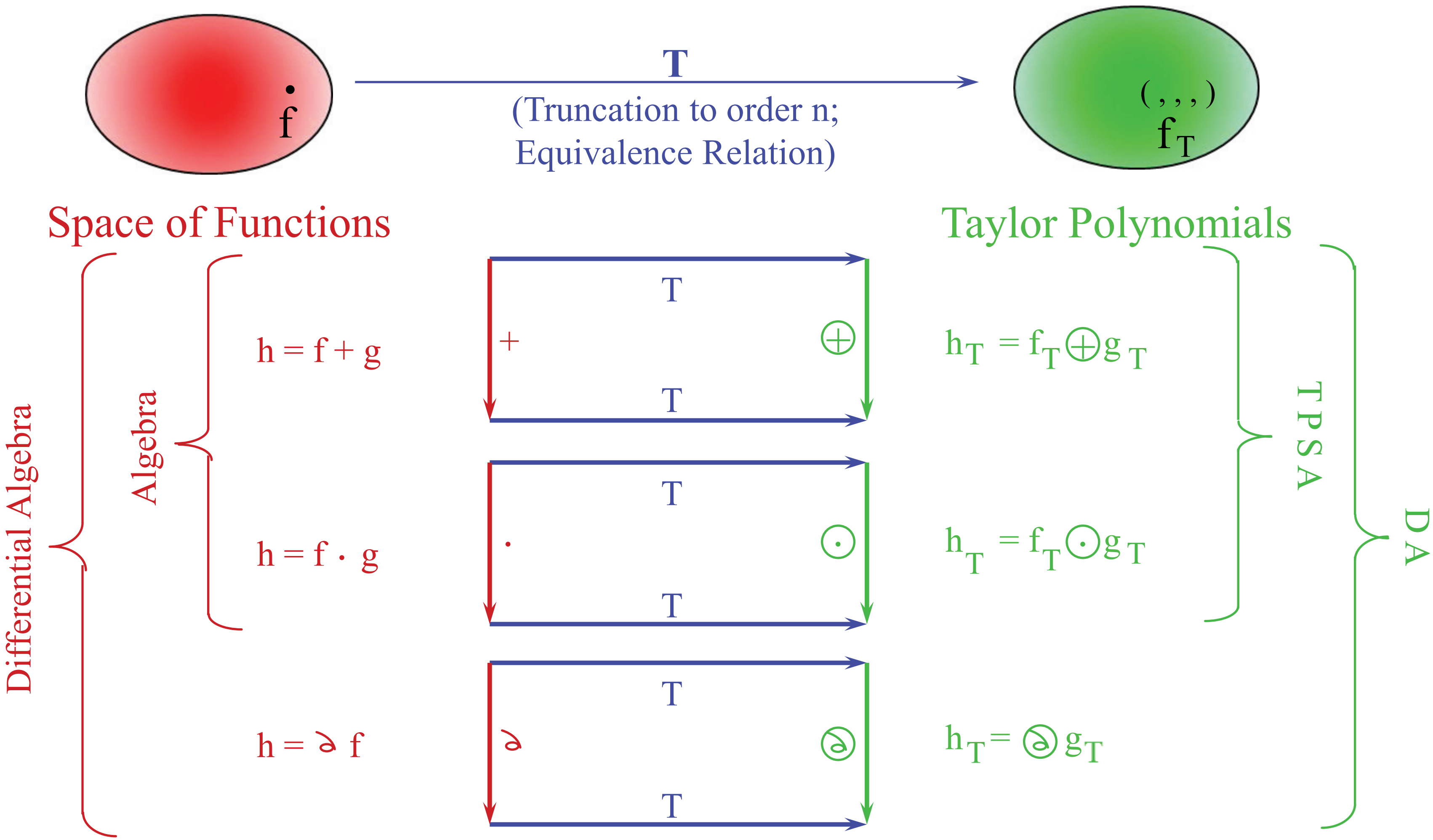
Diagrams commute "approximately"

Field

("approximately")

T: Extracts information considered relevant

Function Algebras



Differential Algebra
(also want "exp", "sin"
etc: Banach DA)

Diagrams commute
exactly

T: Extracts information
considered relevant

Differential Algebra
(even Banach DA)

COSY INFINITY

- Arbitrary order
- Maps depending on parameters (even with mass dependence)
- No approximations in motion or field description
- Large library of elements
- Arbitrary Elements (you specify fields)
- Very flexible input language
- Powerful interactive graphics
- Errors: position, tilt, rotation
- Tracking through maps (with/without symplectification. EXPO)
- Normal Form Methods
- Spin dynamics
- Fast fringe field models using SYSCA approach
- Reference manual (70 pages) and Programming manual (120 pages)
- Currently about 2500 registered users

Field Description in Differential Algebra

There are various DA algorithms to treat the fields of beam optics efficiently.
For example, **DA PDE Solver**

- requires to supply only
  - the midplane field for a midplane symmetric element.
  - the on-axis potential for straight elements like solenoids, quadrupoles, and higher multipoles.
- treats arbitrary fields straightforwardly.
  - Magnet (or, Electrostatic) fringe fields:
    The Enge function fall-off model

$$F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + ... + a_6 \cdot (s/D)^5)}$$

    where D is the full aperture.
    Or, any arbitrary model including the measured data representation.
  - Solenoid fields including the fringe fields.
  - Measured fields: E.g. Use Gaussian wevelet representation.
  - Etc. etc.

