Applications of Kalman State Estimation In Current Monitor Diagnostic Systems

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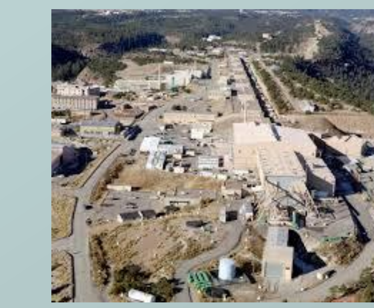

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Control Theory Background

Any linear time-invariant multi-input multi-output system might be represented in a so-called state-space representation

$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t)$

$y(t) = Cx(t) + Du(t) + E_y d(t)$

Where:

x(t) is a vector of dynamical system states

u(t) is a vector of system inputs

d(t) is a vector of system disturbances

y(t) is a vector of system outputs

$\dot{x}(t) := \frac{d}{dt}x(t)$

A is a system dynamics matrix

B is an input scaling matrix

C is an output scaling matrix

D is an input feedthrough scaling matrix

E is a disturbance scaling matrix

E_y is a disturbance feedthrough scaling matrix

State-Estimator Background

In control-theory a **state-estimator** is an auxiliary system providing approximate values for internal variables of the target system using only measurements of inputs to, and outputs from, the target system.

$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t))$

$\hat{y}(t) = C\hat{x}(t) + Du(t)$

We can obtain the error of the estimator, and its first derivative

$e(t) = \hat{x}(t) - x(t)$

$\dot{e}(t) = (A - KC)e$

Target system observability requires rank of O is the same as rank of A

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{N-1} \end{bmatrix}$$

Kalman State-Estimator Background

- The Kalman Filter is a famous model-based state-estimator algorithm providing optimized iterative estimates of system states in the presence of noise, and in the presence of other uncertainties such as imprecise target system model identification.
- Its algorithm is proven to provide mathematically optimal state estimates when errors have known Gaussian stochastic distribution.
- The filter is implemented in two steps; first it produces current system state estimates along with their uncertainties, and second it updates iterative system state estimates using weighted averaging.
- The optimized ***K*** matrix for the Kalman filter is designed when solving the Algebraic Riccati Equation.

Beam Current Transformer Challenges

- Initially this type of simple RLC sensor actually appears to be problematic
  - Pervasive noise
    - physically co-resident with the sensor
  - System identification complications
    - Sensitivity to sensor parasitic elements

Beam Current Transformer Challenges for Auxiliary State Estimator

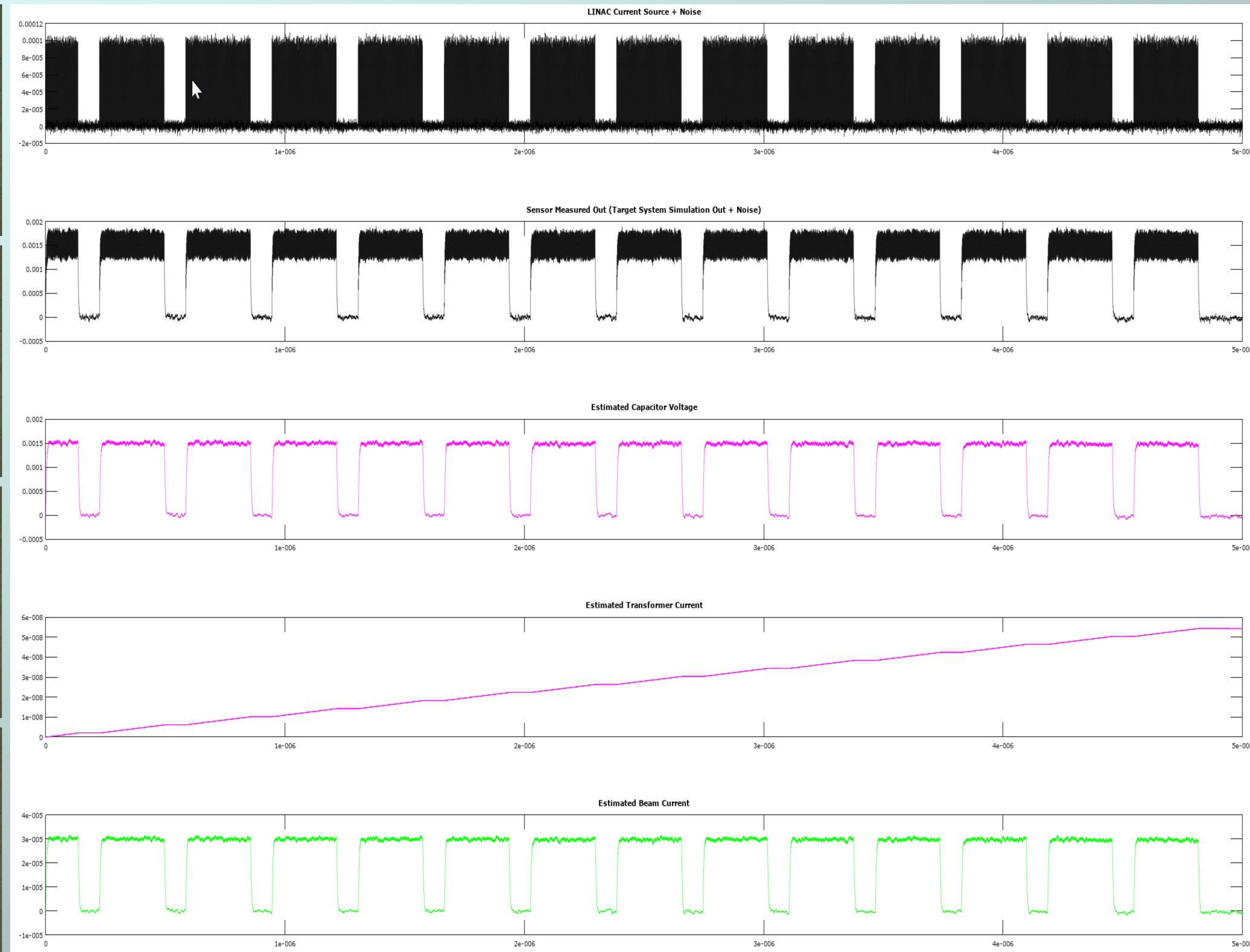
- System with two internal states
  - Only one output port for use by an auxiliary state-estimator to view them
- Beam-current is the sole driving input
  - It must be modelled as a disturbance because it isn't *directly* available to an auxiliary state-estimator as is typically required to obtain state-observability

