Low-energy p-He and mu-He simulation in Geant4

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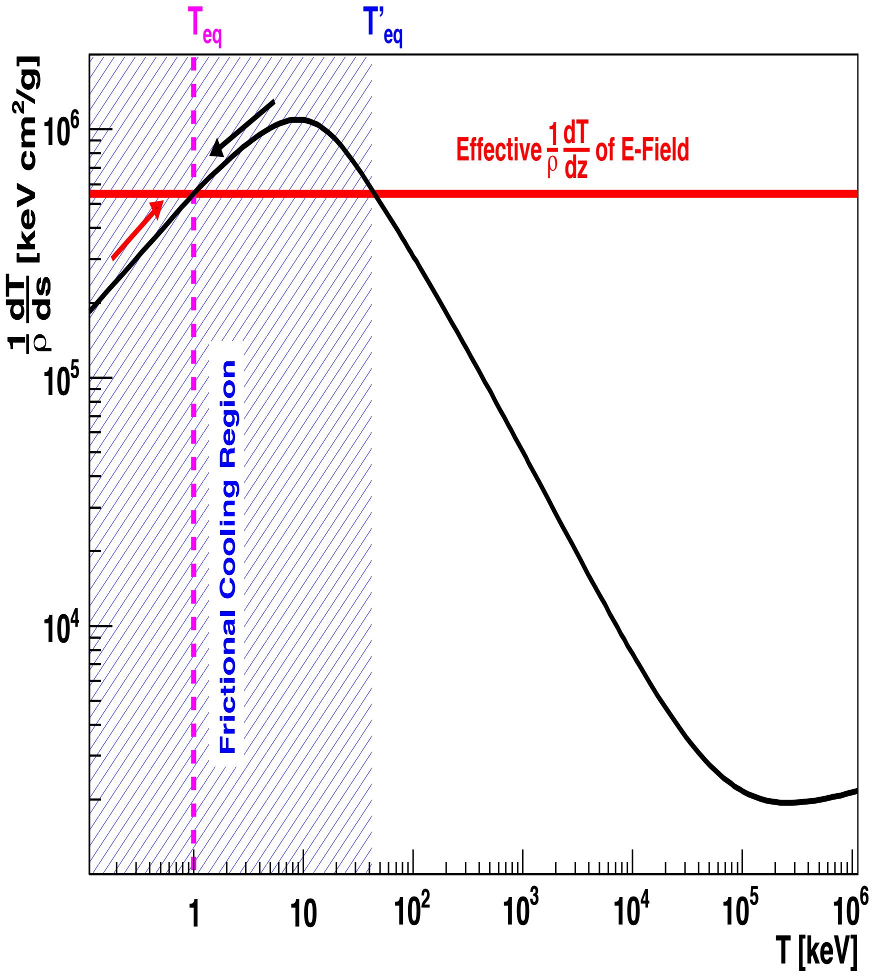
Outline

- Frictional cooling
- Low-energy simulation in Geant4
- Elastic scattering for proton-helium interaction
- Runaway effect
- Conclusion and the code status

Frictional cooling method

- Slow down the beam by a retarding material
- Accelerate the beam by an electric field
- High energy particle loss more energy than the lower energy ones
- Equilibrium energy reached



Frictional cooling utility

- Frictional cooling is a promising method to produce a "cold" beam, which is useful in various experiments.
  - Produce slow muon beams for particle physics.
  - Slow down the anti-proton for anti-matter trapping.
  - Steer the muons in the material to let them decay on the surface
  - Prepare an intensive muon beam for a muon collider or neutrino factory.

Geant4 low-energy processes

Energy Loss:

- **> 1 keV**: Stopping power according to NIST data.
- **10 eV – 1 keV** : free electron gas model

  SP = A *sqrt(Tp)
- **< 10 eV :** Particle killed and tracking stopped

Scattering:

- Multiple- scattering process, no energy loss.

Elastic scattering at low energy

- At energies lower than 1 keV, the cross section of the elastic processes are much larger than the inelastic ones (ionisation and charge exchange processes)
- In this work, the elastic scattering process is implemented into Geant4 at energies lower than 1 keV
- Tracking stops at energy of 0.15 eV
- Geant4 processes are turned off in this range.

Cross sections

- The scattering is described by the differential cross section: $\sigma_d = d\sigma(\theta,\phi)/d\Omega$, representing the rate of scattering into angle $d\Omega$
- The total elastic scattering cross section is the flux of particles scattered in all directions:

$$\sigma_{el} = \int d\Omega \frac{d\sigma(\theta,\phi)}{d\Omega}$$

- The momentum transfer cross section representing the average momentum loss per collision:

$$\sigma_{mt} = \int d\Omega \frac{d\sigma(\theta,\phi)}{d\Omega}(1-\cos\theta)$$

Reference: P. Krstic and D. Schultz, *Atomic and plasma-material interaction data for fusion*, vol. 8, 1998.