DA Fixed Point PDE Solvers

The **DA fixed point theorem** allows to **solve PDEs iteratively** in **finitely many steps** by rephrasing them in terms of a fixed point problem.

Consider the rather general PDE

$$a_1 \frac{\partial}{\partial x}\left(a_2 \frac{\partial}{\partial x} V\right) + b_1 \frac{\partial}{\partial y}\left(b_2 \frac{\partial}{\partial y} V\right) + c_1 \frac{\partial}{\partial z}\left(c_2 \frac{\partial}{\partial z} V\right) = 0,$$

where a_i, b_i, c_i are functions of x, y, z.

The PDE is re-written in **fixed point form** as

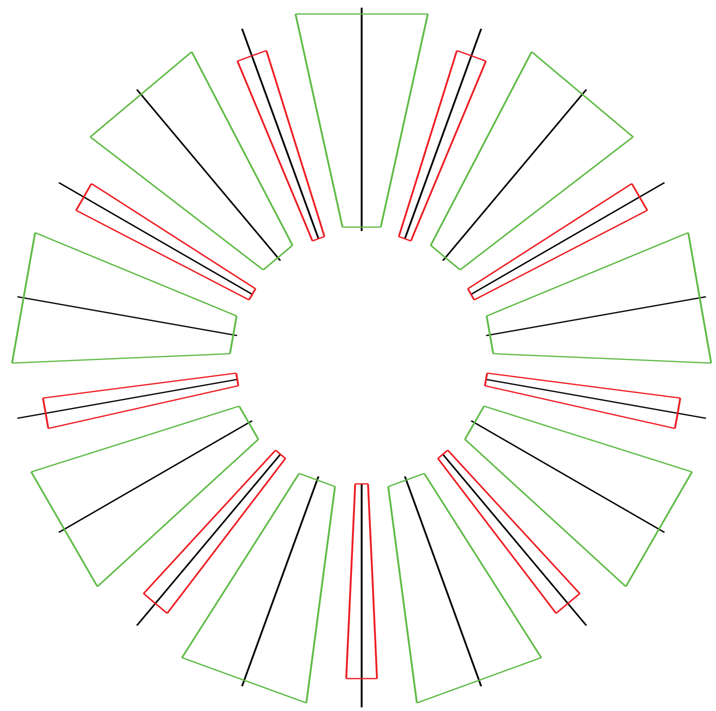
$$V = V|_{y=0} + \int_0^y \frac{1}{b_2}\left(b_2 \frac{\partial V}{\partial y}\right)\Bigg|_{y=0} - \int_0^y \frac{1}{b_2} \int_0^y \left(\frac{a_1}{b_1}\frac{\partial}{\partial x}\left(a_2 \frac{\partial V}{\partial x}\right) + \frac{c_1}{b_1}\frac{\partial}{\partial z}\left(c_2 \frac{\partial V}{\partial z}\right)\right) dy dy.$$

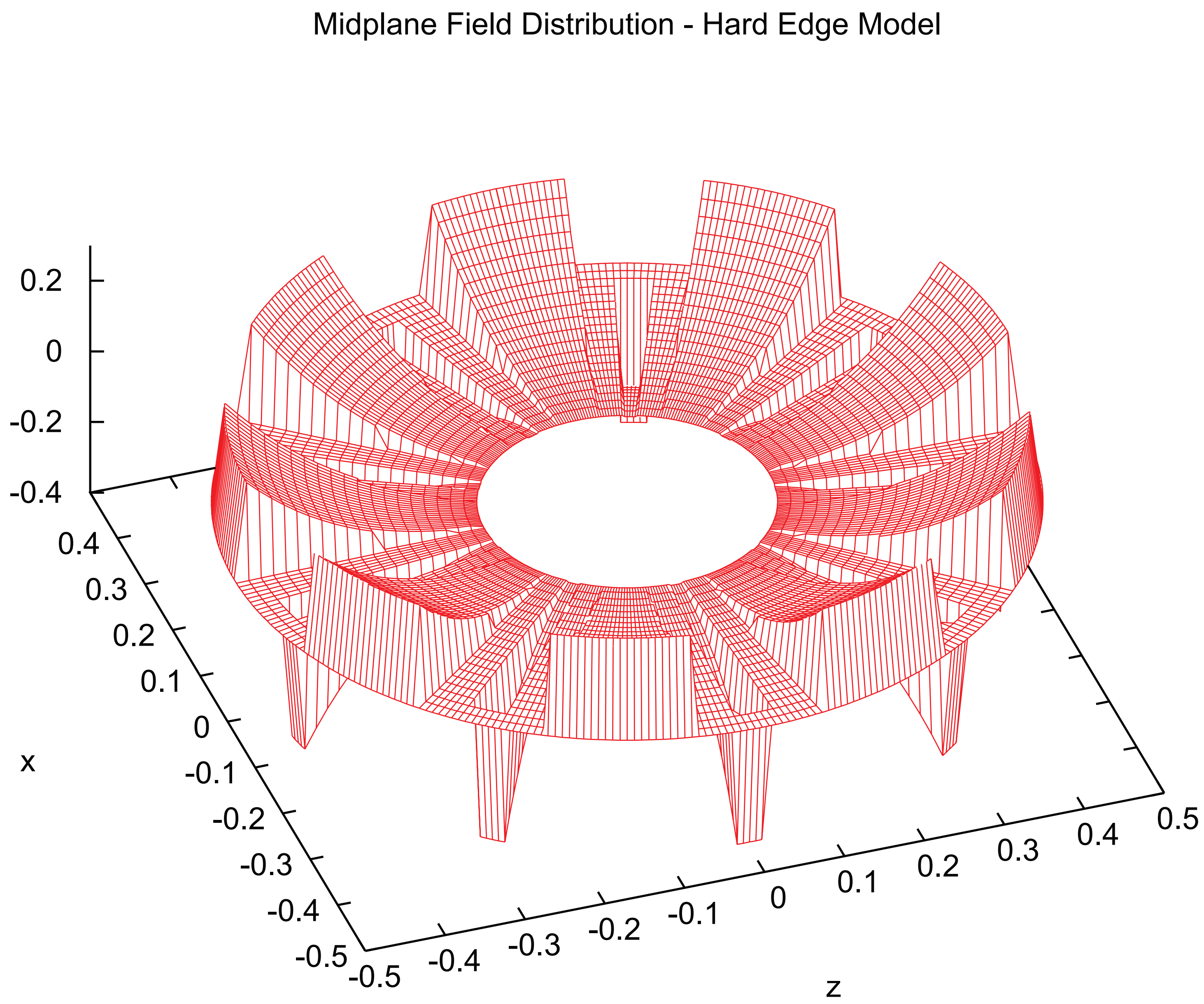
Assume the derivatives of V and $\partial V/\partial y$ with respect to x and z are **known in the plane** $y = 0$. Then the right hand side is **contracting** with respect to y (which is necessary for the DA fixed point theorem), and the various orders in y can be **iteratively** calculated by mere iteration.

