EIGENMODE COMPUTATION FOR FERRITE-LOADED CAVITY RESONATORS *

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Abstract

For acceleration of charged particles at the heavy-ion synchrotron at the GSI Helmholtzzentrum für Schwerionenforschung in Darmstadt two ferrite-loaded cavity resonators are installed within the ring. Their eigenfrequency can be tuned by a properly chosen bias current and thereby modifying the differential permeability of the ferrite material. The goal of the presented work is to numerically determine the lowest eigensolutions of accelerating ferrite-loaded cavities based on the finite integration technique (FIT). The newly developed solver includes two subcomponents: Firstly, a magnetostatic solver supporting nonlinear material for the computation of the magnetic field which is excited by the specified bias current. This enables to linearize the constitutive equation for the ferrite material at the current working point, at which the differential permeability tensor is evaluated. Secondly, a Jacobi-Davidson type eigensolver for the subsequent solution of the nonlinear eigenvalue problem. Particular emphasis is put on the implementation to enable efficient distributed parallel computing. First numerical results for biased ferrite-filled cavity resonators are presented.

INTRODUCTION

Within the heavy-ion synchrotron at GSI two ferrite-loaded cavity resonators are operated to continuously accelerate the injected charged particles. Inside the cavity housing ferrite ring cores are installed around the beam pipe. A magnetic field is established in these rings by means of two different current windings: Firstly, a field constant in time due to the bias current, and, secondly, an additional time-harmonic component induced via radio frequency coupling. During the acceleration phase the resonance frequency has to be adjusted to reflect the increasing speed of the heavy ions. This can be achieved by properly choosing a bias current and thereby modifying the differential permeability of the ferrite material. For the SIS 18 ferrite cavity, biasing enables to alter the resonance frequency in a range from about 0.6 MHz to 5.0 MHz. A detailed description of the SIS 18 ferrite cavity can be found in [1].

In this paper, the main aspects of the applied numerical approach are briefly summarized followed by helpful remarks on efficient parallel computing. After that, the functionality of the solver is demonstrated based on two different simple examples.

* Work supported by GSI
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COMPUTATIONAL APPROACH

The relevant fundamental relations used for the calculation of eigenmodes of biased ferrite-loaded cavity resonators as well as the applied approach for the numerical computation were already discussed in [2]. Hence, here the most important aspects are picked up and summarized.

The dependence of the eigenmodes on the differential permeability leads to several consequences: The static magnetic field generated by the bias current has to be calculated beforehand by means of a nonlinear magnetostatic solver. The constitutive equation is then linearized at the specified working point. Moreover, the frequency dependence of the permeability tensor results in a nonlinear eigenvalue problem. A dedicated solver for parallel computing has been developed to meet the tight requirements. Presently, only Hermitian eigenvalue problems are supported. Consequently, the current version of the solver is applicable for loss-less materials as well as models discretized on equidistant grids.

PARALLEL COMPUTING

The realization of the solver should allow an efficient computation on distributed memory machines. To this end, it aims at a high computation to communication ratio as well as a good load balancing. Regarding the first aspect, the degrees of freedom (DOFs) of the FIT are arranged such that all matrices have only few non-zero components in their far off-diagonal regions. This directly leads to reduced communication between the individual processes and thus to a better parallel efficiency. For illustration, the effect of the re-ordering is shown with the structure of the system matrix of the eigenvalue problem on the left of Fig. 1. Having in mind that the number of computations one processing unit has to perform is approximately proportional to the number of non-zeros of the system matrix which are owned by this process, it can also be seen from this figure that the load balancing is rather poor. The reason for this is that many variables are included that in fact are zero in the FIT because they are allocated on elements outside the computation domain (including perfect electric conductor cells) or due to boundary conditions. Consequently, in the current implementation all the pseudo DOFs are completely removed beforehand and therefore very good load balancing is obtained.

The reduction of the system size automatically results in further benefits, not only for parallel computing. In fact, the beneficial impact on the memory allocation at the time

ISBN 978-3-95450-116-8

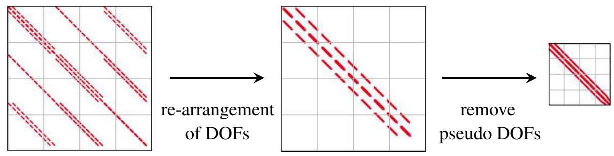

Figure 1: Structure of the system matrix of the eigenvalue problem for arrangement of the DOFs in standard FIT (left), after re-arrangement (middle) and additional removal of pseudo DOFs (right). Non-zero matrix elements are shown in red. The thin horizontal lines indicate how the data is partitioned on the different processes (four in this this example).

the preconditioner is constructed is even more crucial as the preconditioner is computed for the system matrix which is shifted by a scaled unit matrix. Since it is not possible to explicitly preallocate memory for vanishing diagonal elements when using PETSc- (Portable, Extensible Toolkit for Scientific Computation [3]) routines for sparse matrix-matrix multiplication, the consequence is that additional memory has to be allocated multiple times, which is an expensive operation. The complete removal of these elements is one way to solve this issue.

