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kulak

Implementing New Beam Line Elements into a Moment Method Beam Dynamics Code

Toon Roggen

Wave Propagation and Signal Processing Research Group

Department of Physics and Astronomy

KU Leuven, Belgium

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V-Code

Beam Dynamics Code

→ TEMF, TU Darmstadt

→ Moment approach

Particle density distribution fct.:

$$f(\vec{r}, \vec{p}, \tau)$$

Space coordinates: $\vec{r} = (x, y, z)$

Norm. Momentum: $\vec{p} = (p_x, p_y, p_z)$

Equivalent time: $\tau = c \cdot t$

→ Vlasov equation

V-Code

Beam Dynamics Code

→ TEMF, TU Darmstadt

→ Moment approach

→ Vlasov equation

$$\frac{\partial f}{\partial \tau} + \frac{\partial f}{\partial \vec{r}} \cdot \frac{\vec{p}}{\gamma} + \frac{\partial f}{\partial \vec{p}} \cdot \frac{\vec{F}}{m_0 c^2} = 0$$

→ for “slow” varying $\vec{F}$

→ 6D numerically solving ≠ fast

V-Code

- Approach: Discrete set of characteristic moments for $f(\vec{r}, \vec{p}, \tau)$:

  6D → 1D time integration

- Moment definition:

$$<\mu> = \int_{\Omega} \mu\, f(\vec{r}, \vec{p}, \tau)\, \mathrm{d}\Omega \qquad \Omega = \{\vec{r}, \vec{p}\}$$

  – First order: $\mu \in \{x, y, z, p_x, p_y, p_z\}$
  – Higher order: $\mu \in \{(x - <x>)^{l_1} \cdot \ \ldots \ \cdot (p_z - <p_z>)^{l_6}, \ \ldots\}$

V-Code

- Second order:

$$M_{xx} = \sigma_x^2 = \langle (x - \langle x \rangle)^2 \rangle \qquad M_{xy} = \langle xy \rangle_{avc} = \langle (x - \langle x \rangle)(y - \langle y \rangle) \rangle$$

$$M_{yy} = \sigma_y^2 \qquad M_{xz} = \langle xz \rangle_{avc}$$

$$M_{zz} = \sigma_z^2 \qquad M_{yz} = \langle yz \rangle_{avc}$$

$$M_{xx}\ M_{xy}\ M_{xz}\ M_{yy}\ M_{yz}\ M_{zz}$$

$$M_{xp_x}\ M_{xp_y}\ M_{xp_z}\ M_{yp_x}\ M_{yp_y}\ M_{yp_z}\ M_{zp_x}\ M_{zp_y}\ M_{zp_z}$$

$$M_{p_xp_x}\ M_{p_xp_y}\ M_{p_xp_z}\ M_{p_yp_y}\ M_{p_yp_z}\ M_{p_zp_z}$$

Time evolution:

$$\frac{\partial < \mu >}{\partial \tau} = \frac{\partial}{\partial \tau} \int \mu f d\Omega = \int f \frac{\partial \mu}{\partial \tau} + \mu \frac{\partial f}{\partial \tau} d\Omega$$

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$$+ <\frac{\partial \mu}{\partial \vec{r}}\ \frac{\vec{p}}{\gamma}> + <\frac{\partial \mu}{\partial \vec{p}}\ \frac{\vec{F}}{m_0 c^2}>$$

V-Code

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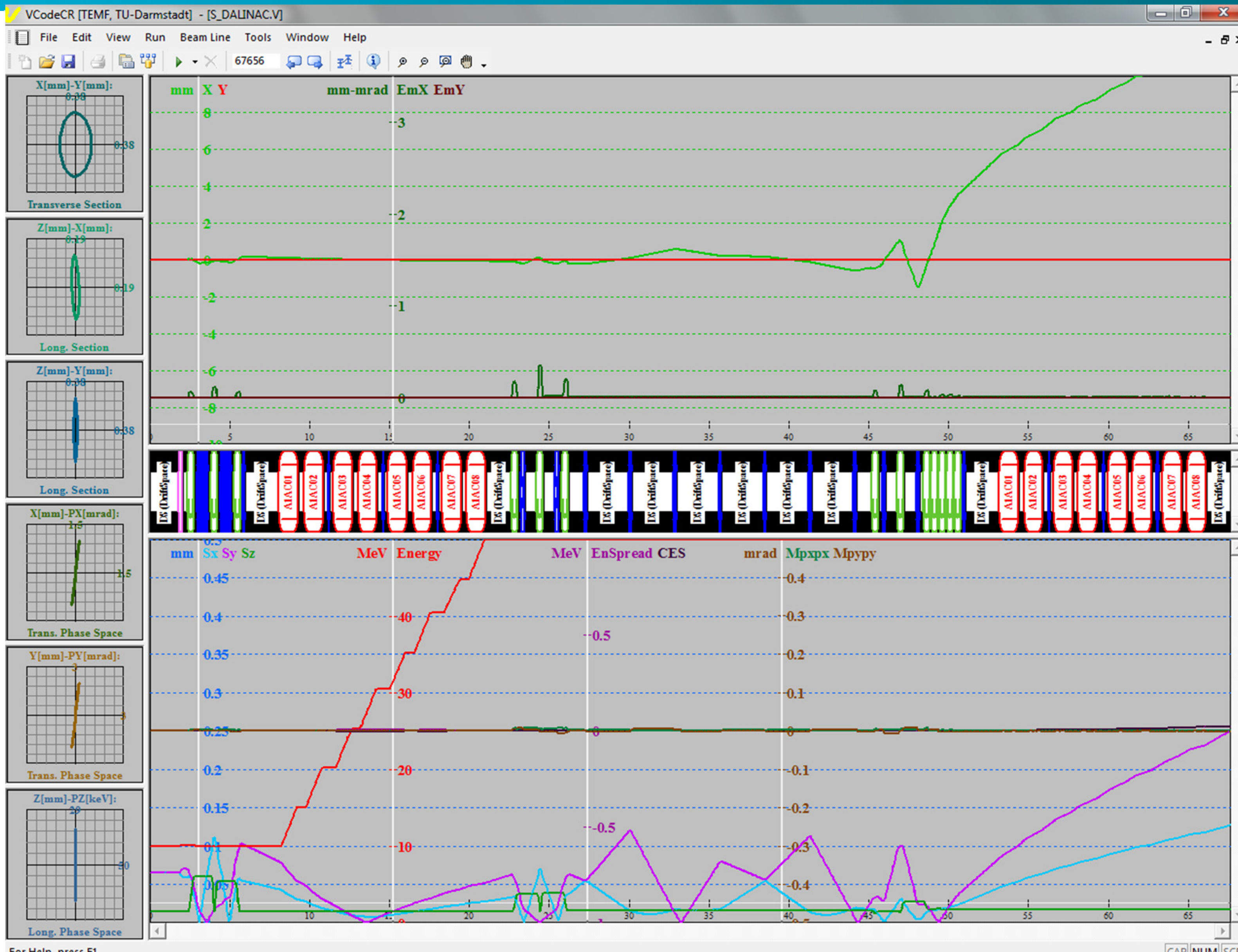
V-Code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File Edit View Run Beam Line Tools Window Help
67656
X[mm]-Y[mm]:
Transverse Section
Z[mm]-X[mm]:
Long. Section
Z[mm]-Y[mm]:
Long. Section
X[mm]-PX[mrad]:
Trans. Phase Space
Y[mm]-PY[mrad]:
Trans. Phase Space
Z[mm]-PZ[keV]:
Long. Phase Space
mm X Y
mm-mrad EmX EmY
mm Sx Sy Sz
MeV Energy
MeV EnSpread CES
mrad Mpxpx Mpypy
For Help, press F1
CAP NUM SCRL