Field Computations, Out-of-Plane Field Expansions

- Example using a nonscaling FFAG model

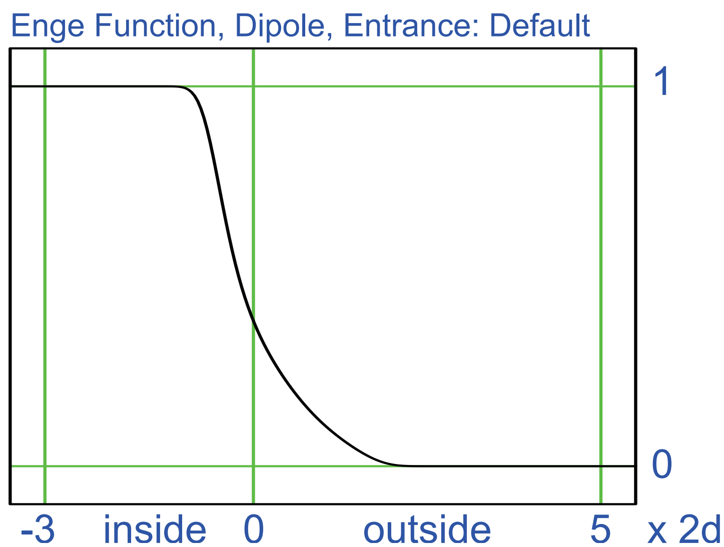
NSFFAG 9 2 full system





Midplane Field Distribution - Hard Edge Model
0.2
0
-0.2
-0.4
x
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
z
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

The Profile of the Default Fringe Field Falloff of Deflectors



A suitable analytical function describing the field falloff – often the Enge function:

