SIMULATION OF BASEBAND BTFs USING A PARTICLE-IN-CELL CODE

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Abstract

A simulation model for transverse bunched beam transfer functions (BTFs) at the base harmonic is presented. It is based on a code including different machine effects, most notably transverse space charge using a two-dimensional (2D) Poisson solver. A simplified model for the simulation of the strong-strong beam-beam effect was implemented using either 2D field data or analytic expressions under the assumption of Gaussian beams for the beam-beam interaction. The validity of the BTF model is verified based on the comparison of BTF and Schottky spectra features with analytic expectations from literature. The simulation model is then applied to the RHIC proton lattice. A linear transfer map is used between interaction points. BTFs including the beam-beam effect are simulated. Measurements are compared to simulation results at machine conditions.

MOTIVATION

Transverse BTFs of coasting beams were shown to give great diagnostic opportunities [1], for example enabling the direct measurement of the incoherent space charge tune shift. Our aim is to investigate the diagnostic opportunities of the transverse BTFs of bunched beams. We decided to start the investigation with the baseband beam transfer functions of bunched beams experiencing the beam-beam effect. The choice of baseband BTFs was made because both at Brookhaven National Laboratory (BNL) as well as GSI a high sensitivity BTF system based on the direct diode detection method developed at CERN [2] is in use. The choice of the beam-beam effect was made because it is localized and reduces overall simulation time.

THE BTF MODEL

In the measurement of BTFs, the beam is transversely excited using a kicker operating at a certain frequency. The response of the beam center of charge oscillation is measured using a pickup. The process is repeated over a range of frequencies and the relative complex response amplitude as a function of excitation frequency gives the beam transfer function. In our simulation model we started from an existing code [3] to implement a simplified baseband beam transfer function: The codebase provides particle tracking based on the 6D transfer maps computed by MADX [4] with the addition of different effects such as chromaticity, self-consistent space charge and others. The BTF simulation consists of a combination of two modifications:

1. In one cell of the tracking lattice, a transverse periodic excitation is added to the momenta of all particles. For the RHIC case where bunch lengths are far below 1/100 of the machine circumference we consider it safe to expect that all particles in one bunch get the same kick amplitude from an excitation signal at a frequency below the revolution frequency.
2. In the cell of the kick we also compute the offset of the center of charge $\langle x \rangle$ from the reference trajectory and store it together with the excitation amplitude and frequency.
3. The particle distribution is initialized and tracked for a few thousand turns to equilibrate possible matching errors due to the beam-beam effect. The equilibrated particle distribution is stored.
4. The simulation is run for a range of frequencies, simulating excitation for 3500 turns. The same number of samples is taken in the real-life BTF measurements. After each run, the equilibrated particle distribution is reloaded in order to save the time needed to simulate until the excited oscillation dies down.
5. In offline analysis, we compute the complex response amplitude and excitation amplitude exactly at the excitation frequency using the discrete time Fourier transform (DTFT). The division of the complex response amplitude by the excitation amplitude gives the BTF.

Testing of the BTF Model

In order to verify our implementation we compare our simulation with analytically accessible scenarios found for example in [5]. A popular example concerns a particle distribution with a Gaussian frequency spread. The analytic result [5] is given in units of the normalized frequency deviation u defined as a function of the mean particle betatron frequency $\overline{\omega}$, the driving frequency Ω and the frequency width of the distribution $\Delta\omega$ via the equation:

$$u = \frac{\overline{\omega} - \Omega}{\Delta\omega}. \tag{1}$$

For a Gaussian frequency distribution the analytic BTF $R(u) = f(u) + ig(u)$ is given by:

$$f(u) = \sqrt{\frac{2}{\pi}} e^{-u^2/2} \int_0^\infty \frac{\mathrm{d}y}{y} e^{-y^2/2} \sinh(uy) \tag{2}$$

$$g(u) = \sqrt{\frac{\pi}{2}} e^{-u^2/2} \tag{3}$$

We replicated the situation of a Gaussian frequency spread in the transverse direction for the PIC simulation of a longitudinally frozen bunch: We made use of Gaussian momentum distribution that acted via chromaticity as a Gaussian tune

