COMPUTATION OF A TWO VARIABLE WAKE FIELD INDUCED BY AN ELECTRON CLOUD *

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Abstract

The instability of a single positron or proton bunch caused by an electron cloud has been studied using analytical and semi-analytical methods which model the influence of the cloud with the wake field to the bunch. Usually this simulations are fast because the transverse wake due to the electron cloud is being pre-computed and then it is being applied to the bunch turn after turn to simulate the head tail instability. The wake field [1] in these cases is computed in the classical sense as excited electromagnetic field that transversally distorts those parts of the bunch trailing a certain transversal offset in the leading part of the same bunch. The transversal wake force depends only on the longitudinal distance between the leading part of the bunch producing the wake force and the trailing parts of the bunch feeling the wake force. However during the passage of the bunch through the electron cloud the density of the cloud near the beam axis changes rapidly. That means that the environment changes in the time as the bunch proceeds through the cloud and therefore it is not sufficient to apply the single variable (the distance) approximation for the wake field.

In this paper pursuing the idea of K. Ohmi [2] we compute the wake forces numerically as two variable function of the position of the leading part of the bunch and the position of the bunch parts trailing the leading offset in the bunch.

INTRODUCTION

In order to simulate a single bunch instability due to the electron cloud (e-cloud) the bunch movement should be followed turn by turn until the synchrotron tune of the bunch has been resolved, which may take some thousands of turns of the bunch in the ring. At each turn along the ring, the bunch interacts with the e-cloud. A fully self-consistent beam – electron cloud interaction simulation at every turn, even with only one interaction point per turn, would inevitably lead to high computational costs. An idea to speed up the single bunch instability simulation would be to pre-compute the transverse kick of the e-cloud on the bunch. Such a pre-computed kick will be later applied on the bunch at each turn during the tracking of the bunch with the appropriate transport matrices.

Because of the nature of the beam – e-cloud interaction there is a dipole kick on the bunch only if a part of the bunch perturbs the cloud, typically if a slice of the bunch has a slight transversal offset at the entrance in the cloud of homogeneously distributed electrons. However, if the bunch enters the e-cloud with no parts transversely displaced, it does not perturbs the e-cloud asymmetrically. During its passage, it only destroys the homogeneous distribution of the electrons because it attracts them towards the beam axis. As a result, the concentration of electrons near the beam axis grows very fast during the bunch passage. As a matter of fact the electrons near the beam axis start oscillating in the beam potential while the electrons from higher radiuses are approaching the beam axis and constantly increase the number of electrons near the beam axis. Thus, if the transversal offset in the bunch occurs in the rare part of the bunch the number of electrons on the beam axis which will be perturbed by the beam offset is very high. Consequently the kick from the cloud on the following bunch slices would be expected to be stronger. On the contrary, if the transversal offset occurs in the front part of the bunch then the number of electrons that will be perturbed is not going to be that large and so the expected transverse kick on the following bunch slices would not be as strong as if the electron perturbation happens later during the bunch passage.

From this very simple consideration it is obvious that the pre-computed kick due to the interaction with the e-cloud, would depend on the longitudinal position of the bunch slice with the transverse offset and the longitudinal position of the slice that receives the transversal kick. Hence it is necessary to pre-compute the matrix of kicks from every transversally slided slab of the bunch to the trailing bunch slabs. The resulting triangular matrix can be used for the single bunch instability tracking.

3D SELF-CONSISTENT PIC SIMULATION

The interaction of two different particle species is being simulated by the particle in cell program MOEVE PIC Tracking [3]. The bunch and the cloud are represented by a 3D distribution of macro-particles in a beam pipe with elliptical cross-section. The macro-particles are defined in the six-dimensional phase space $\Psi(x, p_x, y, p_y, z, p_z)$ and typical values of their number are of order 10^6 for both species. Usually the bunch particles have a Gaussian spacial distribution. The cloud particles are assumed to be homogeneously spreaded in the 3D space bounded by the beam pipe in the transverse plane and with a certain size in longitudinal direction. The interaction is simulated during the bunch passage through the e-cloud. Figure 1 shows the longitudinal profile of the ILC bunch (blue) before it has entered in a 3D homogeneously distributed e-cloud (red).