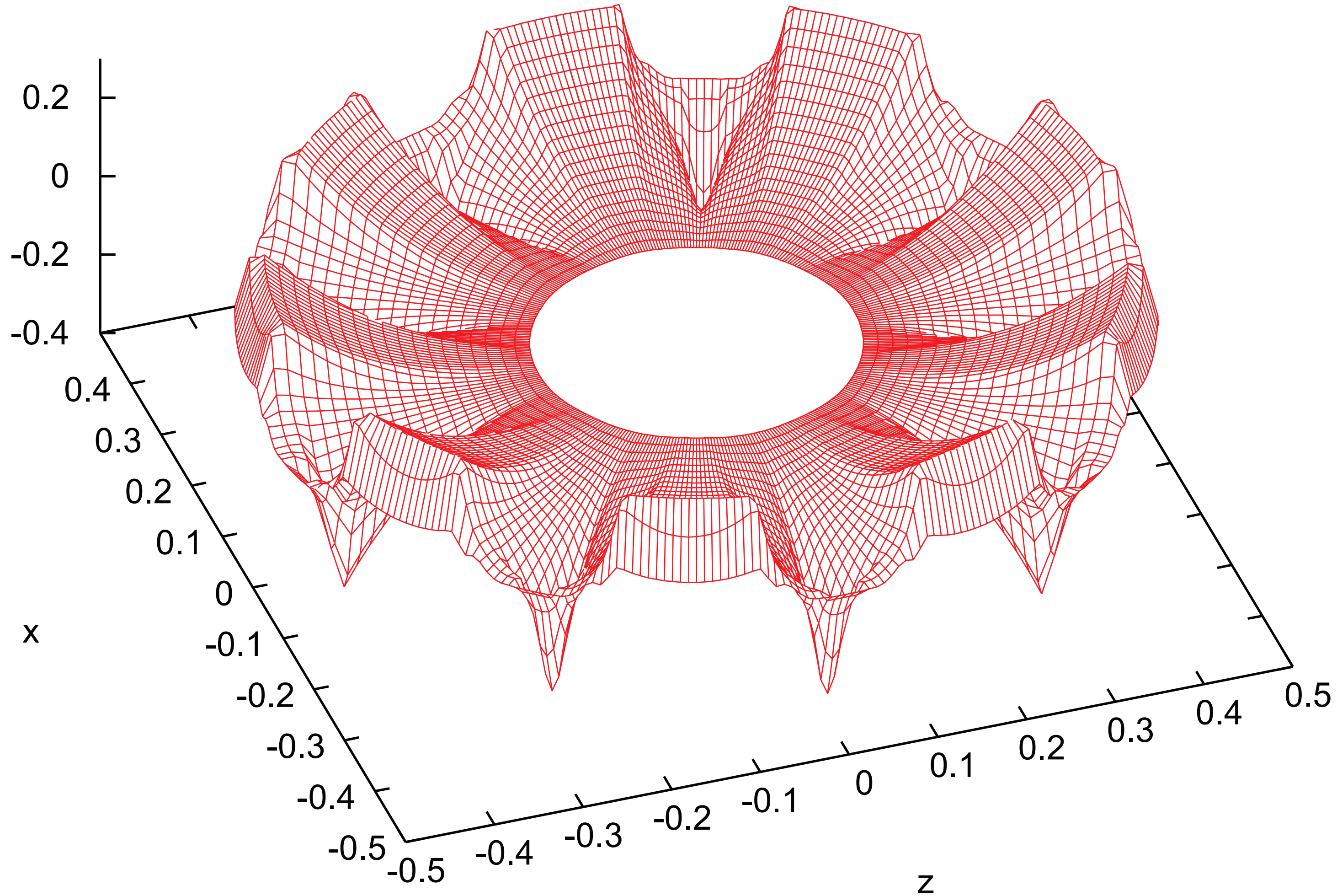
$$F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + \ldots + a_6 \cdot (s/D)^5)}, \qquad D = 2d.$$

– The function varies smoothly from 1 (fully inside) to 0 (fully outside).

– This default model is based on magnet measurement at SLAC.

– The plot is drawn by the **FP** command in COSY INFINITY.

Midplane Field Distribution - COSY default DI Fringe Field Model



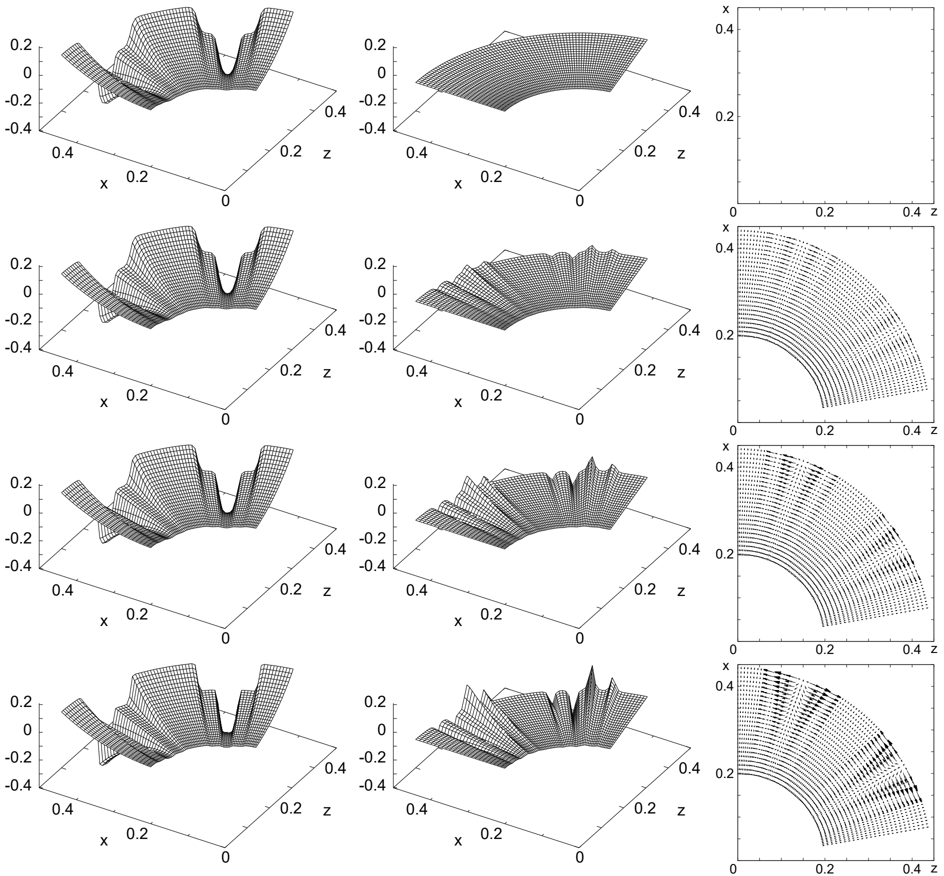


Fig. 9. Field distribution of a 80° section in the x and y planes—in the midplane and at $y =$ 2.5 mm, 5 mm, 7.5 mm planes. The distributions of B_y and B_θ are shown in 3D plots, and the B_r–B_θ behavior is shown by vectors. $0.20 \leq r \leq 0.44$ mm.