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ISBN 978-3-95450-116-8

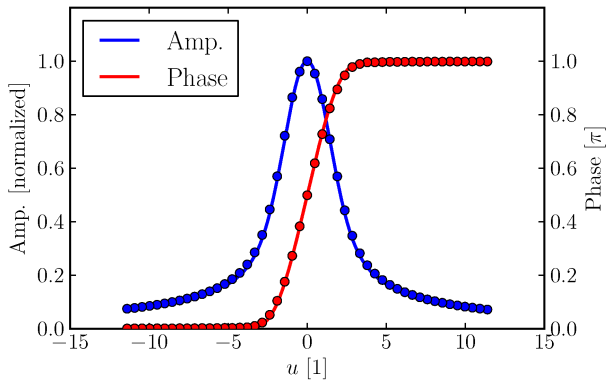

Figure 1: Comparison of amplitude and phase of the analytic prediction for the BTF of a beam with Gaussian tune spread (line) and the PIC BTF simulation results (points). The simulation agrees well with analytic predictions. u is the normalized frequency coordinate given in eq. (1).

spread in the transverse direction. The simulation results are shown in Fig. 1. We can see that our simulation replicates the analytic results well. We display the BTFs as is commonly done by splitting them into the absolute (amplitude) and angular (phase) part of the complex number.

For tune distributions we calculate the mean per-particle tunes for a subset of 10^4 particles using the FFT of the particle coordinates over 2^{11} turns and refine the peak frequencies using the bisection method to maximize the amplitude given by the DTFT.

MODEL FOR THE BEAM-BEAM EFFECT

In order to investigate BTF in RHIC a simplified model of the beam-beam effect was added to the code from [3]. In normal operation in RHIC, sets of six bunches (three per ring) couple with each other via the beam-beam interaction at two interaction points. For this a trivial parallelization scheme was implemented: Six copies of the original code are run, each utilizing the linear transfer map given by MADX [4] for the RHIC 2012 proton proton lattice [6] to translate a single bunch between interaction points. For simplicity we refer to a copy of the original code running for a single bunch simply as bunch. Communication between the bunches happens via the Message Passing Interface (MPI) [7]. At the interaction point two approximations to the beam-beam interaction were implemented and can be used for simulation:

Soft Gaussian approximation The beams are assumed to be of Gaussian transverse shape. Each bunch computes its center of charge position $\mu_{h,v}$ and rms width σ and exchanges them with its collision partner via MPI. The particles momentums are changed according to the analytic field amplitude for a Gaussian charge distribution [8] of given μ and σ.

Two-dimensional Fields Each bunch computes the particle density in the transverse plane on a discretized grid of 128^2 cells and by means of a two-dimensional Fast Fourier Transform (FFT) based Poisson solver the corresponding transverse field strength is derived. In the field calculation, open boundary conditions are assumed [9]. The field data is sent to the partner bunch via MPI and each bunch modifies particle momenta according to the field data.

In both of these the beam-beam interaction is approximated as a single kick on each particle, similar to what would happen if the bunches passed each other with all particles at the longitudinal position of the reference particle. We further assume that in the measurements we want to replicate, the excitation amplitude is sufficiently small to not influence the next BTF sample. This is justified from measurement where BTF can be repeated without significant change on the same beam.

The choice for the two-dimensionality of the interaction is made because of the computational resource limits: For computation of coherent BTFs a few thousand turns per scanned frequency need to be computed. At about 360 ms per turn of the scheme with 6 bunches, a BTF calculation takes quite some time. Simulation of a BTF including the beam-beam effect with 40 frequency samples for 100k particles and 3000 turns per excitation frequency takes approximately 11 hours on 6 CPUs. Usually we would like to have in the range of a hundred frequency samples to resolve subtleties of the spectra. In those cases we employ further trivial parallelization by running multiple simulations for different frequency ranges at once. An obvious extension of the code would include a sliced beam-beam interaction model which would allow for a more realistic simulation of the beam-beam interaction.

For the case with two interaction points, care has to be taken in the BTF model: In the real machine, the three bunches belonging to one beam pass the cells where excitation and recording of the response takes place at a distance of 1/3 of the ring circumference. To model this in simulation a phase shift in the excitation signal depending on the bunch number has to be introduced. Additionally it is necessary to preserve the order of the response data so that the later Fourier transform can easily be done over the combined position data of all three bunches, again corresponding to reality where each bunch contributes to the signal on the pickup.

