

LOW NOISE PARTICLE-IN-CELL SIMULATIONS OF LASER PLASMA ACCELERATOR 10 GeV STAGES*

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Abstract

Because of their ultra-high accelerating gradient, laser plasma based accelerators (LPA) are contemplated for the next generation of high-energy colliders and light sources. The upcoming BELLA project will explore acceleration of electron bunches to 10 GeV in a 1 meter long plasma, where a wakefield is driven by a PW-class laser. Particle-in-cell (PIC) simulations are used to design LPA stages relevant to the upcoming experiments. Simulations in a Lorentz boosted frame are used to gain significant speed up and to simulate the 10 GeV stages at full scale parameters, which are otherwise impractical. As criteria on energy spread and beam emittance become more stringent, PIC simulations become more challenging as high frequency noise artificially increases those quantities. To reduce numerical noise, we consider using a Poisson solve to calculate the beam self-fields. This method allows correct cancellation of the beam transverse self-forces and prevents artificial emittance growth.

INTRODUCTION

Laser-plasma accelerators (LPA) can reach accelerating gradients several orders of magnitude larger than conventional accelerating structures [1], opening the path to more compact light sources and particle colliders. Recently, mono-energetic electron beams have been accelerated to 100 MeV in a 3 mm long plasma [2, 3, 4] and 1 GeV in a 1 cm plasma [5, 6], using 10 and 40 TW lasers respectively. Energy gain is increased by using higher laser power and lower density for longer acceleration length. The BELLA project at LBNL will explore increasing the energy gain to 10 GeV, using a 1 meter long plasma and a PW-class laser [7]. A succession of this type of stages can be used efficiently to build a TeV linear collider [8].

Particle-in-Cell (PIC) simulations are an essential tool to understand the physics inherent to laser-plasma acceleration and to design future devices. As longer stages are used to reach higher energy, simulations become more computationally intensive, since grid size and time step are lim-

ited by the laser wavelength, typically of the order of 1 micron. To make simulations more affordable, reduced models are used such as the Ponderomotive Guiding Center method [9, 10, 11, 12, 13], or envelope model, where the laser wavelength is no longer resolved and the smallest scale length is the plasma wavelength which scales with the stage length and energy gain. The latter method allows order of magnitude speed-up but is limited because it can not model the laser all the way into depletion. Simulations in a Lorentz boosted frame reduce the number of time steps needed for the simulations while still resolving the laser wavelength [14, 15]. This method has proven successful in simulating 10 GeV LPA stages with orders of magnitude speedup [16].

Accurate representation of the accelerated beam is limited by numerical noise in PIC simulations. This becomes more of an issue as the condition on beam energy spread and emittance becomes more stringent. Introduction of higher order particle shapes has allowed significant reduction of numerical noise in LPA simulations with PIC [17], representing beam properties within a few % of what is obtained in experiments, in contrast to results using linear interpolation which differ by an order of magnitude [18]. To more accurately represent beam evolution with % level energy spread and a fraction of mm mrad emittance, as required by applications, we explore using a method commonly used in tracking codes [19, 20, 21] but that has never been used in the context of LPA, in which the beam self-fields are calculated using a Poisson solve in the beam rest frame. This method provides correct relativistic cancellation of the beam transverse self-forces and prevents high frequency numerical noise responsible for artificial emittance growth of the beam.

SIMULATIONS IN A BOOSTED FRAME

Methods presented by J.-L. Vay and collaborators [16] are used in the Vorpil framework [22] to perform simulations in a relativistic boosted frame of 10 GeV LPA stages, relevant to the BELLA project, with parameters similar to those presented in [23]. The boosted frame technique allows the simulation to be performed at full scale, i.e., 1 m long stage at the nominal plasma density $n_0 = 10^{17} \text{ cm}^{-3}$. An externally injected 2 pC beam is used, with normalized emittance $\epsilon_n = 0.5 \text{ mm mrad}$ and initial energy $E = 1 \text{ GeV}$. The evolution of the beam properties is consistent for different values of the simulation frame relativis-