3.3.2 Bending Elements

COSY INFINITY supports both magnetic and electrostatic elements including so called combined function elements with superimposed multipoles.

...

The following commands let an inhomogeneous combined function bending magnet and a combined function electrostatic deflector act on the map:

MS <radius> <angle> <aperture> $< n_1 > < n_2 > < n_3 > < n_4 > < n_5 >$;

E S <radius> <angle> <aperture> $< n_1 > < n_2 > < n_3 > < n_4 > < n_5 >$;

The radius is measured in meters, the angle in degrees, and the aperture is in meters and corresponds to half of the gap width. The indices n_i describe the midplane radial field dependence which is given by

$$F(x) = F_0 \cdot \left[1 - \sum_{i=1}^{5} n_i \cdot \left(\frac{x}{r_0} \right)^i \right],$$

where r_0 is the bending radius. Note that an electric cylindrical condenser has $n_1 = 1$, $n_2 = -1$, $n_3 = 1$, $n_4 = -1$, $n_5 = 1$, etc, and an electric spherical condenser has $n_1 = 2$, $n_2 = -3$, $n_3 = 4$, $n_4 = -5$, $n_5 = 6$, etc. Homogeneous dipole magnets have $n_i = 0$.

There are various specialized electrostatic deflectors.

...

ECL <radius> <angle> <aperture> ;

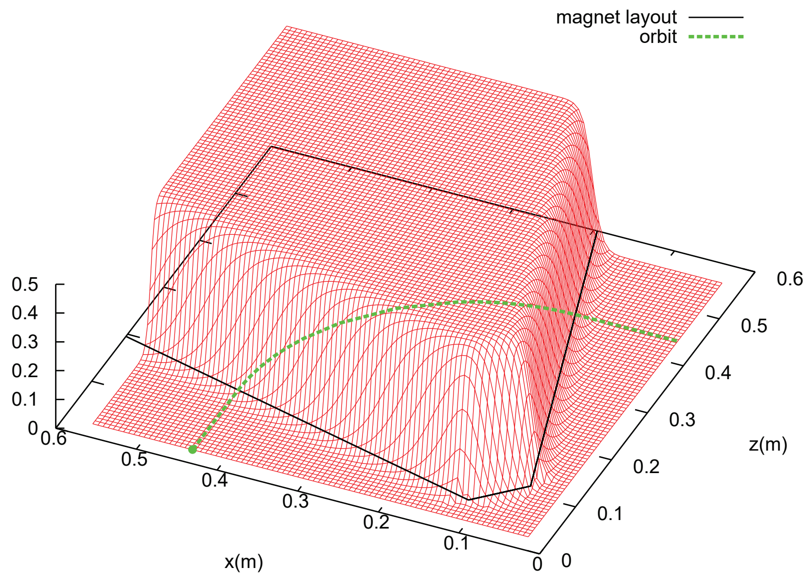
invokes an electrostatic cylindrical deflector, and the element

ESP <radius> <angle> <aperture> ;

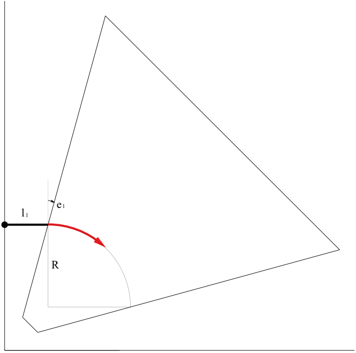
invokes an electrostatic spherical deflector.

A Subtle Problem to Deal with Fringe Fields

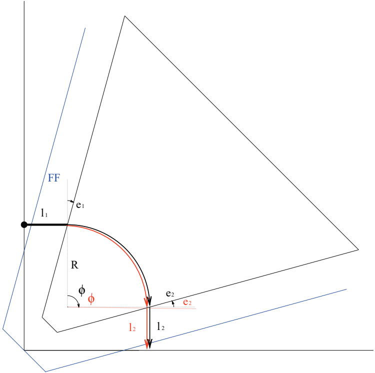
The trajectory differs from that of the idealized one.
If a hard edge model is used at the initial planning stage,
one *would* encounter the necessity to adjust the system.
– The degree of the severity depends on the system parameters.



An illustrating example:
A homogeneous bending magnet with fringe fields.



e1
l1
R


FF
l1
e1
R
ϕ
ϕ
e2
e2
l2
l2

Fringe Fields and Their Nonlinearities

- Fringe fields are often the main source of (non-deliberate) nonlinearities
  - In main fields, one of course attempts very carefully to keep the field constant in direction of reference orbit, and imposes specific axial dependencies
  - In fringe fields, there is natural nonlinearity due to unavoidable curvature of electric or magnetic field lines.
  - These curvatures of fields affect particles at different distances from reference orbit differently, and because of curvature, they do so nonlinearly.
  - All these things are unavoidable; they are a direct consequence of Maxwell's equations.

The Pain with Electrostatic Elements

- Unless one is very careful, there will be various undesirable effects:
  - The motion from before to after the element satisfies energy conservation, but the integrator does not know this
  - Repeated small violations of energy conservation can lead to either oscillations, or big long term effects
  - Particular problem: due to offset of reference orbit, it is very useful to re-align elements. This is normally done after each part:
  - After entrance fringe field, after main field, after exit fringe field
  - Each re-alignment causes small change in geometry, and hence small change in potential!