NUMERICAL EXAMPLES

Numerical results for two examples, both biased cavity resonators, are presented. Whereas the first one aims at the verification of the nonlinear eigensolver, the second one is a simple model of a cavity filled with ferrite ring cores.

Biased Cylinder Resonator

In the following, a lossless, ferrite-filled cylindrical cavity resonator longitudinally biased by a homogeneous static magnetic field is considered. Assuming that its magnetic properties can be described by the Polder tensor [4], a characteristic equation determining the resonance frequencies can be formulated analytically [5, 6]. The parameters of the test model are the same as for the one described in [2]. Yet, here the numerical computation is carried out for different orientations of the cylinder axis to the coordinate axes keeping the external magnetic field aligned with the cylinder axis. This way the construction of the permeability tensor for an arbitrarily oriented magnetic field is tested additionally. The eigenmodes for the lowest eigenfrequencies are compared with the ones obtained numerically with the implementation of the nonlinear eigensolver. Note that for a reliable verification it is crucial to assign the modes correctly. If one sorts the eigenmodes simply in ascending order with respect to the eigenvalues, by comparing the field solutions it shows that the order of the modes is different compared to the semi-analytical calculation. This is, however, only the case for discretizations on a rather coarse mesh. By a proper refinement of the grid, the expected order is retained. Taking this into account, good accordance of the numerical values with the analytical results is observed for all tested orientations, which is shown as an example in Fig. 2 for the orientation as depicted in Fig. 3. Moreover, both the accuracy at a given number of DOFs and the convergence order coincide with the results obtained for the cylinder axis aligned with the z-axis (cf. [2]).

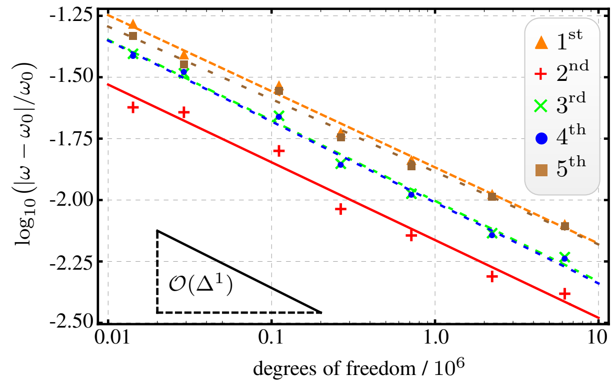

Figure 2: Relative deviation of the numerically obtained value ω to the analytical result ω_0 as a function of the DOFs for the five lowest eigenfrequencies for a lossless, longitudinally biased, ferrite-filled cylindrical cavity resonator. The cylinder is oriented as shown in Fig. 3.

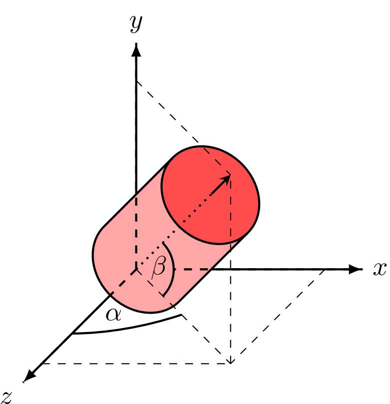

Figure 3: Orientation of the cylinder in the coordinate system. For the convergence study shown in Fig. 2, the cosine of β, i.e. the angle between the cylinder axis and the x-z-plane, is $\sqrt{2/3}$; the angle α between the projection of the cylinder axis onto this plane and the z-axis is $45°$.

For the model of the rotated cylinder the usage of the reduced system matrix as explained in the previous section is of particular importance. Since for the discretization of the model the cylinder is embedded in a larger box with the remaining space filled up with perfect electric conductor cells, the total number of mesh cells is much larger than the actual cells of the cylinder. For instance, for the orientation shown in Fig. 3 the used mesh for the finest discretization consists of approximately $22 \cdot 10^6$ cells whereas the DOFs, which are truly non-zero, amount to about only $6.2 \cdot 10^6$. Yet, thanks to the reduction of the system size not the total number of mesh cells but only the number of non-zero DOFs matters.