V-Code

Data input:

- Ensemble file

V-Code

Data input:

- Ensemble file
- Field parameter file

V-Code

Data input:

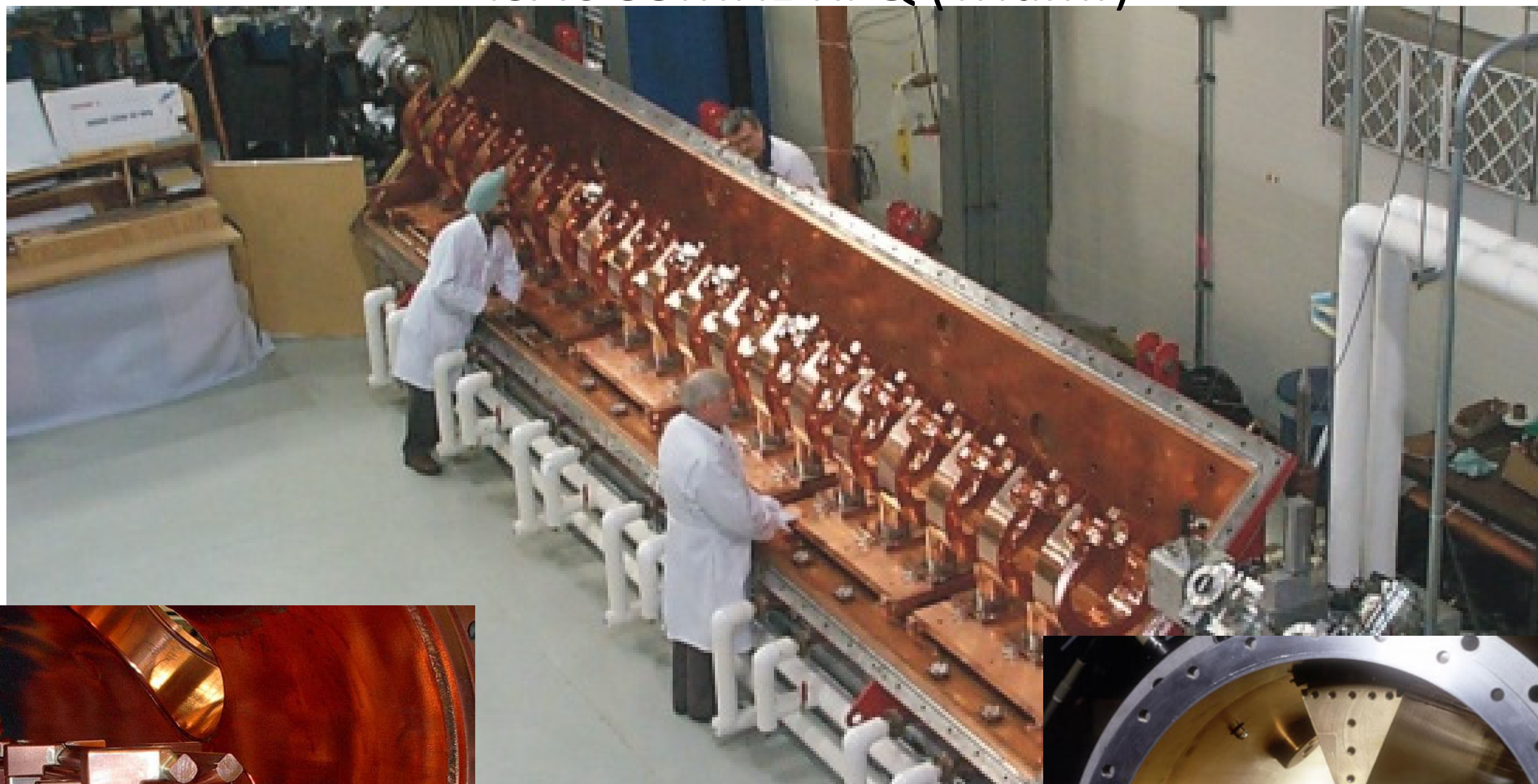
- Ensemble file

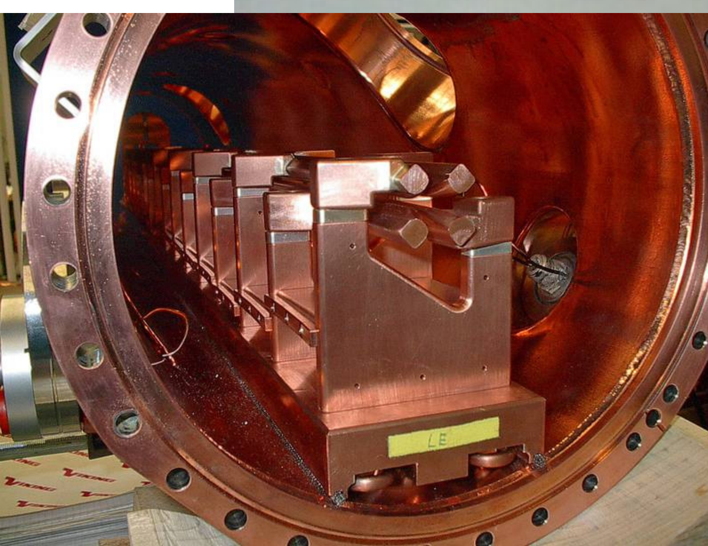
- Field parameter file
- Beam line file

We Th Fr Sa Su Mo Tu We Th Fr Sa Su Mo Tu We Th Fr Sa Su Mo Th

Radio Frequency Quadrupole

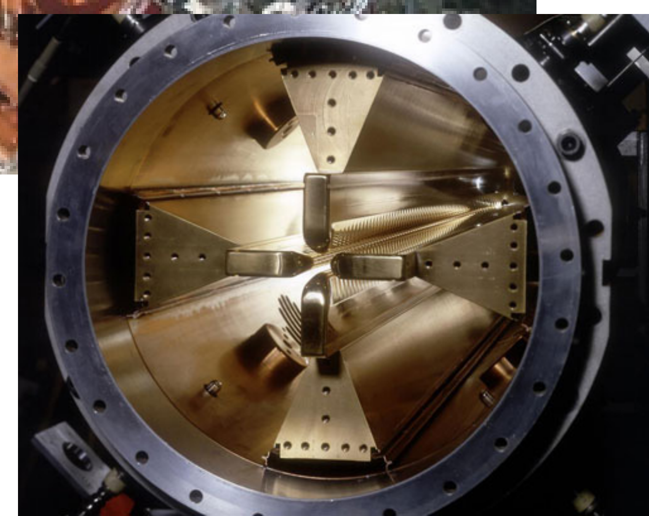
ISAC 35MHz RFQ (Triumf)