Simplified Beam Current Transformer Circuit Model

- A passive pulsed-beam current transformer has a parallel RLC simplified equivalent circuit with a band-pass behaviour transfer function
- There are two system internal states
  - Capacitive and inductive
- In the schematic we consider some additional details with C_{sp}, R_{scp}, and R_{slp} modelling the parasitic elements of transformer secondary Inductor L_s

Kalman Estimator Applicability

- Initially, the Kalman Estimator's optimal noise reduction and system component variability error reduction appear to be quite promising for addressing at least two of the previously identified challenges, associated with current transformer sensors
- With conventional Kalman filter design state-observability is also contingent upon driving input history being known to the state-estimator
  - Therefore, a specialized algorithm, the so-called Unknown Input Observer, is required



Unknown Input Estimator (UIE) Structure

With an unknown input estimator the *input to the system, in this context the beam current, is modeled as a disturbance* $\mathbf{d(t)}$

$\dot{x}(t) = Ax(t) + Ed(t)$

$y(t) = Cx(t) + E_y d(t)$

The estimator has a state space structure as follows.

$\dot{z}(t) = Fz(t) + Ky(t)$

$\hat{x}(t) = z(t) + Hy(t)$

To complete the estimator we need to derive time invariant matrices F, K, and H

Deriving the Unknown Input Estimator

A fundamental assertion of UIE design is that the error $\mathbf{e(t)}$ is *defined* to approach zero asymptotically independent of the presence of an unknown input $\mathbf{d(t)}$, and that the estimation process can be decoupled from the disturbance

The estimator system state-space matrices can be solved for algebraically based on the dual conditions that the error approaches zero asymptotically, and the estimator structure requires $\dot{e}(t) = Fe(t)$