Electrostatic Elements – Study Case –

A cylindrical element with fringe fields (22.5, R=24.7m, $d = 5$cm)

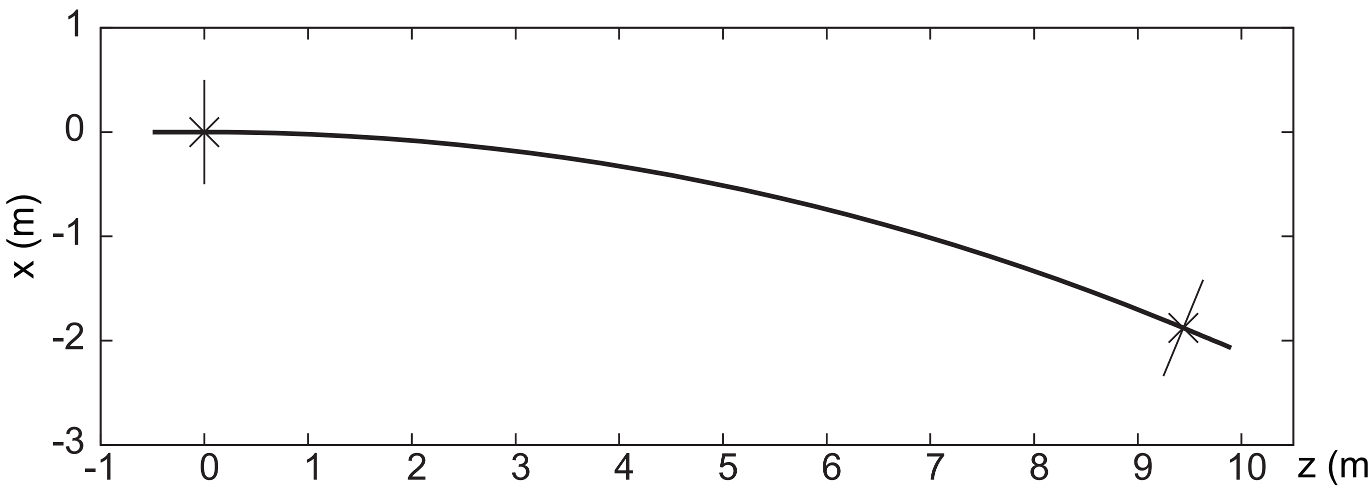
• Fields, especially fringe fields

• Reference orbits

• COSY fringe field computation modes FR 3 and FR 2.5

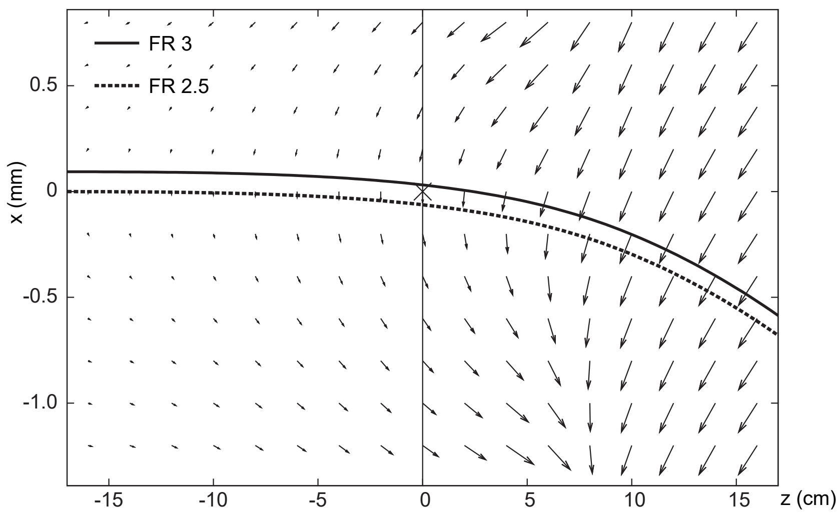
— FR 2.5: Computes through the main and the fringe fields

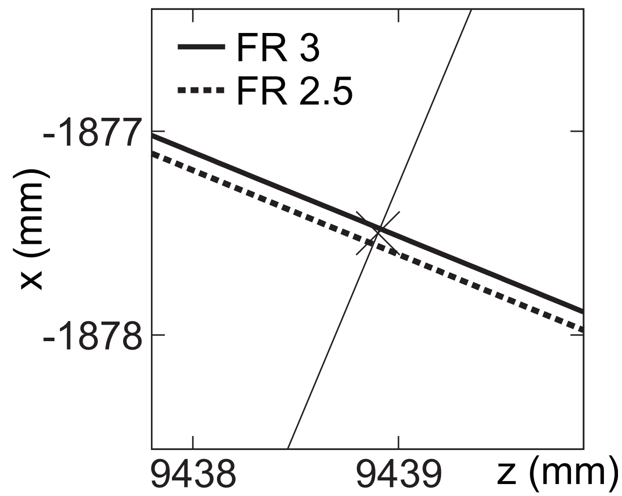
— FR 3: Further, preserves the mirror symmetry



The effective field boundaries, the crossing points expected by the hard edge model are marked by lines, and $\times$'s.

Reference Trajectories at the Entrance and the Exit Fringe Fields





Entrance: The horizontal field components (E_z, E_x) in the midplane are shown; E_z is multiplied by 10^4, and the z and x axis scales differ by a factor of 100.

