LOW-ENERGY p-He AND mu-He SIMULATION IN GEANT4

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Abstract

The frictional cooling method is one of the most promising methods on cooling a muon beam. Several frictional cooling schemes have been simulated in Geant4 to be efficient to produce intense muon beams. Frictional cooling works at a low energy range, where the energy loss (momentum transfer) from elastic collision is not negligible. In this paper, the p-He collision process is implemented into Geant4 and the simulation results are compared to the literature data. The process is then scaled for mu-He interaction, which will provide more accurate Geant4 simulations at low energies.

INTRODUCTION

Frictional cooling is one promising method to produce a “cold” beam. It balances the energy loss to a material with energy gain from an external electric field, so that the beam reaches an equilibrium energy and the energy dispersion is reduced. Several cooling schemes based on frictional cooling were outlined for various experiments [1–4]. In most of these schemes, low density helium gas is chosen as the retarding material for its high effective charge [5]. For accurately simulating the transport of the particles in the frictional cooling energy range in the helium gas, the low energy physics processes are needed.

Geant4 is a powerful toolkit for simulating the particle-material interations. The energy loss of the particle is handled by the ionisation process according to the stopping power from the NIST table down to 1 keV. In the energy range between 10 eV and 1 keV the model of a free electron gas [6] is used, in which the energy loss is calculated proportional to the velocity of the particle. When the particle energy gets lower than 10 eV, it’s treated as “stopped” and if there is no “AtRest” process the tracking of the particle will be terminated. The scattering of the particles are simulated in Geant4 by the multiple-scattering method, which has been proved to have the same accuracy as the single-scattering simulations, and the energy loss due to elastic scattering can be neglected at high energies.

At energies lower than 1 keV, the cross sections of the elastic processes are much larger than the inelastic ones. The energy loss due to the elastic scattering plays the dominant role in the particle transport. These processes were investigated in the plasma physics decades ago and were summarized by P. S. Krstic and D. R. Schultz in the reference [7].

In this work the elastic scattering process of the p-He interaction is implemented into Geant4 and is scaled for the μ^+-He interaction. The multiple scattering and the ionisation processes in Geant4 are turned off at energies lower than 1 keV, and the elastic scattering process is used in the range between 1 keV and 0.1 eV. When the energy goes lower than 0.1 eV, the tracking is terminated.

THEORY

The scattering is usually described by the differential cross section $d\sigma(\theta,\phi)/d\Omega$, defined as the ratio of the number of particles scattered per unit time into an element of solid angle $d\Omega = sin\theta d\theta d\phi$, per unit solid angle, to the flux of incoming particles.

The total (elastic) scattering cross section is the flux of particles scattered in all directions, defined as:

$$\sigma_{el} = \int d\Omega \frac{d\sigma(\theta,\phi)}{d\Omega} = 2\pi \int_0^\pi sin\theta |f(\theta)|^2 d\theta \quad (1)$$

and the momentum transfer cross section is defined as:

$$\sigma_{mt} = \int d\Omega \frac{d\sigma(\theta,\phi)}{d\Omega}(1-cos\theta) = 2\pi \int_0^\pi sin\theta |f(\theta)|^2 (1-cos\theta) d\theta \quad (2)$$

in which $f(\theta)$ is the amplitude of the scattered wave. The differential cross section is only a function of the scattering angle θ for a certain particle velocity:

$$\sigma_d(\theta,v) = \frac{d\sigma(\theta,\phi)}{d\Omega} = |f(\theta)|^2 \quad (3)$$

$f(\theta)$ and all these cross sections are computed in the center-of-mass (CM) reference frame in Ref. [7] for ten points per energy decade at $E_{CM} = 10^{0.1j-1} eV, j = 0, 30$ (Data available for $j \leq 50$ online[1]). Both the differential and the total cross sections are obtained from extensive quantum-mechanical calculations and can be regarded as having very high accuracy.

To see how the momentum transfer cross section relates to the particle transport, consider the elastic scattering of a particle labled a from a material atom. In the CM frame the momentum of the particle is simply μv_a, where μ is the reduced mass of the ion-atom pair and v_a is the drift velocity of particle a. Hence the momentum loss is $\mu v_a(1-cos\theta_{CM})$ and σ_{mt} is the average momentum transfer in a

[1] http://www-cfadc.phy.ornl.gov/elastic/homeh.html

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collision. So σ_{mt} can be used to define the energy loss per collision and the corresponding mean free path.

