16th Int. Conf. on Accelerator and Large Experimental Control Systems ICALEPCS2017, Barcelona, Spain JACoW Publishing
ISBN: 978-3-95450-193-9 doi:10.18429/JACoW-ICALEPCS2017-THPHA128

# APPLICATIONS OF KALMAN STATE ESTIMATION IN CURRENT MONITOR DIAGNOSTIC SYSTEMS*

J. Hill, LANL/LANSCE, Los Alamos, U.S.A

*Abstract*

Traditionally, designers of transformer-based beam current monitor diagnostic systems are constrained by fundamental trade-offs when reducing distortion in time-domain beam-pulse facsimile waveforms while also attempting to preserve information in the frequency-domain. When modelling the sensor system with a network of linear time-invariant passive components, and a state-based representation based on first-order differential equations, we identify two internal dynamical states isolated from each other by the parasitic resistance in the transformer windings. They are the parasitic capacitance voltage across the transformer's windings, and the transformer inductor current. These states are typically imperfectly observed due to noise, component value variance, and sensor component network topology. We will discuss how feedback-based Kalman State Estimation implemented within digital signal-processing might be employed to reduce negative impacts of noise along with component variance, and how Kalman Estimation might also optimize the conflicting goals of beam-pulse facsimile waveform fidelity together with preservation of frequency domain information.

## CONTROL THEORY BACKGROUND

Any linear time-invariant multi-input multi-output system might be represented in a so-called state-space representation [1], see equations 1 and 2.

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \quad (1)$$
$$y(t) = Cx(t) + Du(t) + E_y d(t) \quad (2)$$

Where:

- ***x(t)*** is a vector of dynamical system states
- ***u(t)*** is a vector of system inputs
- ***d(t)*** is a vector of system disturbances
- ***y(t)*** is a vector of system outputs
- $\dot{\boldsymbol{x}}(\boldsymbol{t}) := \frac{d}{dt}\boldsymbol{x}(\boldsymbol{t})$
- ***A*** is a system dynamics matrix
- ***B*** is an input scaling matrix
- ***C*** is an output scaling matrix
- ***D*** is an input feedthrough scaling matrix
- ***E*** is a disturbance scaling matrix
- $\boldsymbol{E_y}$ is a disturbance feedthrough scaling matrix

The ***A*** matrix determines the systems *dynamical* behaviour while the ***B***, ***C***, ***D*****,** and ***E*** time-invariant matrices determine how the system interacts with its external environment. A canonical form of the state-based representation has the main diagonal elements of the time-invariant ***A*** matrix populated with the system's Eigenvectors and other elements zero.

In control-theory a **state-estimator** is an auxiliary system providing approximate values for internal variables of the target system using only measurements of inputs to, and outputs from, the target system. It is often possible to provide optimized system diagnostics, and also optimized system control, when enhanced estimates of the, often not directly measurable, internal states of the system are available. Equations (3) and (4) show the typical state-space representation for an axillary model-based state estimator. The emphasis with state estimator design is to formulate matrix ***K*** for stable feedback minimizing error in equation (5), which must satisfy differential equation (6).

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t)) \quad (3)$$
$$\hat{y}(t) = C\hat{x}(t) + Du(t) \quad (4)$$
$$e(t) = \hat{x}(t) - x(t) \quad (5)$$
$$\dot{e}(t) = (A - KC)e \quad (6)$$

A system is said to be **state-observable** if estimates for all internal states as time progresses can be provided contingent on knowledge of a model for the linear time-invariant system, initial conditions for its states, history of system inputs, and history of system outputs. In control-theory a necessary and sufficient condition for successful state-estimator design is that the rank of ***O*** in equation (7) based on the state-space representation of the target system must be the same as the rank ***N*** of ***A***.