The entrance and the exit positions in the hard edge mode (FR 0) are marked by $\times$'s.

The reference orbits for the fringe field modes FR 3 (solid) and FR 2.5 (dash): The offset of ~ 0.1mm is seen.

mode	x_f	a_f	y_f	b_f	l_f	dep.
FR 3	0.876407	-0.024984	0	0	-0.343123	x_i
	9.282307	0.876407	0	0	-1.697380	a_i
	0	0	1.000588	0.000121	0	y_i
	0	0	9.686087	1.000588	0	b_i
	1.697380	0.343123	0	0	1.712180	δ_i
FR 2.5	0.87640**8**	-0.024984	0	0	-0.34312**0**	x_i
	9.282**277**	0.876407	0	0	-1.6973**52**	a_i
	0	0	1.000588	0.00012**2**	0	y_i
	0	0	9.686**121**	1.0005**90**	0	b_i
	1.6973**64**	0.34312**0**	0	0	1.7121**64**	δ_i
FR 2.5 Reverse	0.876407	-0.024984	0	0	-0.34312**0**	x_i
	9.282**277**	0.87640**8**	0	0	-1.6973**64**	a_i
	0	0	1.0005**90**	0.00012**2**	0	y_i
	0	0	9.686**121**	1.000588	0	b_i
	1.6973**52**	0.34312**0**	0	0	1.7121**64**	δ_i
FR 0	0.876**045**	-0.02**5052**	0	0	-0.343**058**	x_i
	9.2823**48**	0.876**045**	0	0	-1.6973**94**	a_i
	0	0	1.000**000**	0.000**000**	0	y_i
	0	0	9.68**5965**	1.000**000**	0	b_i
	1.6973**94**	0.343**058**	0	0	1.712**239**	δ_i

Table 1. Transfer maps of a cylindrical electric deflector with fringe fields (FR 3, 2.5) and without fringe fields (FR 0). Digits differing from the FR 3 case are marked by bold face. $(l_f|l_i) = 1$ for all cases; omitted to save space.

Testing Electrostatic Elements

- The Spherical Capacitor is an excellent test case for studying the performance of codes:
  - Observe that just in the gravitational case, the motion follows Kepler orbits
  - Set up a “lattice” consisting of n spherical capacitors of angle $2\pi/n$
  - Compute dynamics for one revolution, it should be the identity
  - In particular, in map picture, this gives a nice effect because ALL transfer matrix elements have to vanish
  - Very important: need non-relativistic motion for testing! (Remember precession of perihelion of Mercury)

Since there is frequently various confusion about electric elements and their properties, we describe a few elementary consistency tests and observations that are useful for checking purposes. First, an electric cylindrical condenser is invariant under translation along the y axis, and the y motion behaves like a drift. Furthermore, although the x motion depends on y and b, an offset in the y direction does not alter the x motion. Thus, a map produced by **ECL**, $\mathcal{M}_{ECL}$, and a y offset map $\mathcal{M}_{\triangle y}$, need to commute, i.e. we must have $\mathcal{M}_{ECL} \circ \mathcal{M}_{\triangle y} = \mathcal{M}_{\triangle y} \circ \mathcal{M}_{ECL}$. A particularly powerful way to check this within the transfer map concept of COSY is to make the offset Δy a DA parameter, and correspondingly use the command "**SA** 0 DA(5) ;".

A similar consistency test can be performed for an electric spherical condenser, for which any transfer map must be invariant under any rotation around an axis through the center. One of the meaningful tests based on this observation for which the geometry is easily worked out is a tilt of the plane of motion through the condenser along a central axis that is parallel to the reference orbit. A corresponding tilting map $\mathcal{M}_{\triangle}$ consists of moving to the center, rotating, and moving back; expressed in COSY's axis manipulating tools, it can be achieved for example by "**SA** -R 0 ; **RA** DA(5) ; **SA** R 0 ;". Now choosing a spherical condenser with a deflection of 180° entails that the reference orbit after the device is parallel with the the one before the device, but points in the opposite direction, so that the necessary back rotation requires the same (and not the opposite) angle. So, letting $\mathcal{M}_{ESL180}$ denote the spherical deflector, we must have that $\mathcal{M}_{\triangle} \circ \mathcal{M}_{ESL180} \circ \mathcal{M}_{\triangle}$ agrees with $\mathcal{M}_{ESL180}$. Yet another test is based on the observation that the motion is that of a Kepler problem, which entails that the motion should return to the exact original state after one full revolution, independent of initial conditions. Thus $\mathcal{M}_{ESL360}$, or a combination of k copies of $\mathcal{M}_{ESL(360/k)}$ for any k must lead to an identity map.

The Kepler Test: A Spherical Electrostatic Deflector of 360 deg

The transfer map of a spherical deflector of 360 should be an identity map.
For the test, the fringe fields must not be included.

The high-order transfer map is computed by the short code in COSY INFINITY:

```
OV 5 2 0 ; RPP 1e-6 ; UM ; ESP 1 360 0.1 ; PM 6 ;
```

x_f	a_f	y_f	b_f	dep.
1	6.696550e-9	0	0	x_i
-6.696550e-9	1	0	0	a_i
0	0	1	-4.702209e-15	y_i
0	0	4.702209e-15	1	b_i
-3.061968e-12	-6.688603e-16	0	0	x_i^2
-1.339527e-8	4.364584e-16	0	0	$x_i a_i$
...	...	...	...	...
0	0	2.084504e-8	-4.517182e-5	b_i^5

A “lattice” consisting of n spherical deflectors of angle $360/n$ also produces the same result.