In this paper the interaction with the electron cloud is

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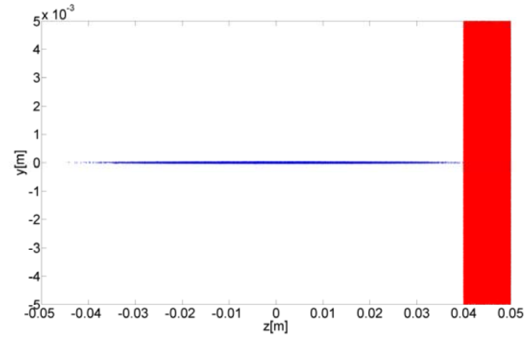

Figure 1: Longitudinal profile of the ILC bunch and the e-cloud, before the bunch enters in the cloud.

simulated for a region without external magnetic field, hence only the fields of the beam and the e-cloud are acting on the both particle species. Beside the strong transversal electrical field $\mathbf{E_b}$, the beam, being ultra relativistic $\mathbf{v_b} \approx c$, evokes also a strong transversal magnetic field $\mathbf{B_b} = (\mathbf{v_b} \times \mathbf{E_b})/c^2$. The e-cloud produces only it's own space-charge field $\mathbf{E_e}$. Because of the cancelling of the own magnetic and the electric forces the only force that affects the bunch particles $\mathbf{F_b}$ is the space-charge force of the e-cloud:

$$\mathbf{F_b} = q(\cancel{\mathbf{E_b}} + \mathbf{E_e} + \cancel{\mathbf{v_b} \times \frac{\mathbf{v_b} \times \mathbf{E_b}}{c^2}}).$$

On the other hand, the electrons feel their own and the space-charge forces of the beam:

$$\mathbf{F_e} = q(\mathbf{E_b} + \mathbf{E_e} + \cancel{\mathbf{v_e} \times \frac{\mathbf{v_b} \times \mathbf{E_b}}{c^2}}).$$

Since the electrons are relatively slow (approximately $\mathbf{v_e} \approx 0.01c$) the magnetic force from the beam can be neglected. Consequently in order to integrate the trajectory of the both species it is necessary to compute the electrical fields of the beam $\mathbf{E_b}$ and of the e-cloud $\mathbf{E_e}$. The time integration of the particle trajectory is computed for a discrete time step with typical values of $dt = 1$ ps. Before the particles are pushed, following algorithmic steps are performed at every time step in the simulation:

- Definition of the 3D laboratory frame grids for the distributions of the bunch and the cloud which is followed by weighting the particle charge on the grid nodes as an input for the discretized Poisson equation.
- The computation of the Poisson equation for the ultra-relativistic bunch takes place on it's centre-of-mass system which does not correspond with the laboratory frame grid in which the grid Poisson equation for the e-cloud will be solved. Hence two separate computations of the grid Poisson equation for both species are performed in parallel.
- Interpolation of the grid fields $\mathbf{E_b}$ and $\mathbf{E_e}$ on each particle position in the space and optionally superposition with external fields if present.

Once the forces are computed for every macro-particle of the bunch and the cloud, their trajectories will be all pushed by the leap-frog method for one time step dt further. The total time simulated equals the time the bunch needs to cross the thickness of the defined e-cloud.

In order to evaluate the interaction we first simulate a symmetrical passage of the ILC bunch (parameters given in Table 1) through a homogeneous e-cloud with a density $\rho_e = 10^{12}\mathrm{m}^{-3}$. The bunch is represented by 10^6 and the 10 mm thick e-cloud slab by $0.5 \cdot 10^6$ macro-particles.

Figure 2 shows a vertical stripe of electrons in the transverse plane gathered during the bunch passage through the initially homogeneous e-cloud. Figure 3 represents the vertical phase space of the initially static electrons after the bunch passage. It displays qualitatively two types of electron motion during the bunch passage. The electrons from the periphery are attracted to the bunch centroid position. The more they approach the beam axis the more energy they win. By the time they reach the beam axis the bunch moved further and these particles continue non-braked until they hit the chamber wall on the opposite side.