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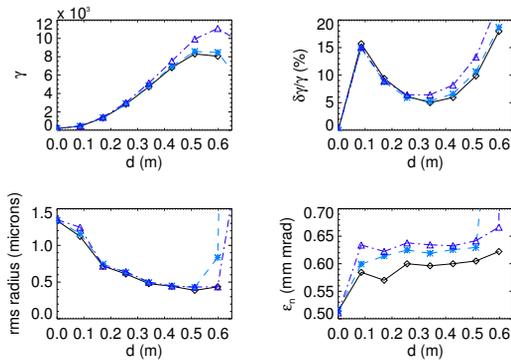


Figure 1: Evolution of electron beam mean γ , rms energy spread, rms radius, and normalized emittance as a function of propagation distance for $\gamma_{\text{boost}} = 25$ (diamonds), $\gamma_{\text{boost}} = 50$ (stars) and $\gamma_{\text{boost}} = 75$ (triangles).

tic factor γ_{boost} as shown in Fig. 1. A factor of 3, 500 times speedup is achieved when performing the simulation with $\gamma_{\text{boost}} = 75$. Numerical instabilities arise when performing the simulation with a high γ_{boost} , which are mitigated by using smoothing with large stride on the current deposited by the particles [24]. Although beam energy gain, energy spread and radius evolution are converged, higher resolution is needed at high γ_{boost} to accurately represent the beam emittance, limiting the effective speedup to 550-fold in this case, the simulation being performed in 4,500 processor hours.

BEAM FRAME POISSON SOLVE

Because of numerical noise in PIC simulations, high resolution is often needed to accurately represent the evolution of the accelerated electron beam. Beam emittance can particularly be affected by the high frequency noise inherent to the point like representation of the macro-particles. In simulations of conventional accelerator with tracking codes, it is common to use a Poisson solve in the beam rest frame to calculate the beam self-fields, however this method has never been employed in PIC simulations of LPA. The beam charge density is Lorentz transformed in the frame where the beam is at rest, and a Poisson solve is used to calculate the corresponding electrostatic field, which is then Lorentz transformed back into the laboratory frame to obtain the beam electro-magnetic beam self-fields. This is shown in the blue boxes of the diagram in Fig. 2. This method assumes that simultaneity in the laboratory frame implies simultaneity in the beam frame (the particle time transformation is neglected), and that all the particles of the beam are non-relativistic in the beam rest frame, and hence can only be used in the case of compact, low energy spread, low divergence beams. This is usually the case when designing LPA for future applications. In the following, we refer to this method as *Beam Frame Poisson Solve* (BFPS).

This method allows the beam fields in the laboratory frame to be calculated at the same position on the grid, and

hence allows correct relativistic cancelation of the beam transverse self-forces. In the PIC algorithm, where a Yee update of the electro-magnetic fields is generally used, the \mathbf{E} and \mathbf{B} fields are staggered on the grid, leading to different interpolation errors to the macro-particle position. This translates into an inaccurate cancellation of the \mathbf{E} and $\mathbf{v} \times \mathbf{B}$ transverse forces, which in turn leads to increase of the radius of an otherwise matched beam.

The BFPS also mitigates artificial beam emittance growth of a matched beam in the presence of a linear focusing force. Numerical emittance growth, when using the Yee algorithm, can be reduced by using higher resolution and several pass of smoothing, with larger strides, on the current and the forces applied to the beam, but is sometimes not sufficient to converge to the right answer, especially for very low emittance beams (~ 0.1 mm mrad).

The BFPS method can also be used in the plasma wakefield, thanks to the linearity of Maxwell's equations, at the condition that the beam is initially in the vacuum region. The method, implemented from the Vorpil input file, is shown in Fig. 2. The beam self-fields are calculated as explained above, using a Poisson solve in the beam frame. In parallel, the fields of the plasma are calculated self-consistently using the Yee advance normally used in PIC simulations. The fields are then combined to push particles of both beam and plasma, which then deposit charge and current densities, respectively, to perform the next field advance.