Average momentum loss

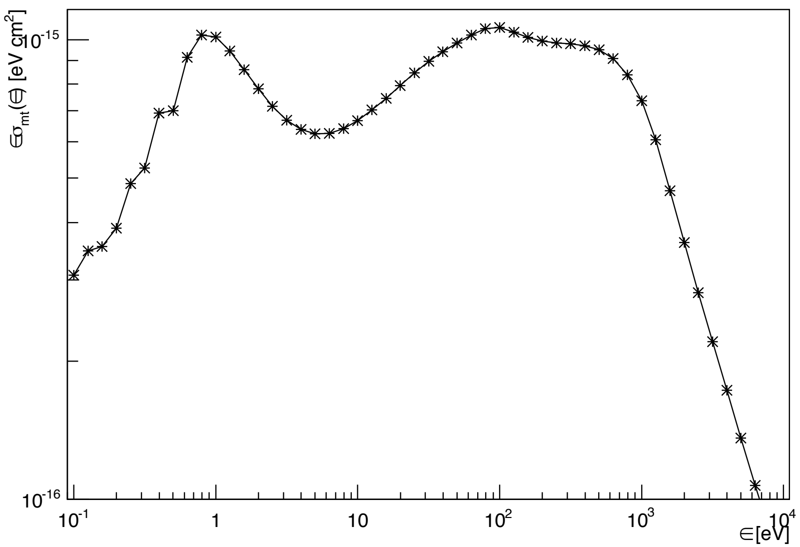
- In the center of mass (CM) frame, the momentum is μv_a and the momentum loss is $\mu v_a(1-\cos\theta)$
- The average number of scatters into an angle theta per unit time is $N\bar{v}2\pi\sigma_d(\theta,\bar{v})d\theta$
- Multiply the above two and integrate over all the angles, the average momentum loss per unit time is:

$$\mu v_a N\bar{v}\sigma_{mt}$$

- Relate the v_a and $\bar{v}$ by $m\bar{v}^2/2 = mv_a^2/2 + Mv_a^2/2$ we get the average momentum loss:

$$2N[m/(m+M)]^{1/2}\varepsilon\sigma_{mt}(\varepsilon)$$

Collisional energy loss

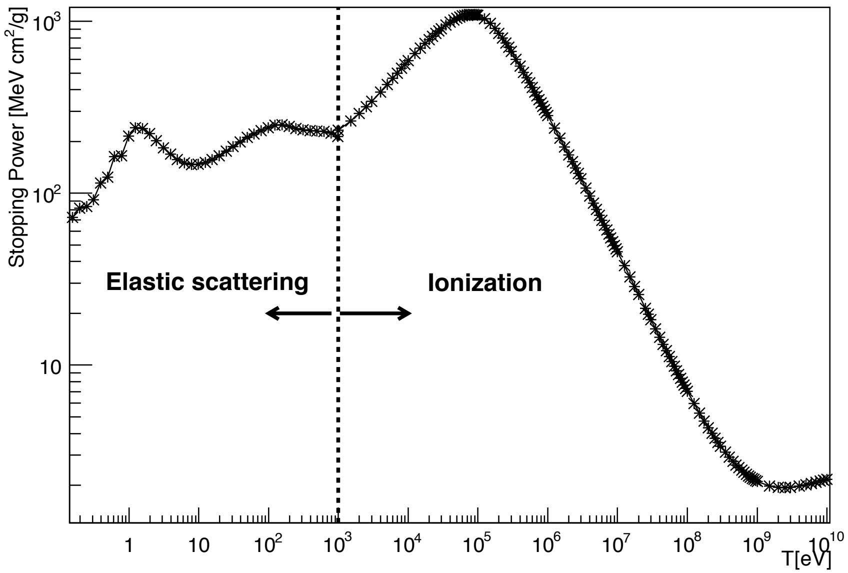


Collisional momentum loss per unit time as a function of relative collision energy for p-He collision

- Normally $\varepsilon\sigma_{mt}(\varepsilon)$ continues to rise with increasing ε and does not have an absolute maximum.
- But it is never rises higher than 110*10^-17 eVcm2 for proton in helium.
- It gets down rapidly after 1 keV

Stopping power and "Runaway"