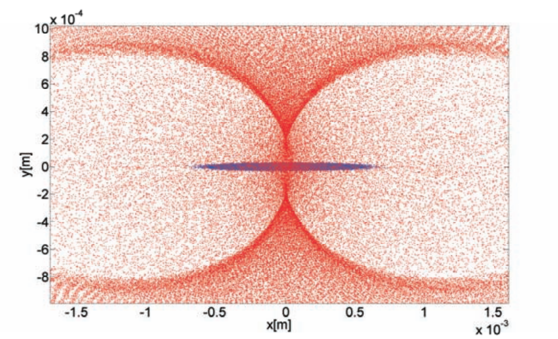

Figure 2: Transversal profile of the ILC bunch (blue dots) and the e-cloud (red dots) after the bunch passes through the cloud.

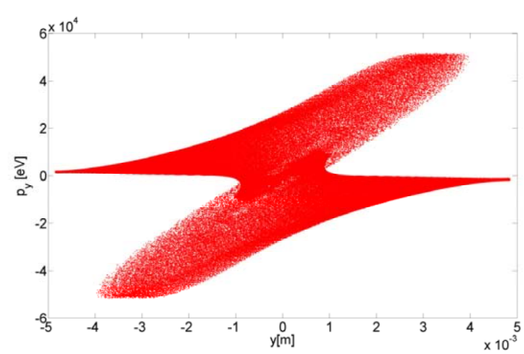

Figure 3: Electron distribution in the transversal (y) phase space after the passage of the ILC bunch.

On the other hand the fine time discretization of the simulation allows us to monitor the oscillations of the electrons of the cloud near the beam axis. First those electrons are attracted in the beam potential. Later as the electron concentration in a very small space grows, the repelling own space-charge force prevails over the beam attractive force and the electrons would disintegrate. This attraction and repulsion of the electrons to and from the beam centroid position in the transverse plain continues until the whole bunch passes by that certain longitudinal position. The frequency and the amplitude of this oscillations depend mainly on the longitudinal charge profile of the bunch, of course the longer the bunch the higher the number of oscillations will be. An indication of the e-cloud oscillations around the beam axis can be also seen at Figure 4, where the transversal momentum of the bunch particles is displayed in the function of the bunch length. The transversal momentum of the bunch particles at the beginning of the interaction is set to zero. As the bunch passes through the cloud the head particles receive a kick towards higher radiuses. As the time passes and the electron concentration on the beam axis grows the bunch particles will be kicked towards the beam axis. The envelope of the distribution of the transverse impulse of the bunch particles along its length reflects the e-cloud oscillations. Nevertheless in the case of a symmetrical bunch passage through an initially homogeneous cloud the net transverse force on the bunch over its length remains around zero (red line in Figure 4).

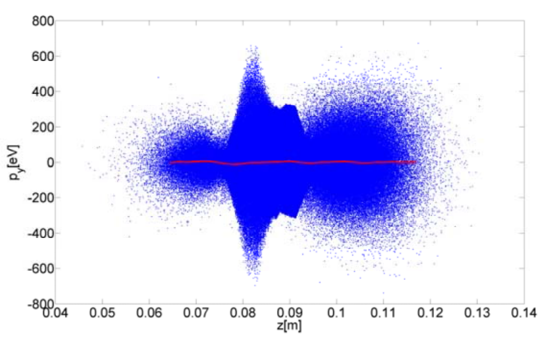

Figure 4: Transversal impulse (p_y) of the ILC bunch particles after the passage through the e-cloud.

Simulation with parts of the bunch offset

The following interaction simulations are done with a bunch which has a 3D slab shifted in the transversal plane. The offset in the y-plane for the examples presented in Figure 5 and 7 is equal to σ_y of the ILC bunch.

Figure 6 displays the transversal impulse of the bunch particles after the passage through an initially homogeneous e-cloud as shown in Figure 1. The red line is the average value of the transversal impulse of the particles at a certain longitudinal position in the bunch. It can be observed that the particles of the offset part got a vertical kick towards a higher radius of the pipe. This is due to the fact that the electrons from the higher radiuses started moving towards the beam axis attracted from the previous head parts of the bunch. Since this electrons did not had the time to reach the beam axes until the offset part arrived in the e-cloud, the offset part has been attracted towards the approaching electrons from the higher radii.