The average number of scatters into an angle θ per unit time is $N\overline{v}2\pi\sigma_d(\theta,\overline{v})d\theta$, where $\overline{v}$ is the mean relative particle-atom speed and N is the number of atoms per volume in the material. Multiplying it by the momentum loss in one collision and integrating over all angles, the average momentum loss per unit time is $\mu v_a N\overline{v}\sigma_{mt}$. v_a and $\overline{v}$ can be related by the partition of the total energy:

$$\frac{m\overline{v}^2}{2} = \frac{mv_a^2}{2} + \frac{Mv_a^2}{2} \tag{4}$$

in which m is the mass of the sample particle and M is the mass of the material atom or molecular. Then the average momentum loss can be written as: $2N[m/(m+M)]^{1/2}\epsilon\sigma_{mt}(\epsilon)$, where ϵ is the relative energy of the collision: $\epsilon = \mu\overline{v}^2/2$.

If an electric field E is applied, an equilibrium between the acceleration and the collision deceleration can be reached: $eE = 2N[m/(m+M)]^{1/2}\epsilon\sigma_{mt}(\epsilon)$ and the kinetic energy of the particle should be fixed at the equilibrium energy.

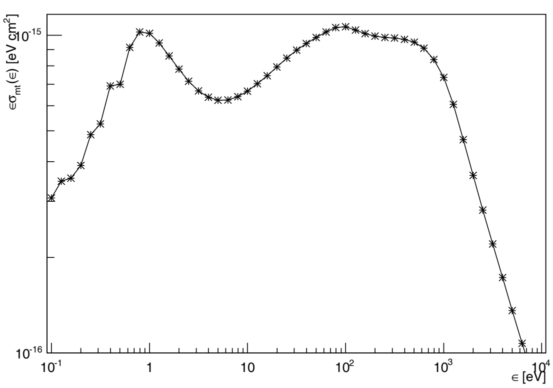

Figure 1: Collisional momentum loss as a function of relative collision energy for p-He collision.

This is true in most of the cases. Normally the parameter $\epsilon\sigma_{mt}$ continues to rise with increasing ϵ and does not have an absolute maximum in the energy range lower than the Bragg peak. This confirms the free electron gas model in Geant4. However, this is not the case for the proton-helium interaction. Figure 1 shows the curve of $\epsilon\sigma_{mt}$ versus ϵ for proton in the helium gas. The curve never rises higher than about 110×10^{-17} eVcm^2 at the energies lower than 1 keV, so if the electric field strength is higher than the energy loss the particle will eventually runaway from the low drift velocity and reach a higher kinetic energy. This "runaway" effect is first predicted by S. L. Lin [8] and experimentally observed by F. Howorka [9].

In order to compare the collisional energy loss and the ionisation energy loss, the $\epsilon\sigma_{mt}$ curve is converted to the energy loss per distance (the stopping power) in the laboratory reference frame. Figure 2 shows the energy loss

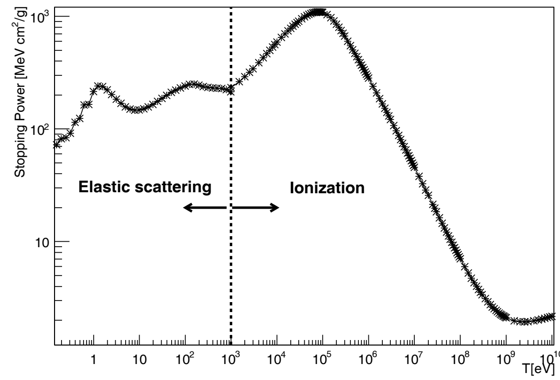

Figure 2: Stopping power of proton in helium. Left side of the dash line is the energy loss from elastic scattering, and the right side of the line is the ionisation energy loss according to the NIST data.

of the two processes matches good at the energy of 1 keV. From Fig. 1 we know that at energies higher than 1 keV the collisional energy loss rapidly gets down to zero at 6 keV. It's much smaller compared to the ionisation energy loss. In the energy range between 1 eV and 1 keV the stopping power is roughly the same. In case an electric field is applied, the equilibrium energy is hard to reach in this energy range. The particle will run away from the low energy to above 1 keV as electric field raises.