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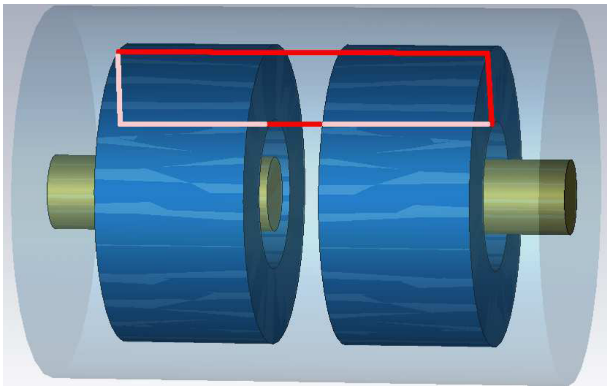

Figure 4: CAD model of a simple biased cavity with ferrite ring cores modelled with CST STUDIO SUITE® [7]. The current winding is indicated by the solid red lines.

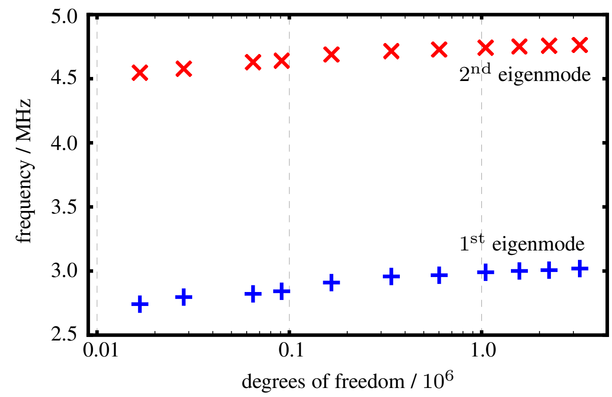

Figure 5: Eigenvalues as a function of the DOFs for the two lowest eigenmodes computed using the fully nonlinear approach for the simple cavity model.

Biased Cavity With Ferrite Ring Cores

The second example may be regarded as a simplified model of the GSI SIS 18 ferrite cavity. Inside the cavity housing ferrite ring cores are installed around the beam pipe, one of them on each side of the centric gap (cf. Fig. 4). The bias magnetic field is excited by a current winding around these rings. For simplicity, the two parts of the beam pipe are modelled as cylinders filled with perfect electric conducting material. The parameters of the model are as follows: The total length of the structure is 140 cm, its radius 50 cm; the radius of the beam pipe is 10 cm, the inner and outer radius of the ring cores are 20 cm and 40 cm, respectively; the length of each ring is 40 cm; the length of the gap is 20 cm; finally, the bias current is 2 kA. The ferrite material is characterized by the constitutive relation

$$B(H) = \mu_0\, 2.5 \cdot 10^4 \tanh\left(H \cdot 10^{-2} \frac{\mathrm{m}}{\mathrm{A}}\right) \frac{\mathrm{A}}{\mathrm{m}} + \mu_0 H \quad (1)$$

and a relative permittivity of $\epsilon_r = 1$. The accuracy of the magnetostatic solver is chosen such that the maximum change of the relative permeability between subsequent nonlinear iterations in any of the mesh cells does not exceed 10^{-4} in the final iteration, which corresponds to a relative change of the norm of the magnetic field in the order of 10^{-6}. Moreover, for the nonlinear eigenvalue solver the accuracy is set as in the previous example. With these settings the fully nonlinear computation of the nine lowest eigenmodes, which are in the range from approximately 87 MHz to 300 MHz and all below magnetic resonance, is performed. The lowest mode is found to be suitable for acceleration. In order to test the reliability of the solver, the whole computation is repeated multiple times with a more and more refined mesh. For illustration the results for the first two eigenmodes are shown in Fig. 5.

Additionally, the effect of the nonlinearity is estimated by comparing the results with those of a linear calculation. In order to obtain reasonable results for the linear approach, the value of the relative permeability of the ferrite material is set homogeneously such that the lowest eigenfrequency of the linear and the nonlinear computation is approximately identical. Despite the fact that the linear solver is a priori provided with this accurate data, the relative deviation of the obtained eigenvalues of both approaches is still up to about 5% for the first nine eigenmodes.

SUMMARY AND OUTLOOK

The computation of eigenmodes of biased ferrite-cavities based on the FIT has been demonstrated. For this purpose, a new solver is developed for the evaluation of the permeability tensor at the working point defined by the bias magnetic field and the solution of the subsequent nonlinear eigenvalue problem. Particular emphasis is put on efficient computing on distributed memory machines. Whereas first numerical results for lossless biased cavity resonators have been presented, the support of lossy materials is planned in future implementations.

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ISBN 978-3-95450-116-8