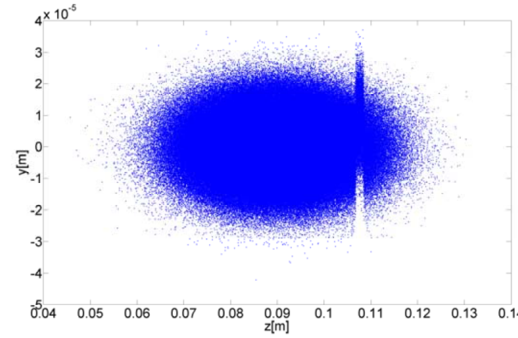

Figure 5: Profile of the ILC bunch with an offset slab in the leading part of the bunch.

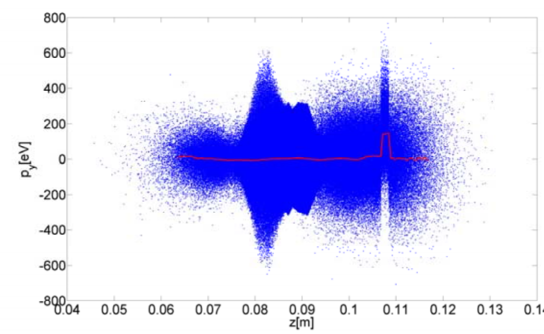

Figure 6: Transversal impulse (p_y) of the ILC bunch particles from Figure 5 after the passage through the e-cloud.

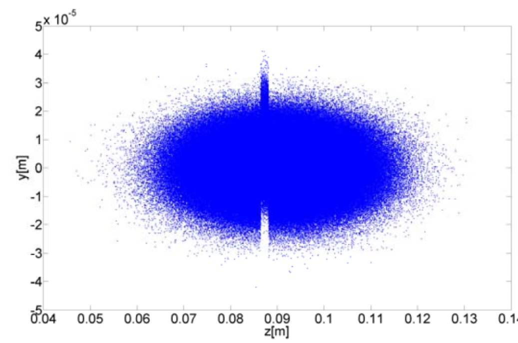

Figure 7: Profile of the ILC bunch with an offset slab in the middle part of the bunch.

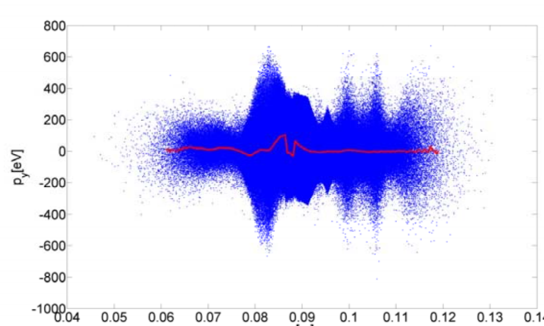

Figure 8: Transversal impulse (p_y) of the ILC bunch particles from Figure 7 after the passage through the e-cloud.

On the other hand, Figure 8 displays the transversal impulse of the bunch for the case that it's offset part is in it's middle as shown on the Figure 7. Here by the time the offset part arrives in the e-cloud enough electrons gathered on the beam axis to pull the offset part down towards the beam axis. The offset particles receive an impulse in the negative direction as it can be seen from the trend of the red line at the longitudinal position of the offset part. In the same time the electrons on the beam axis will be perturbed and shifted towards the centroid position of the offset part of the bunch. Thus the following bunch parts will receive an transversal kick in the positive direction (towards the centroid position of the previous offset part). This sort of a head-tail coupling between the bunch part perturbing the cloud and the following bunch parts happens on a very short time scale, due to a very rapid movement of the electrons in the transversal plane around the beam axis. The transversal movement of the beam particles is comparatively slow.

TWO VARIABLE WAKE FIELD INDUCED BY AN ELECTRON CLOUD IN KEKB-LER AND ILC E^+ DR

Parameter	symbol	KEKB-LER	ILC e+ DR
Circumference	L	3016 m	6695 m
Beam energy	E_b	3.5 GeV	5 GeV
Population	Nb	$3.3 \cdot 10^{10}$	$2.0 \cdot 10^{10}$
Charge	Q	5.28 nC	3.22 nC
Length (rms)	σ_z	6 mm	9 mm
Beam size(rms)	σ_x	420μm	156μm
	σ_y	60μm	7.8μm
Damping time	$\tau_{x(y)}$	4000 turns	1150 turns

Table 1: Bunch parameters of the low energy ring of the KEK B-factory and the ILC positron damping ring.