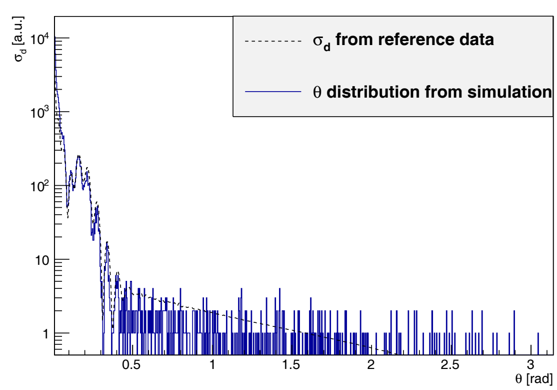

Figure 3: Simulated θ distribution (solid curve) and the differential cross section from reference data (dash curve). $T = 10eV$

Except for the average calculation, the energy loss for each single scattering can be accurately calculated from the differential cross section. The scattering angle θ is firstly randomly sampled according to the particle kinetic energy T and the differential cross section σ_d. Figure 3 shows an example of simulated θ distribution at $T = 10$ eV compared with the differential cross section data from reference [7]. Most of the time the particles are scattered in very small angles, whereas the large angle scatterings are

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important for the energy loss. Considering the kinetic energy and momentum conservation in the elastic process, the energy loss dT of the particle and the scattering angle Φ in the laboratory frame are then obtained:

$$dT = \frac{2mM}{(m+M)^2}(1 - cos\theta)T \qquad (5)$$

$$\tan\Phi = \frac{sin\theta}{cos\theta + m/M} \qquad (6)$$

Equation (5) shows that the energy loss of each collision dT is strongly correlated to the scattering angle θ by the factor $(1 - cos\theta)$. The small angle scatterings have minimum effect on the energy loss. The energy loss is mostly from the large angle scattering. The randomly generated θ causes a large fluctuation in dT.

Instead of using momentum transfer cross section σ_{mt} as in the average calculation, the transport cross section σ_{el} is used for determin the mean free path in the accurate simulations. Because σ_{el} is much larger than σ_{mt}, the computing time is much longer in the accurate simulation.

SIMULATED "RUNAWAY" EFFECT

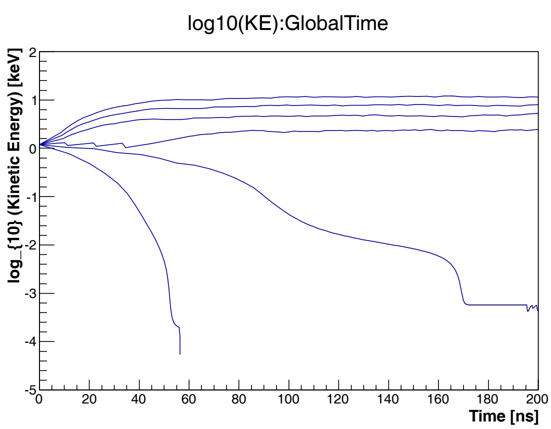

Figure 4: Run away of protons in helium with a density of $0.01\ \mathrm{mg/cm^3}$. Under various electric fields the protons reach different equilibrium energy. As the electric field increases linearly from $100\ \mathrm{kV/m}$ to $600\ \mathrm{kV/m}$, the equilibrium energy does not raise linearly but jumps from $1\ \mathrm{eV}$ to $1.5\ \mathrm{keV}$, indicating the run away effect.

By applying various electric fields, different equilibrium energies are expected. Because of the "runaway" effect the equilibrium energy should have a jump as the electric field changes linearly. Figure 4 shows this jump. The protons are injected to a helium gas (with a density of $0.01\ \mathrm{mg/cm^3}$) tube with the same initial energy. The kinetic energy changes with time under 6 different electric fields from $100\ \mathrm{kV/m}$ to $600\ \mathrm{kV/m}$. A field strength of $100\ \mathrm{kV/m}$ is not enough to compensate the energy loss. The kinetic energy goes lower than the low limit and the particle is killed by the program. With $200\ \mathrm{kV/m}$ the particle is slowly reaching the equilibrium energy around $1\ \mathrm{eV}$. When the electric field reaches $300\ \mathrm{kV/m}$, the equilibrium energy jumps to $1.5\ \mathrm{keV}$, and as the field strength raises linearly to $600\ \mathrm{kV/m}$, the equilibrium energy goes up linearly to $10\ \mathrm{keV}$.

CONCLUSION AND THE CODE STATUS

The p-He elastic scattering process has been implemented into Geant4 at energies lower than $1\ \mathrm{keV}$. Based on this process, the "runaway" effect is successfully simulated.

Currently the simulation for muons is still under testing. Several scaling methods are considered. Because of the lack of the muon scattering study in literature, we still need to check the muon simulation with our experiment at the Paul Scherrer Institut.

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