Unknown Input Estimator Design

$H = E[CE]^{-1}$

$A_1 = (I - HC)A$

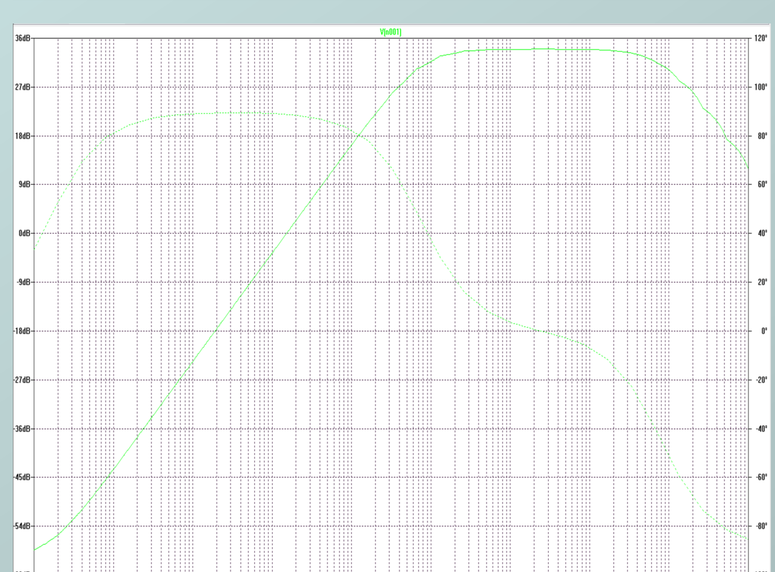
$F = A_1 - K_1 C$

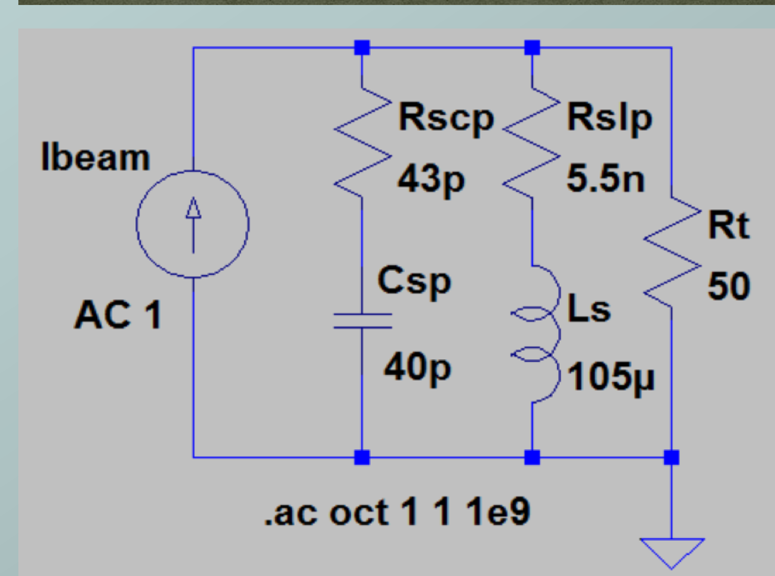
$K = K1 + K2 = K1 + FH$

Unknown Input Estimator Design

The Octave pole placement or lqr functions can be used to solve for a K matrix producing a stable estimator feedback $F = A_1 - K_1 C$

Unfortunately, with this approach we can obtain stable feedback, and can also adjust estimator performance for various metrics, but in contrast to the famous Kalman algorithm, mathematical error reduction optimality for noise and or system errors with known Guassian statistics can no-longer be guaranteed





Obtaining Sensor State-Observability

The sensor has two internal states, one input, and one output

- The voltage across Csp
- The current in Ls
- The sensor input is Ib
- The sensor output is across Rt

Obtaining sufficient observability for the beam current transformer sensor requires modelling some additional parasitic components

- In particular, Rslp and Rscp

Considering these components ensures that our model for the circuit output voltage is based on *both* of the internal states of the sensor, inductive and capacitive

Simulation Results

- *First*, a simulated beam current source was created by removing the negative excursions of a sine function, next imposing a LANSCE mini-pulse time structure, and finally adding Gaussian distribution noise.
- *Second*, the target beam current transformer sensor system was simulated, using the simulated beam current source as its input, and capturing its output.
- *Third*, Gaussian distribution noise was added also to the target beam current transformer sensor system simulated output.
- *Fourth*, the UIE system was simulated, using the simulated sensor system output plus added noise as its input, and capturing its output.

- Initial simulation results are positive with the estimator appearing to exhibit good performance removing sensor and process noise as anticipated.

Future Work

- At this time, mismatching the UIE's knowledge of the target sensor components and the target sensor simulation's knowledge of sensor components has not been tried. This would require some substantial modifications to the simulation script.
- Modern tools exist for converting Octave (MATLAB) models into a physical FPGA-based signal processing system, and the next step would be to test the algorithm on a physical system.
- Initial results appear to indicate that we could do more to eliminate droop distortion possibly by resetting the UIE initial conditions at the end of each mini pulse, or possibly implementing **d(t)** estimation enhancements

Conclusions

- The current transformer sensor was characterized in SPICE considering additional parasitic component values.
- A novel approach for beam current transformer data acquisition was designed and simulated.
- The specific Kalman algorithm was found to be incompatible with this context and instead a related estimator algorithm, the so-called the Unknown Input Estimator (UIE), was evaluated, for beam current transformer sensor systems.
- Initial Octave simulation results are positive with this estimator appearing to exhibit good performance reducing sensor and process noise.
- We hope to test the UIE based data acquisition on a physical system in the near future.
- The UIE implementation also appears to have potential for removing droop distortion.

Unknown Input Estimator Design

After designing the estimator then the next step is to produce an estimation of the unknown disturbance input d(t), in this context the beam current, from the state estimates. This must be approached carefully so that we obtain a meaningful and accurate result.

$$\hat{d}(t) = [CE]^{-1}[\hat{y} - CA\hat{x}(t)]$$