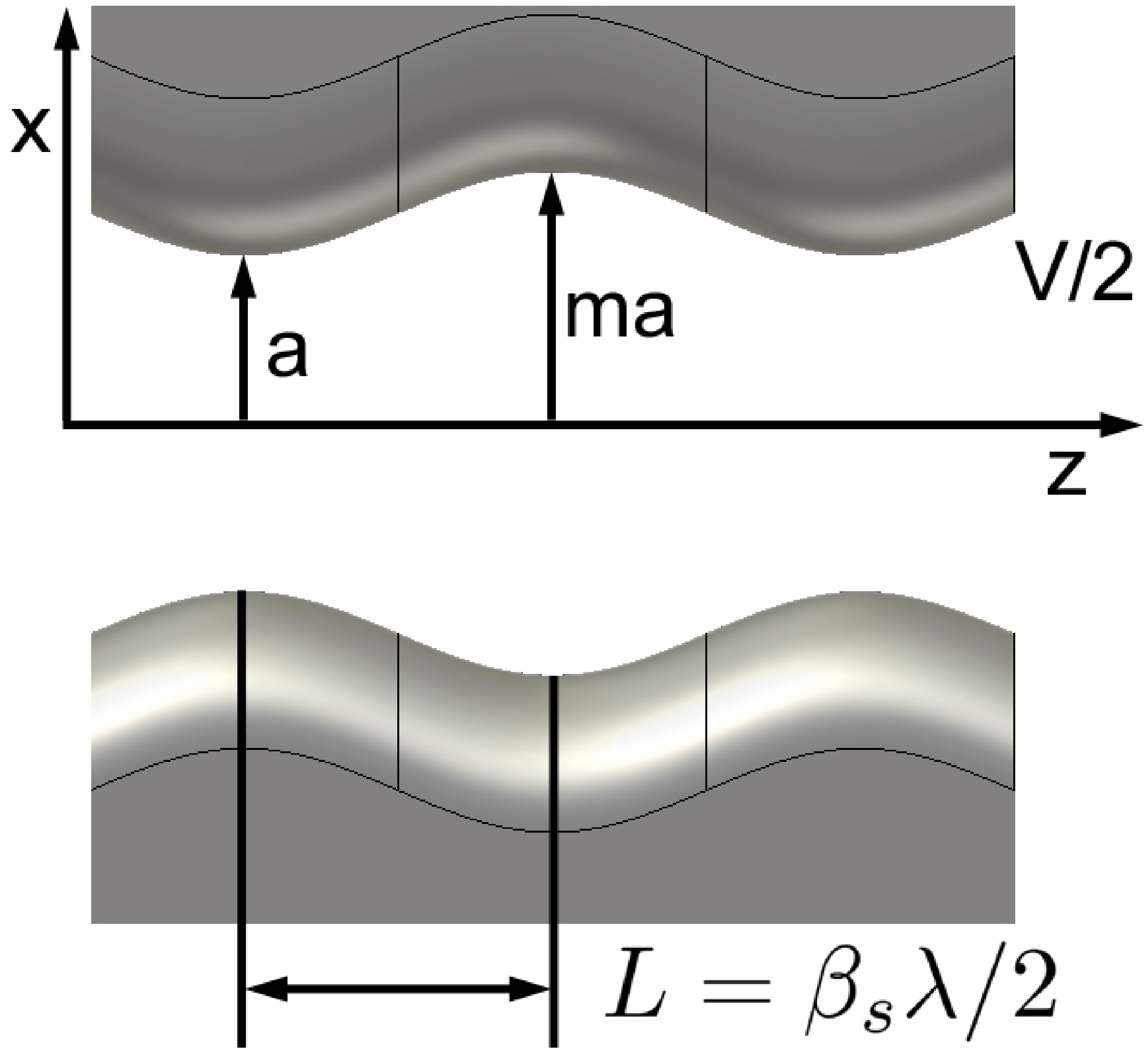


← 4 rod type

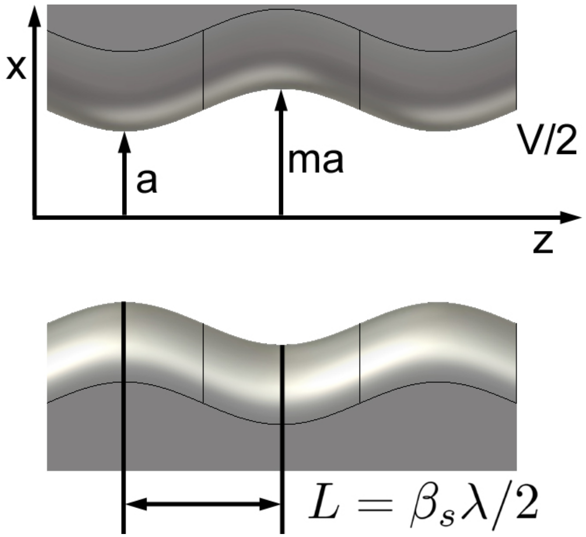
4 vane type →

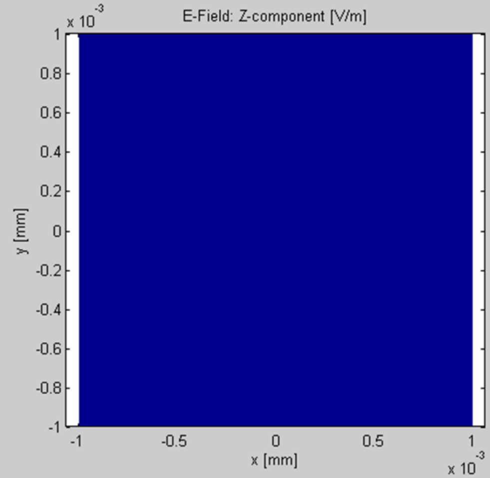


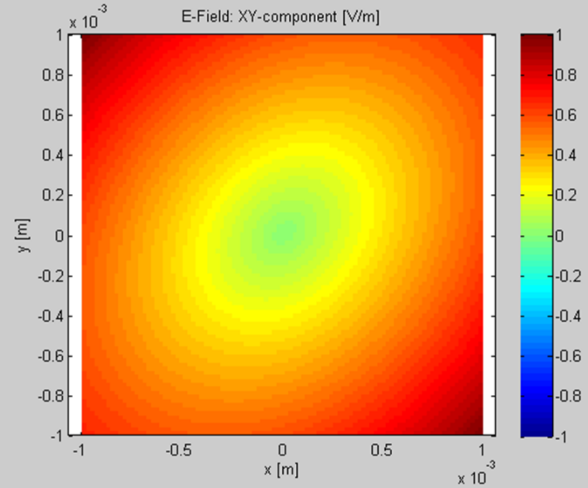
Radio Frequency Quadrupole

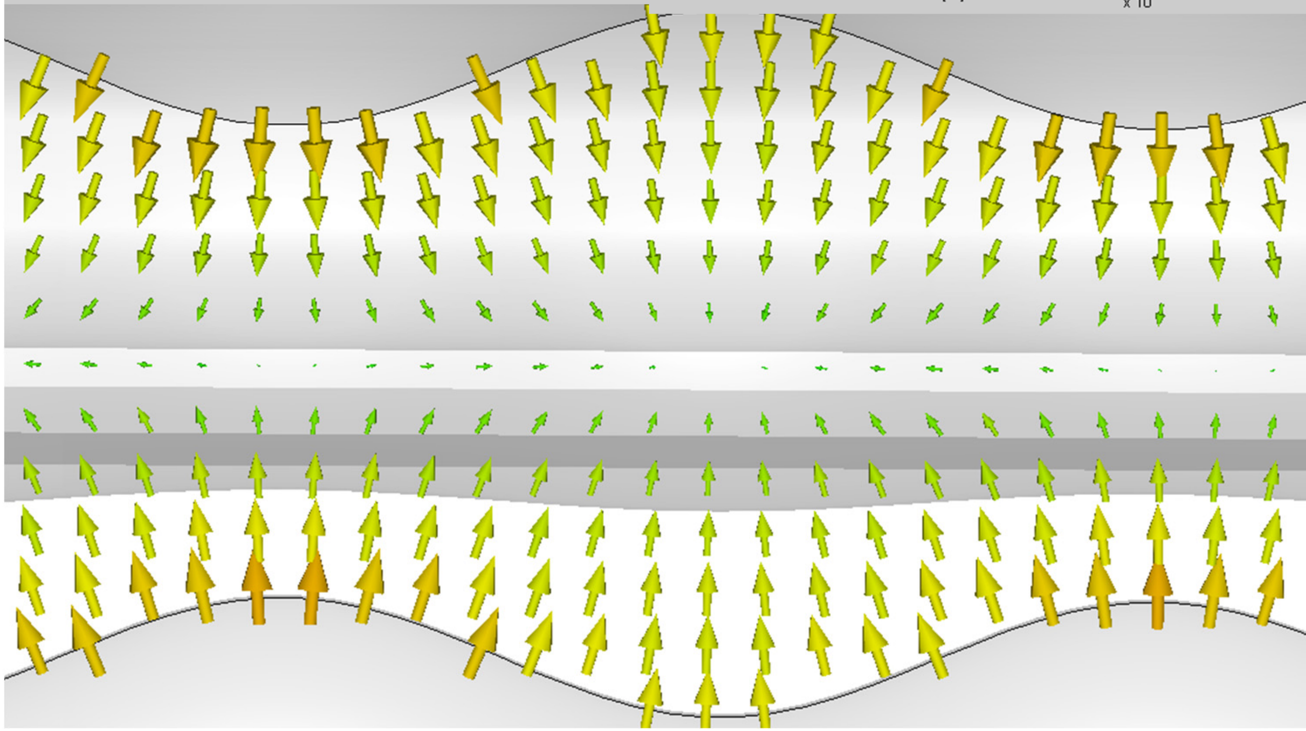