The interaction simulations presented in Figure 6 and 8 showed that the transverse kick of the e-cloud on the bunch depends strongly on the longitudinal position of the shifted part of the bunch z_i. In order to compute the kick from a transversally slided slab i on the trailing bunch slabs j ($z_i > z_j$), both bunches (ILC DR and KEKB LER) are longitudinally divided into N_s slabs. Thus we performed a series of N_s interaction simulations. For each simulation a single slab i ($i = 1 \ldots N_s$) was shifted by $\Delta y_i = \sigma_y$ and sent through an initially homogeneous e-cloud.

As a result we obtained a matrix with the average dipole kicks $\Delta p_y(j,i)$, from every transversally shifted slab i, on the trailing bunch slabs j ($z_i > z_j$). This matrix is triangular with non-zero entries for $j = i \ldots N_s$ and $i = 1 \cdots N_s$. The equation of motion for the beam in the y-plane is given by

$$\frac{\Delta p_y(j,i)}{p_b} = \frac{F_y T_t}{p_b} = \frac{\gamma E_e q T_t}{m_o c \gamma} = r_e W_1(z_j, z_i) \Delta y_i N_i. \quad (1)$$

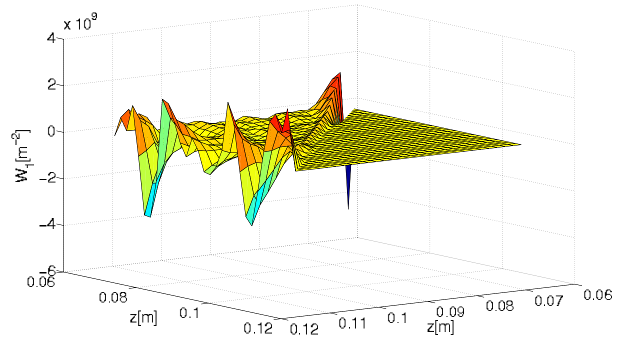

Figure 9: Two variable wake field $W_1(z_j, z_i)$ for the ILC damping ring, $N_s = 50$ slabs.

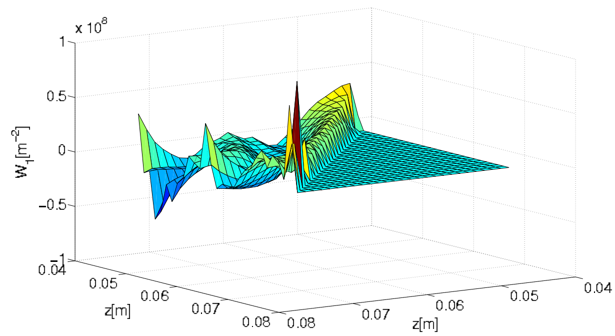

Figure 10: Two variable wake field $W_1(z_j, z_i)$ for the KEKB, $N_s = 50$ slabs.

The computed kick $\Delta p_y(j,i)$ is extrapolated for the total length L of the corresponding ring, thus T_t is the revolution period of the bunch in the ring. On the left hand side the change of the transverse impulse $\Delta p_y(j,i)$ is given relative to longitudinal impulse of the bunch p_b. On the right hand side the classical electron radius r_e is multiplied by the wake function $W_1(z_j, z_i)$ and $\Delta y_i N_i$. The parameter $\Delta y_i N_i$ represents a measure of how many particles N_i are shifted for a Δy_i in the slab i. Note that because of the Gaussian profile of the particle distribution in longitudinal direction N_i varies considerably. Finally the wake function

$$W_1(z_j, z_i) = \frac{\Delta p_y(j)}{p_b r_e \Delta y_i N_i} \quad (2)$$

was computed for both rings as shown in Figure 9 and 10.

Since the computed two variable wake matrix scales linearly with the transversal shift Δy in the next step it could be applied during the tracking simulation with the program of K. Ohmi to investigate the single bunch instability.

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