Beam-beam Implementation Testing

In order to verify the correct implementation of the beam-beam effect we chose to simulate Schottky-type beam spectra. The simulation was run using the nominal RHIC lattice with identical tunes in the two rings, named Blue and Yellow of $Q_h = 28.695, Q_v = 29.685$. The beam-beam interaction was simulated but no BTF applied. We recorded the transverse position of the simulated bunch centres of charge every turn. Schottky spectra were derived using the FFT. In the resulting spectra, the positions of the coherent beam-beam modes were observed. The well-known expectations [10]

ISBN 978-3-95450-116-8

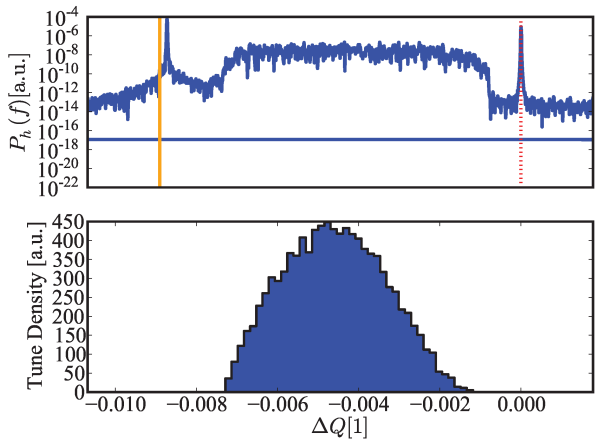

Figure 2: Example of a Schottky simulation of two bunches with one interaction point and identical tunes in both rings, the analytically expected position for the π-mode is shown in orange, the lattice tune (and expected position of the σ-mode) is indicated by a dashed line. The bottom plot shows the tune distribution. The simulation results correspond to the expectation. Similar results were obtained for the lattice with two interaction points when the beam-beam parameter was multiplied by the number of interactions per turn.

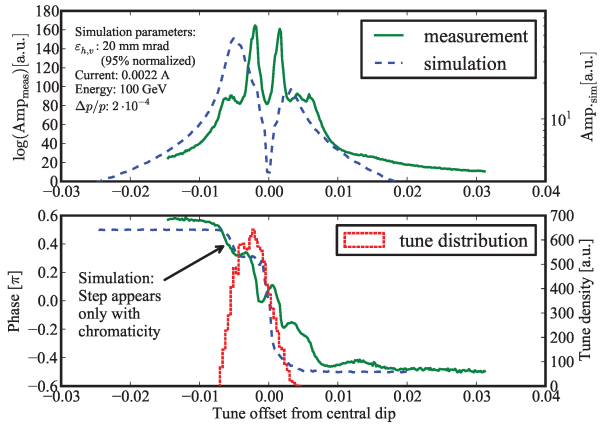

Figure 3: Comparison between simulated and measured BTF for the split tune conditions (tunes given in the text). The position of the central dip was set to zero to align measurement and simulation results.

for beams of identical tunes were met by simulation (the slight deviation is expected for beams with a beam-beam parameter above 0.01 [11]). As shown in fig. 2, the σ mode was found at the lattice tune, the π-mode was found at a the expected position for round beams of 1.215ξ with ξ the beam-beam parameter [10].

The beam-beam simulation using the Gaussian approximation gives a slightly smaller displacement of the π-mode with respect to the σ mode. This also is in accordance with expectation [11].

Comparison with Measurements

In March 2012 during the RHIC polarized proton-proton run there were test runs by S. White et al. [12] using a lattice different from the nominal one: The tunes in the blue ring ($Q_h = 28.691$, $Q_v = 29.689$) were chosen far from the tunes in the yellow ring ($Q_h = 28.735$, $Q_v = 29.725$). Measured BTF in the vertical plane showed an unusual peak under these conditions. We tried to replicate these settings using our simulation. The results are shown in fig. 3. While in the measurement the structure only shows up in one plane, in the simulation it is visible in every plane. However this is not too surprising: While in the real ring we have nonlinearities possibly damping coherent modes, our model only takes into account the beam-beam effect, chromaticity and the linear transfer matrices between the interaction points. Varying chromaticity we noticed that the step to the left of the phase jump in fig. 3 only appears after introduction of a finite chromaticity, the plot is for a chromaticity $\Delta Q/(\delta p/p)$ of 2. A rigid bunch model simulation [12] agrees in the prediction of coherent modes in all planes.

We consider the remaining differences to measurement are caused by the simplifications of our simulation, where we assume a linear lattice apart from the beam-beam effect.

OUTLOOK

With our BTF simulation tool we will investigate possible diagnostic opportunities of the BTF with respect to the incoherent tune distribution. This is desirable as diagnostics for the electron lens for head-on beam-beam compensation [13] to be installed at RHIC in the near future.

ACKNOWLEDGEMENTS

P. Görgen wishes to thank M. Blaskiewicz for many helpful suggestions during the implementation.

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ISBN 978-3-95450-116-8