Radio Frequency Quadrupole



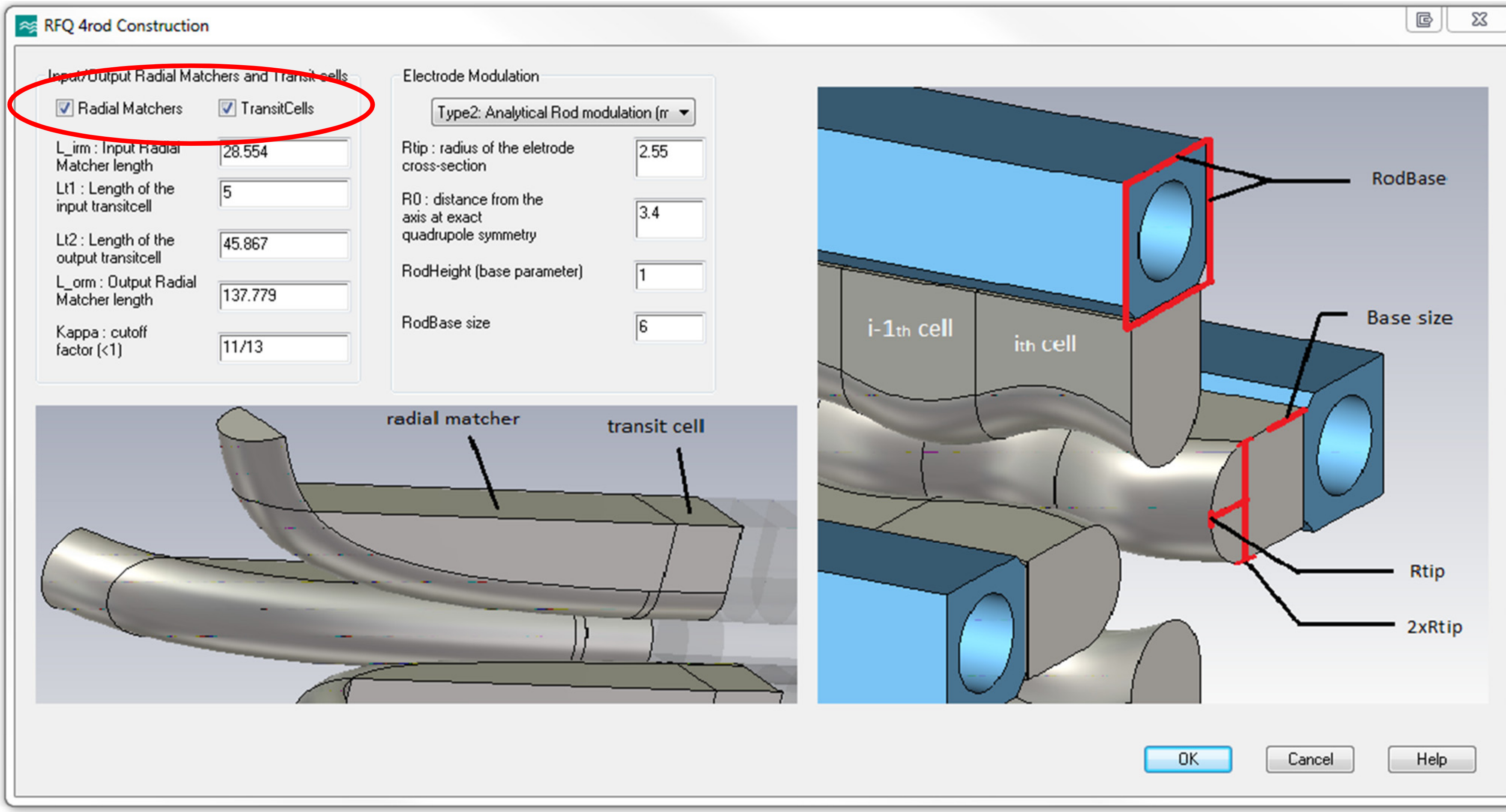




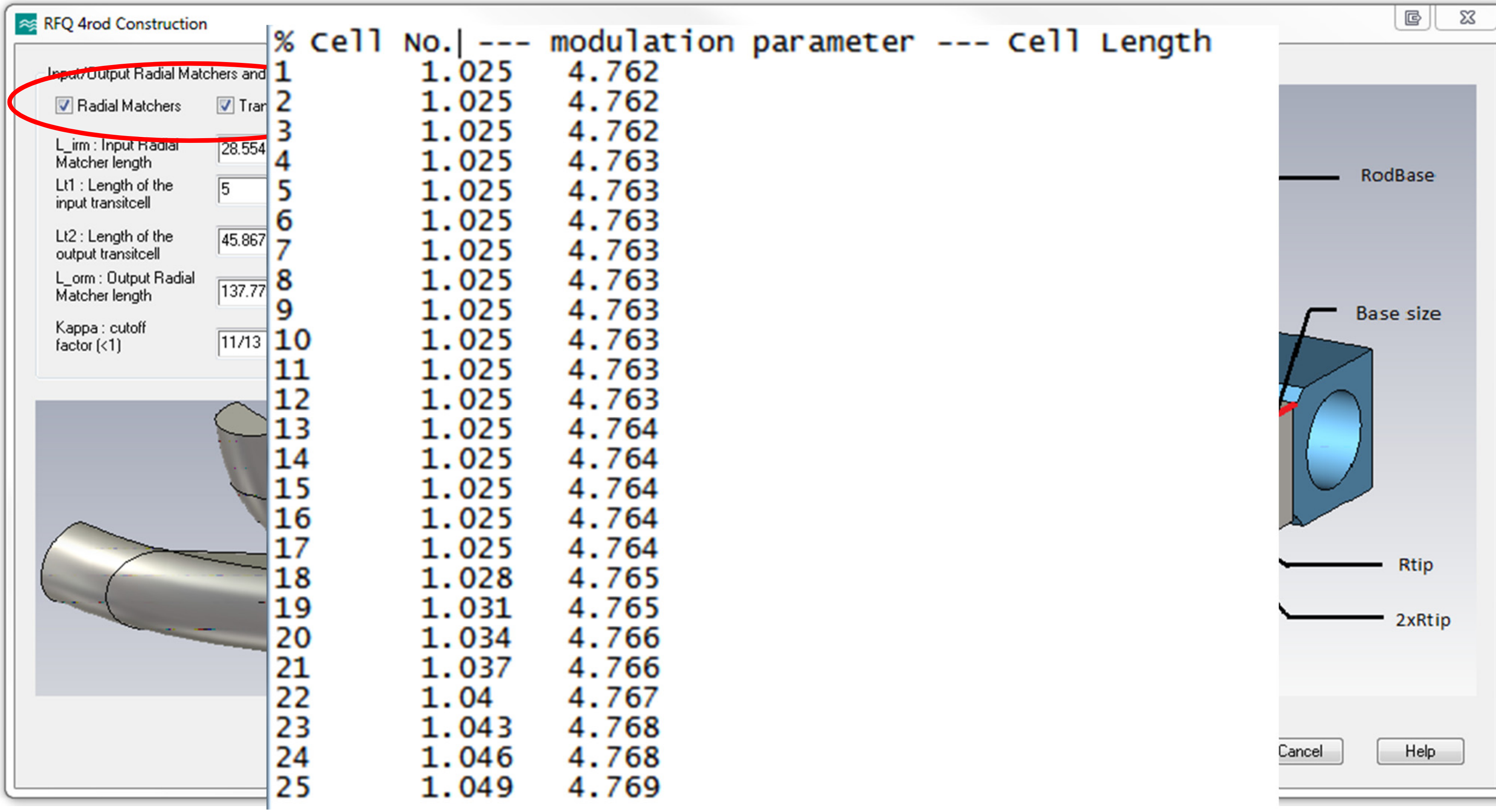


Radio Frequency Quadrupole

CST Studio Suite



CST Studio Suite



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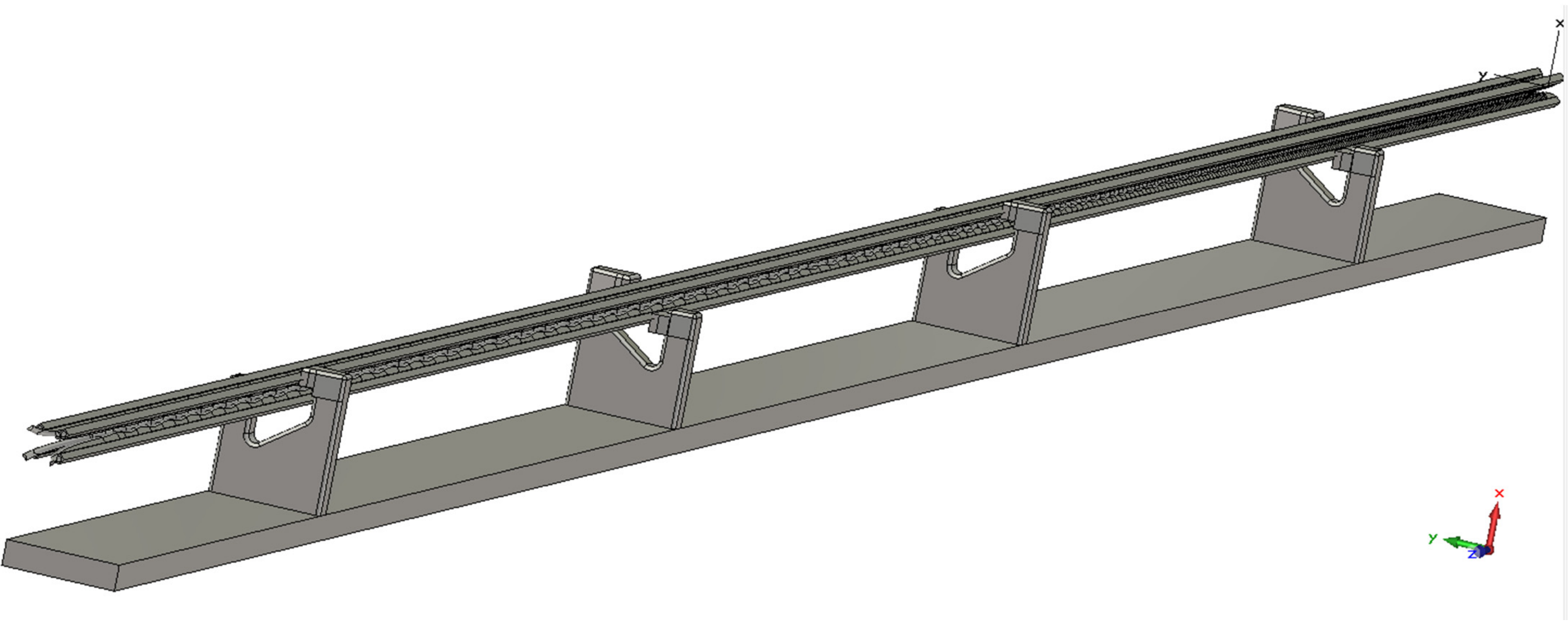
7	1.025	4.763
8	1.025	4.763
9	1.025	4.763
10	1.025	4.763
11	1.025	4.763
12	1.025	4.763
13	1.025	4.764
14	1.025	4.764
15	1.025	4.764
16	1.025	4.764
17	1.025	4.764
18	1.028	4.765
19	1.031	4.765
20	1.034	4.766
21	1.037	4.766
22	1.04	4.767
23	1.043	4.768
24	1.046	4.768
25	1.049	4.769