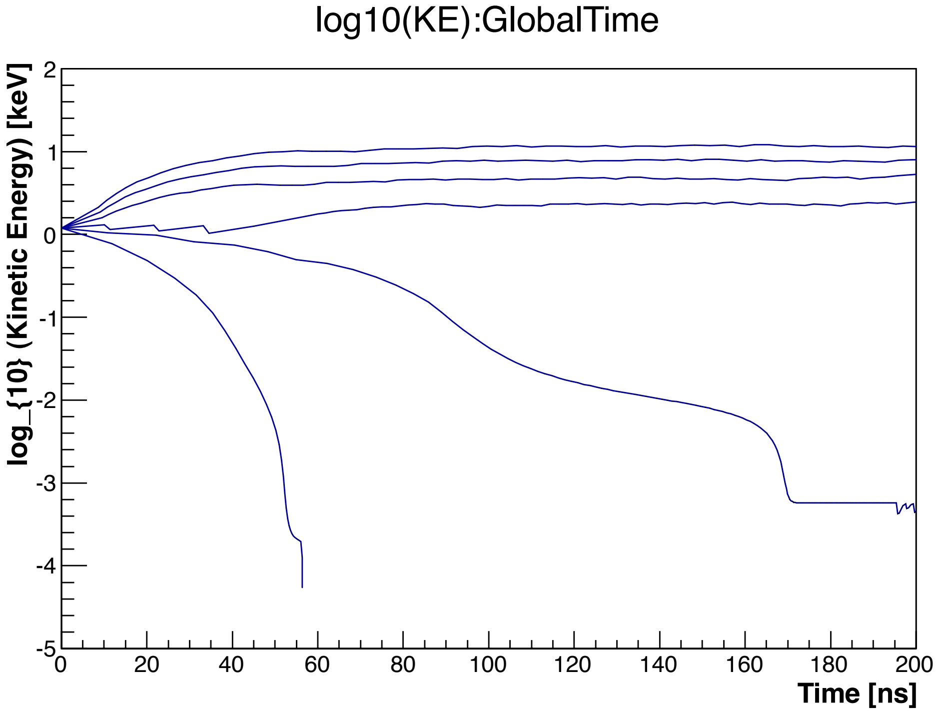
- The $\varepsilon\sigma_{mt}(\varepsilon)$ curve is then converted to the stopping power in the laboratory reference frame.
- The stopping powers of the elastic scattering and the ionisation process matches at the energy of 1 keV.
- In case an electric field is applied, a stable equilibrium is hard to reach in energy between 1 eV to 1 keV. As the electric field increases, the particle will run away from the low energy region



Stopping power of proton in helium. Left side is the energy loss from elastic scattering; right side is the ionisation from NIST data.

Simulated run away protons

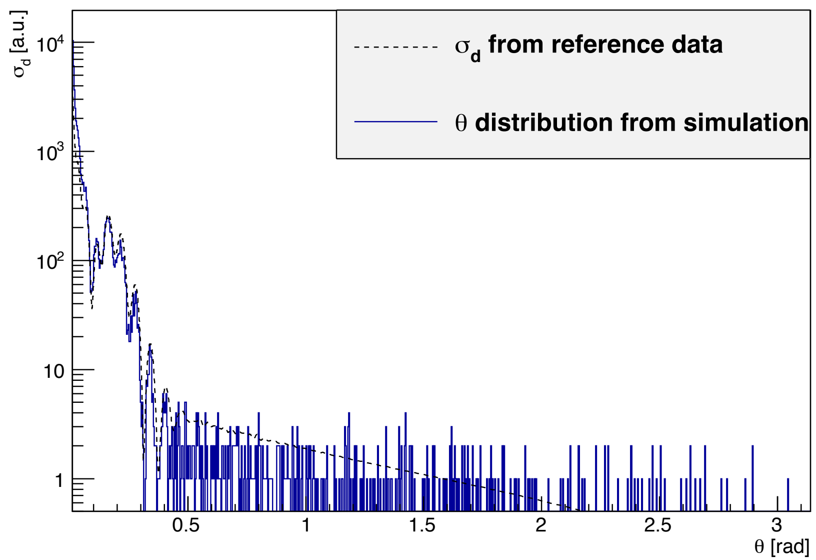
- 6 different electric fields are applied in a helium gas tube at a density of 0.01 mg/cm3.
- Field strength increases from 100 kV/m to 600kV/m with 100kV/m steps
- Equilibrium energy jumps from 1 eV to 1.5 keV.



Run away of protons in helium

Accurate calculation (1) Sampling Angular

- Except for the average calculation, the energy loss for each single scattering can be accurately calculated from the σ_d .
- The scattering angle θ is randomly sampled according to the particle kinetic energy T and the differential cross section σ_d
- Most of the time the particles are scattered in very small angles, whereas the large angle scattering are important for the energy loss



Simulated θ distribution (solid curve) and the σ_d from reference data. T = 10 eV

Accurate calculation (2) Energy loss and angle in lab frame

- Considering the kinetic energy and momentum conservation in the elastic process, the energy loss dT and the scattering angle Φ in the lab frame are:

$$dT = \frac{2mM}{(m+M)^2}(1-\cos\theta)T$$

$$\tan\Phi = \frac{\sin\theta}{\cos\theta + m/M}$$

- The dispersion of theta brings a large fluctuation in dT
- In the case m<<M, $\Phi = \theta$

Accurate calculation (3) Advantage and Disadvantage

- The **advantage** of the accurate calculation is obvious: it correlates the scattering angular and the energy loss, makes the simulation precise, especially in the low density gas and small geometries.
- The **disadvantage** is the computing time is much longer than the average calculation, because σ_{el} is much larger than σ_{mt} .

Conclusion and Code status

- The elastic scattering process has been implemented into Geant4 at energies lower than 1 keV
- The run away effect has been simulated for proton in helium gas
- Several scaling methods for muons are considered. Because unsurprisingly there is no muon data in the literature in this energy range, we have to compare the simulation with our experiment at the PSI.

Thanks!