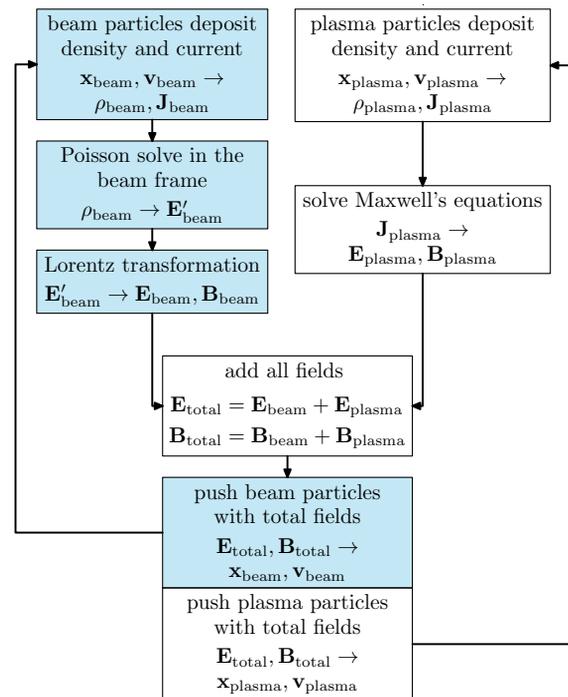


Figure 2: Diagram of the BFPS algorithm when the beam is used inside a plasma wakefield. The blue boxes indicate the algorithm for the beam alone.

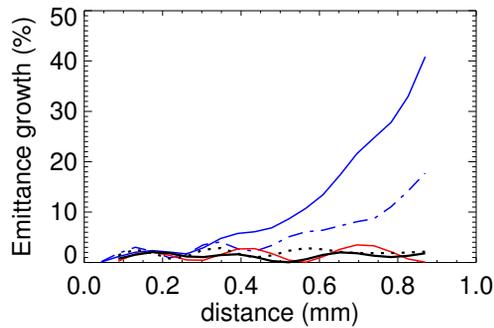


Figure 3: Percentage emittance growth of a quasi-matched electron beam in a plasma wakefield, using the BFPS (black) at the resolution $dx = \lambda_0/24$ (solid line) and $dx = \lambda_0/48$ (dotted line), and using the Yee algorithm (blue) at the resolution $dx = \lambda_0/24$ (solid line) and $dx = \lambda_0/48$ (dash-dotted line), using the Yee algorithm with an additional 4-pass (1,2,1) filter with compensator of stride 2 at the resolution $dx = \lambda_0/24$ (red line).

Fig. 3 shows the beam emittance evolution of a quasi-matched Gaussian beam in a plasma wakefield. The wakefield is driven by a laser pulse of normalized intensity $a_0 = 1.4$ with a Gaussian profile both longitudinally and transversely with dimensions satisfying $k_p L = 1$ and $k_p w_0 = 5.3$, similar to what is used in [23]. Here, $k_p = \sqrt{4\pi n_0 e^2 / mc^2}$ is the plasma wave number, a density $n_0 = 10^{19} \text{ cm}^{-3}$ on axis is used with a parabolic profile transversely. A 10 pC electron beam with a Gaussian profile is loaded behind the laser pulse, with rms dimensions verifying $k_p \sigma_L = 0.1$ and $k_p \sigma_r = 0.35$, initial normalized emittance $\epsilon_n = 0.5 \text{ mm mrad}$ and energy $E = 100 \text{ MeV} \pm 1\%$. The acceleration of the beam is turned off by setting $E_x = 0$, where x is longitudinal direction. The solution with the BFPS is shown in black for $dx = \lambda_0/24$ and $dx = \lambda_0/48$ (dotted line), where λ_0 is the laser wavelength, showing similar evolution of the beam emittance, which oscillates but does not grow overall. The solution using the standard Yee algorithm is shown in blue at the same resolution of the BFPS runs, showing obvious emittance growth, which is reduced when using higher resolution. Using an additional 4-pass smoothing filter with compensator with stride 2 on the current converges toward the BFPS response.

The BFPS algorithm can be used in the boosted frame. We verified that the beam evolution is similar in the previous case for $\gamma_{\text{boost}} = 13$. This allows for simulation full-scale 10 GeV stages with accurate emittance of the beam.

CONCLUSION

Boosted frame simulations are used in the Vorpahl framework to simulate full-scale meter-long 10 GeV gain LPA stages. Evolution of an externally injected beam is consistent for different values of γ_{boost} .

Numerical noise is reduced by calculating the beam self-fields using a Poisson solve in the beam rest frame. This allows correct relativistic cancelation of the beam transverse self-forces and prevents artificial emittance growth of the beam. This method can be used inside the plasma wakefield if the beam is initialized in the vacuum region. A quasi-matched beam in the focusing field of the wake shows no spurious emittance growth using the BFPS, contrary to the standard Yee algorithm in the same conditions.

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