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{N-1} \end{bmatrix} \quad (7)$$

The so-called Kalman Filter [2] is a famous model-based state-estimator feedback algorithm providing optimized iterative estimates of system states in the presence of noise, and in the presence of other uncertainties such as imprecise target system model identification. Its algorithm is proven to provide mathematically optimal state estimates when errors have known Gaussian stochastic distribution. The filter is implemented in two steps; first it produces current system state estimates along with their uncertainties, and second it updates iterative system state estimates using weighted averaging. The optimized ***K*** matrix for the Kalman filter is designed when solving the Algebraic Riccati Equation [3].

## SIMPLIFIED CIRCUIT MODEL

A passive pulsed-beam current transformer has a parallel RLC simplified equivalent circuit with a band-pass behaviour transfer function [4]. In figure 1 we consider some additional details with $\boldsymbol{C_{sp}}$, $\boldsymbol{R_{scp}}$, and $\boldsymbol{R_{slp}}$ modelling the parasitic elements of transformer secondary Inductor $\boldsymbol{L_s}$[5]. The resistor $\boldsymbol{R_t}$ is added across the sensor's voltage

* Supported by US Depart of Energy contract DE-AC52-06NA25396.

ISBN: 978-3-95450-193-9 doi:10.18429/JACoW-ICALEPCS2017-THPHA128

output leads potentially matching impedance with the transmission line path to signal amplification. The beam current input is modelled in figure 1 as an AC ideal current source.

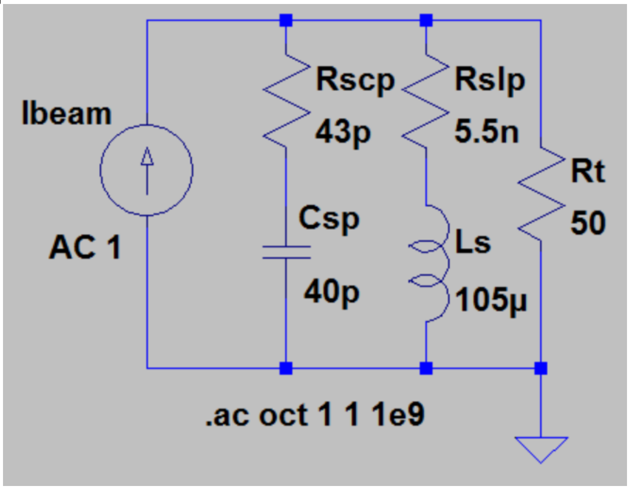


Figure 1: Simplified Passive Current Transformer

Modelling this system in SPICE, we can confirm that this type of two pole parallel RLC circuit behaves fundamentally like a bandpass filter, see figure 2 where the solid line is amplitude and the dotted line phase in the circuit's frequency response.

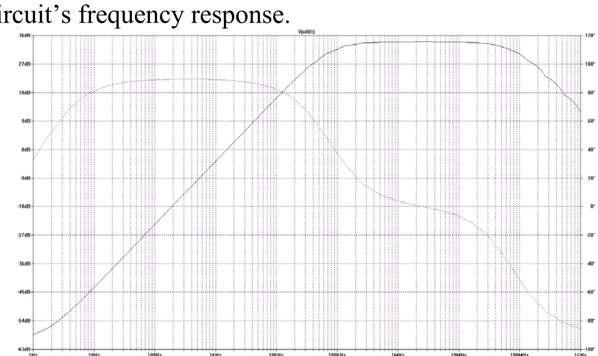

Figure 2: Bandpass Response in SPICE

Two dynamical states can be identified in this simplified model; they are the voltage across $C_{sp}$ and the current through $L_s$. Our state-space representation for the current transformer sensor system in figure 1 is in equations (8) through (16). In these equations $v_{C_{sp}}$ is the voltage across $C_{sp}$ referenced to ground, and $i_{L_s}$ is the current flowing through $L_s$ towards ground. The circuit input ***d(t)*** is the current source **Ibeam** in figure 1, and the output ***y(t)*** is measured across $R_t$ referenced to ground.