Radio Frequency Quadrupole



Implementation

General potential function (Quasistatic):

$$U(r,\theta,z,t) = \sin(\omega t + \phi)\Bigg[$$

$$\sum_{p=0}^{\infty} A_{0,2p+1} r^{2(2p+1)} \cos(2(2p+1)\theta) +$$

$$\sum_{n=1}^{\infty}\sum_{s=0}^{\infty} A_{n,s} I_{2s}(knr)\cos(2s\theta)\cos(knz)\Bigg]$$

n+s = 2p+1

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Implementation

General potential function (Quasistatic):

$$U(r,\theta,z,t) = \sin(\omega t + \phi)\Bigg[\sum_{p=0}^{\infty} A_{0,2p+1} r^{2(2p+1)} \cos(2(2p+1)\theta) + \sum_{n=1}^{\infty}\sum_{s=0}^{\infty} A_{n,s} I_{2s}(knr)\cos(2s\theta)\cos(knz)\Bigg]$$

Laplace

n+s = 2p+1

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Implementation

General potential function (Quasistatic):

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$$\sum_{p=0}^{\infty} A_{0,2p+1} r^{2(2p+1)} \cos(2(2p+1)\theta) +$$

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$$\sum_{n=1}^{\infty}\sum_{s=0}^{\infty} A_{n,s} I_{2s}(knr)\cos(2s\theta)\cos(knz)\Bigg]$$

$$k = 2\pi/2L$$

$$2L = \beta_s \lambda$$

n+s = 2p+1

Implementation

General potential function (Quasistatic):

$$U(r,\theta,z,t) = \sin(\omega t + \phi)\Big[$$

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n+s = 2p+1

Implementation

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n+s = 2p+1

A. Letchford, A Comparison of 4-Rod and 4-Vane RFQ-Fields, EPAC98

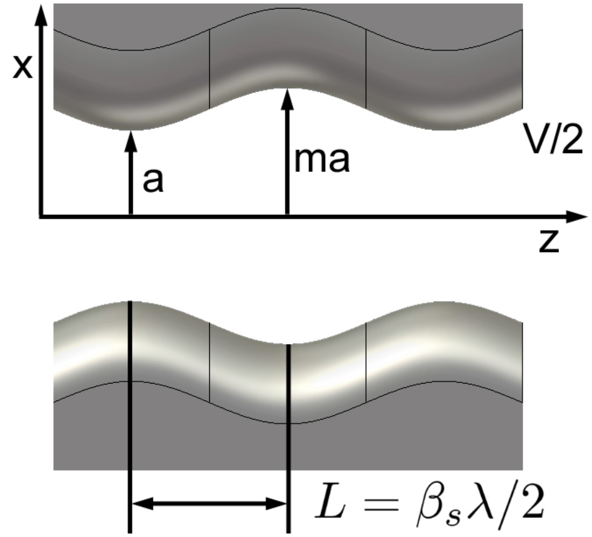
Implementation

Two-Term Potential Function:

$$U(r,\theta,z,t)=\sin(\omega t+\phi)\Big[A_{0,1}r^2\cos(2\theta)+A_{1,0}I_0(kr)\cos(2\theta)\cos(kz)\Big]$$

Collocation points:

- $U=V_0/2, r=a, \theta=0, z=0$
- $U=V_0/2, r=ma, \theta=\pi/2, z=0$



Multipole coefficients:

$$A_{0,1}=\frac{V_0}{2a^2}\frac{I_0(ka)+I_0(kma)}{m^2I_0(ka)+I_0(kma)}=\frac{V_0}{2a^2}X$$

$$A_{1,0}=\frac{V_0}{2}\frac{m^2-1}{m^2I_0(ka)+I_0(kma)}=\frac{V_0}{2}B$$

Implementation

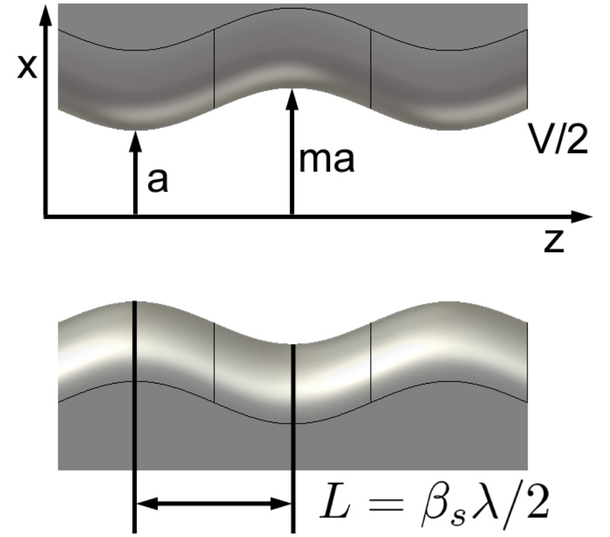
Two-Term Potential Function:

$$U(r,\theta,z,t) = \sin(\omega t + \phi)\Big[$$

$$A_{0,1} r^2 \cos(2\theta) + A_{1,0} I_0(kr)\cos(2\theta)\cos(kz)\Big]$$

Collocation points:

- $U = V_0/2, r = a, \theta = 0, z = 0$
- $U = V_0/2, r = ma, \theta = \pi/2, z = 0$



Multipole coefficients:

$$A_{0,1} = \frac{V_0}{2a^2}\frac{I_0(ka) + I_0(kma)}{m^2 I_0(ka) + I_0(kma)} = \frac{V_0}{2a^2}X$$

$$A_{1,0} = \frac{V_0}{2}\frac{m^2 - 1}{m^2 I_0(ka) + I_0(kma)} = \frac{V_0}{2}B$$

Implementation

Two-Term Potential Function:

$\rightarrow \quad \vec{E} = -\nabla U$

$$E_r = \sin(\omega t + \phi)\frac{V_0}{2}\left[X\frac{2r}{a^2}\cos(2\theta) + BkI_1(kr)\cos(kz)\right]$$

$$E_\theta = \sin(\omega t + \phi)\frac{V_0}{2}\left[-X\frac{r}{a^2}2\sin(2\theta)\right]$$

$$E_z = \sin(\omega t + \phi)\frac{V_0}{2}\left[-BkI_0(kr)\sin(kz)\right]$$

Implementation

Two-Term Potential Function:

$\rightarrow \quad \vec{E} = -\nabla U$

Implementation:

$\rightarrow$ $I_n(kr)$: Taylor expansion

$\rightarrow E_{MAX} \sin(\omega t + \phi)$

E_θ: $\frac{dE_\theta}{dr}$ at $\pi/4$

E_z on axis

E_r: E_θ, E_z and $\nabla.\vec{E} = 0$

Implementation

Four-Term Potential Function:

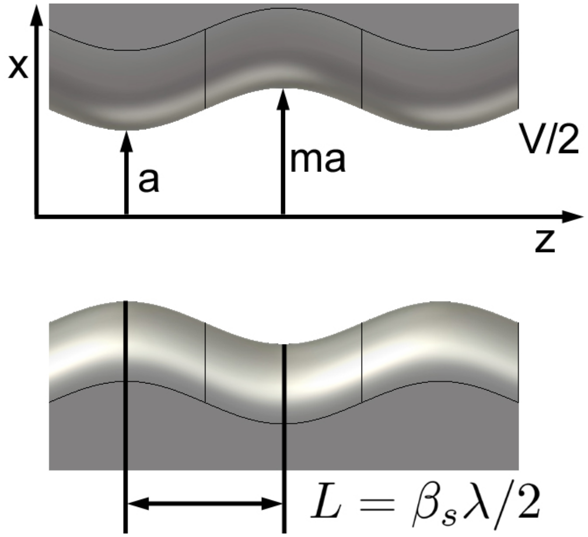
$$U(r,\theta,z,t) = \sin(\omega t + \phi)\Big[$$

$$A_{0,1} r^2 \cos(2\theta) + A_{0,3} r^6 \cos(6\theta)$$

$$+ A_{1,0} I_0(kr)\cos(kz) + A_{1,2} I_4(kr)\cos(kz)\cos(4\theta)\Big]$$

Collocation points:

	U	z	r	θ
1	$V_0/2$	0	a	0
2	$-V_0/2$	0	ma	$pi/2$
3	$V_0/2$	0	$\sqrt{(a+\rho)^2+\rho^2}$	$\cos^{-1}(\frac{a+\rho}{r})$
4	$-V_0/2$	0	$\sqrt{(ma+\rho)^2+\rho^2}$	$\cos^{-1}(\frac{ma+\rho}{r})$



Implementation

Four-Term Potential Function:

$\rightarrow \quad \vec{E} = -\nabla U$

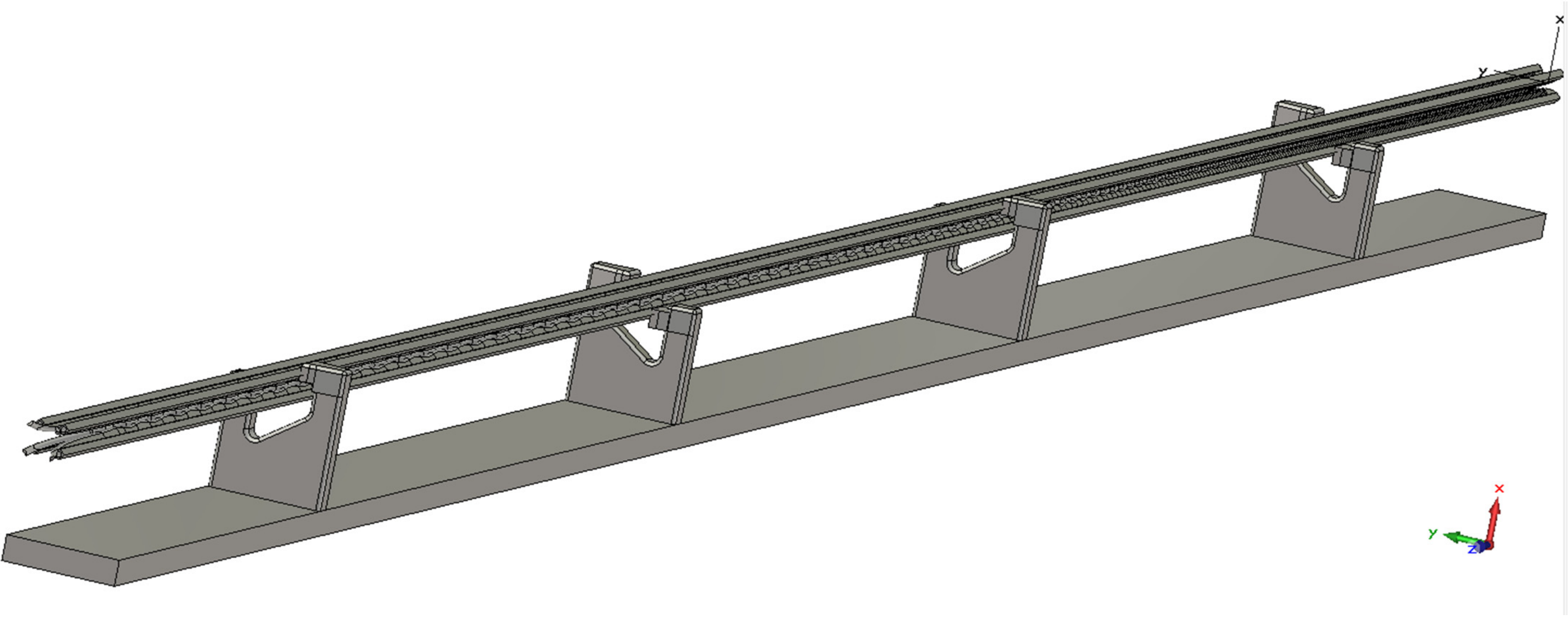
$$E_r = \sin(\omega t + \phi)\Big[2A_{0,1} r \cos(2\theta) + 6A_{0,3} r^5 \cos(6\theta) + kA_{1,0} I_1(kr)\cos(kz) + kA_{1,2} I_5(kr)\cos(kz)\cos(4\theta)\Big]$$

$$E_\theta = \sin(\omega t + \phi)\Big[-2A_{0,1} r \sin(2\theta) - 6A_{0,3} r^5 \sin(6\theta) - 4A_{1,2} I_4(kr)\cos(kz)\sin(4\theta)\Big]$$

$$E_z = \sin(\omega t + \phi)\Big[-kA_{1,0} I_0(kr)\sin(kz) - kA_{1,2} I_4(kr)\sin(kz)\cos(4\theta)\Big]$$

3D Field Maps

Four-Term Potential Function:



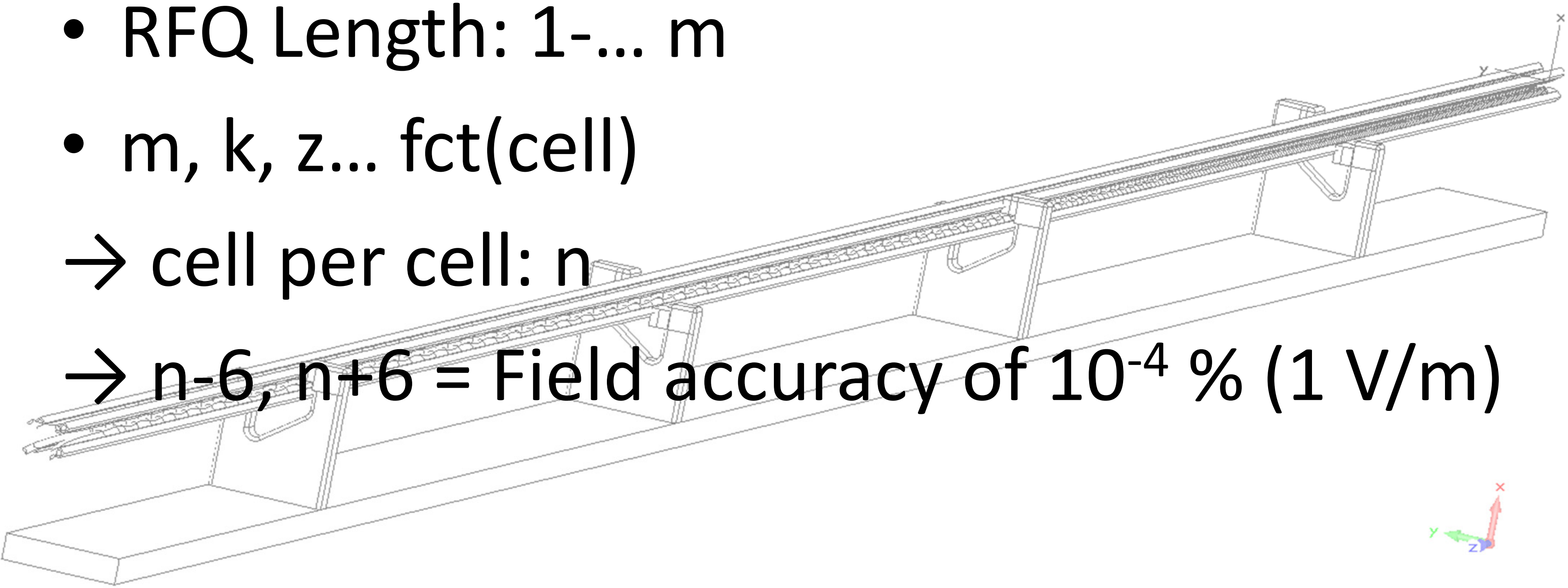
3D Field Maps

Four-Term Potential Function:

- RFQ Length: 1-... m
- m, k, z... fct(cell)

→ cell per cell: n

→ n-6, n+6 = Field accuracy of 10^{-4} % (1 V/m)

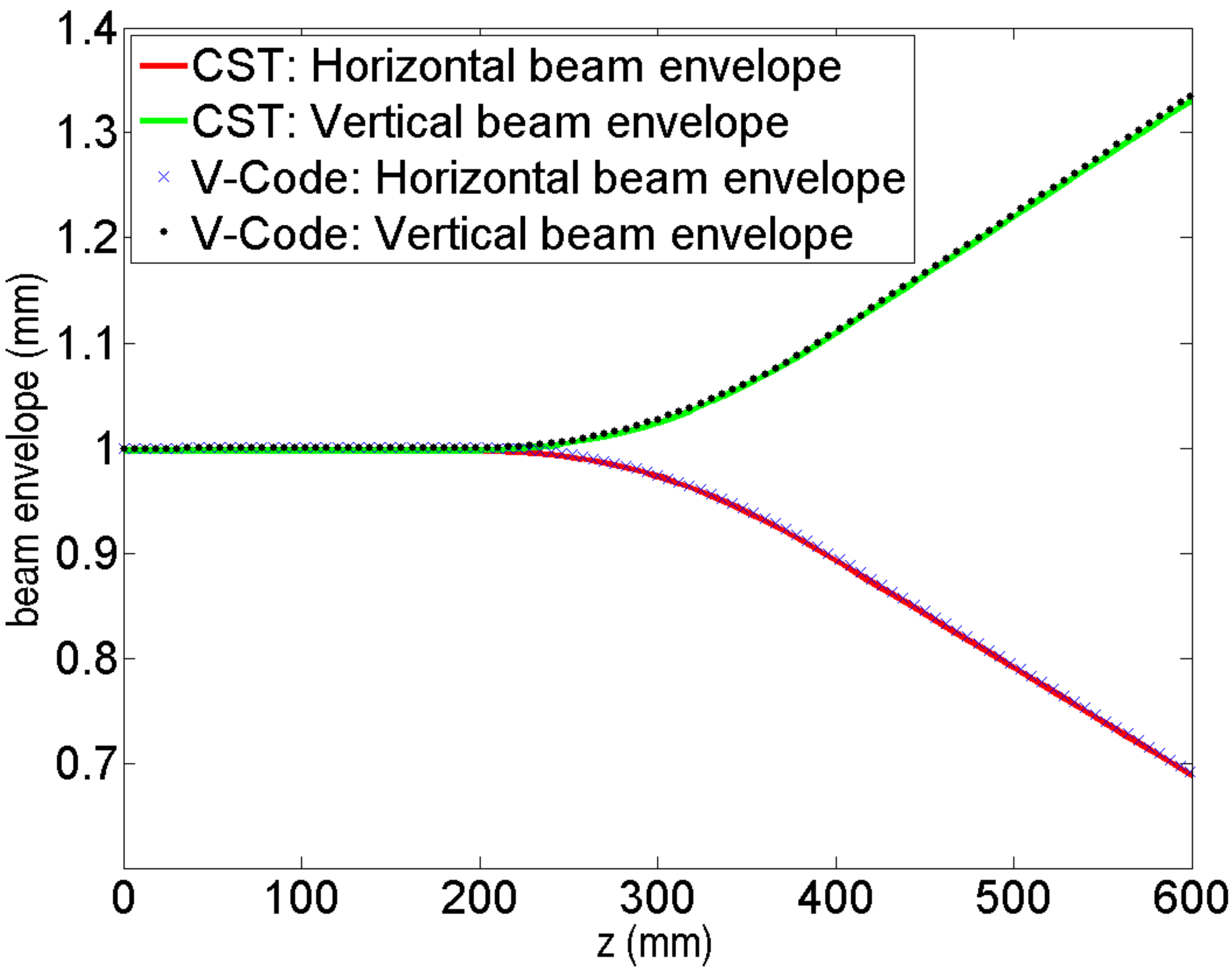


Beam Dynamics Comparison

Implementation verification:

- V-Code
- CST Particle Tracker
- Track
- Toutatis
- ...

Beam Dynamics Comparison



0-166 mm: DT

166-434 mm: Quad

434-600 mm: DT

< 1%

Conclusions

RFQ implementation in V-Code:

- Quasistatic approach
- Two-term potential
- Cell-by-cell components
  - Accurate field parameters
  - Additional geometry parameter
- Beam dynamics comparison
- Four-term potential