COSY Repetitive Tracking

Using the nonlinear transfer map $\mathcal{M}(\vec{z}_i, \vec{\delta})$, perform tracking to estimate
– the dynamic aperture,
– presence of resonances, etc.

There are various repetitive tracking methods in COSY INFINITY.

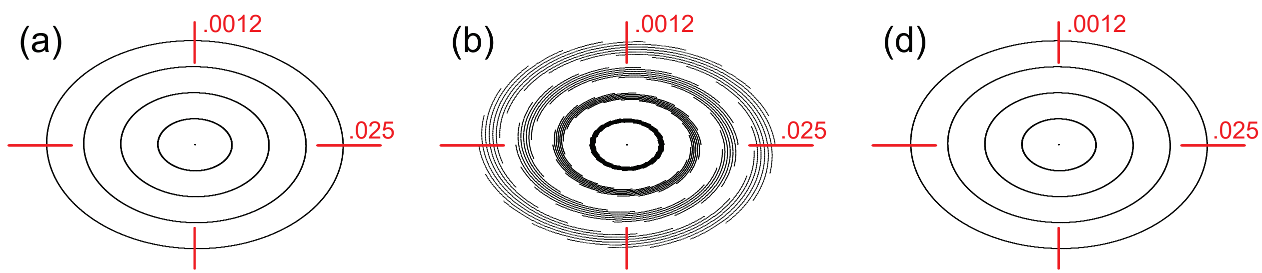
- Several modes of symplectic tracking
  - EXPO with minimal modifications, etc.
  - no symplectification
- Tracking in normal form coordinates.

Long Term Tracking of an Electrostatic Cylindrical Deflector

A cell consisting of an electrostatic cylindrical deflector of 22.5 deflection, 24.7m radius, $d = 5$cm, and a drift 0.6m.

Compare various computation orders, the fringe field computation modes FR 0, 2.5, 3, and the classical piece-by-piece computation mode.

$x_{IC} = 5, 10, 15, 20$mm. 6D tracking. x-a plots for every revolution for 5000 turns.



mode	FR 0	piece-by-piece	FR 2.5 / FR 3
order	3rd / 7th	3rd / 7th	3rd / 7th

Violations of the energy conservation are observed in the piece-by-piece method due to the energy mismatch; the bulk already manifests itself in low orders and which is not remedied by using higher orders.

Summary

- Basic principles of charged particle motions
  - Exceptions for special cases
    Searches for EDMs (electric dipole moments) are preferably done in purely electrostatic rings
- Methods of studying charged particle motions
  - Transfer maps
  - The DA (Differential Algebraic) method
  - The DA PDE solver for obtaining electromagnetic fields
  - Repetitive trackings
- Treatment of fringe fields and special cautions
- Examples
  - Out-of-plane expansions of the fields
  - Fringe field treatments
  - A bench mark test of a electrostatic spherical deflector
  - Long-term repetitive trackings

An Introduction to Beam Physics

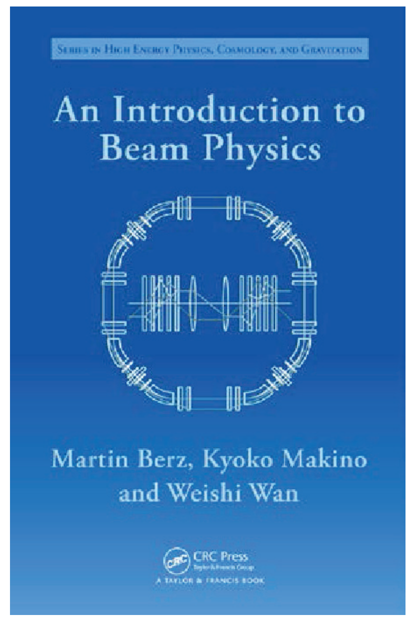
Authors/Affiliations

Martin Berz, Michigan State University, East Lansing, USA
Kyoko Makino, Michigan State University, East Lansing, USA
Weishi Wan, Lawrence Berkeley National Laboratory, Berkeley, USA

SAVE 20%

This book provides an introduction to the physics of beams. This field touches many areas of physics, engineering and the sciences. In general terms, beams describe ensembles of particles with initial conditions similar enough to be treated together as a group, so that the motion is a weakly nonlinear perturbation of that of a chosen reference particle. Applications of particle beams are very wide, including electron microscopes, particle spectrometers, medical radiation facilities, powerful light sources, astrophysics, to name a few, and reach all the way to the largest scientific instruments built by man, namely large synchrotrons and storage rings like LHC at CERN.

The text is based on lectures given at Michigan State University's Department of Physics and Astronomy, the online VUBeam program, the US Particle Accelerator School, the CERN Academic Training Programme, and various other venues. Selected other material is added to round out the presentation and cover other significant topics. It is accessible to students of physics, mathematics and engineering at the beginning graduate or upper division undergraduate level.



Contents

Catalog no. IP127
November 2014, 314 pp.
ISBN: 978-0-7503-0263-0
$129.95 / £82.00

SAVE 20% when you order online and enter Promo Code AZP98
FREE standard shipping when you order online.