$$x(t) = \begin{bmatrix} v_{C_{sp}} \\ i_{L_s} \end{bmatrix} \tag{8}$$

$$d(t) = i_{beam} \tag{9}$$

$$R_{eff} = \frac{1}{1+\frac{R_{scp}}{R_t}} \tag{10}$$

$$A = \begin{bmatrix} \frac{-Reff}{C_{sp}R_t} & \frac{-R_{eff}}{C_{sp}} \\ \frac{1-\frac{R_{scp}R_{eff}}{R_t}}{L_s} & \frac{-(R_{scp}R_{eff}+R_{slp})}{L_s} \end{bmatrix} \tag{11}$$

$$B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{12}$$

$$C = \begin{bmatrix} R_{eff} & (R_{eff}-1)R_t \end{bmatrix} \tag{13}$$

$$D = [0] \tag{14}$$

$$E = \begin{bmatrix} \frac{Reff}{C_{sp}} \\ \frac{R_{scp}R_{eff}}{L_s} \end{bmatrix} \tag{15}$$

$$E_y = [(1-Reff)R_t] \tag{16}$$

## CHALLENGES

Initially this type of simple RLC sensor actually appears to be problematic. Current transformer diagnostics systems are challenged by pervasive noise physically coresident with the sensor in the typical accelerator transport environments [6]. System component value identification is also substantially complicated due to the sensor's sensitivity to parasitic elements in the transformer's secondary windings which typically vary over several identically procured transformers. A secondary issue is that the transformer coupled sensor results in some undesirable droop distortion in response to pulsed input.

Furthermore, when evaluating options for using a state-estimator with this type of sensor, we are circumspectly aware that this is a system with two internal states, but there is only one output port for a state-estimator to utilize when observing them. A more substantial concern is that in this application the beam-current is the sole driving input, but it must be modelled for state-estimation purposes as a disturbance ***d(t)*** because it isn't *directly* available to an auxiliary state-estimator, as is typically required for inputs to systems characterized as sufficiently **state-observable**.

## KALMAN ESTIMATION APPLICABILITY

Initially, the Kalman Estimator's optimal noise reduction and system component variability error reduction appear to be quite promising for addressing at least two of the previously identified challenges, associated with current transformer sensors. Recall however that, in this context, the beam-current is the sole driving input to the target system must be modelled as a disturbance ***d(t)***, only indirectly measurable by an external state-estimator for a beam current transformer sensor target system. With conventional Kalman filter design the necessary and sufficient condition for state-observability, calculated in (7), is also contingent upon driving ***d(t)*** input history being known to the state-estimator. Therefore, application of conventional Kalman state-estimation algorithms in current monitor sensor systems will not be successful. Instead, a specialized state-estimator formulation, the so-called Unknown Input Observer (UIO) with associated theory investigated since the 1970's [7][8][9][10][11] is required. To remain consistent in this paper we will use the name unknown input estimator (UIE) for this algorithm.

## SENSOR SYSTEM OBSERVABILITY

Initially we assume that the external state-estimator *does* have explicit knowledge of all forcing inputs and

ISBN: 978-3-95450-193-9 doi:10.18429/JACoW-ICALEPCS2017-THPHA128

focus instead on the more antecedent issue of obtaining observability of all internal states of the target sensor system using only its single output port. Recall that we must obtain necessary and sufficient state-observability as calculated in equation (7). Commonly employed models for a non-ideal inductor omits component $\boldsymbol{R_{scp}}$ possibly because it has minimal impact on the inductor's circuit behaviour when operating at lower frequencies. However, when attempting to estimate *both* $\boldsymbol{v_{C_{sp}}}$ and $\boldsymbol{i_{L_s}}$ we must model also parasitic component $\boldsymbol{R_{scp}}$ so that $\boldsymbol{v_{C_{sp}}}$ does not become the only internal state variable algorithmically linked with the circuit's output voltage. Furthermore, parasitic component $\boldsymbol{R_{slp}}$ must also be modelled so that $\boldsymbol{i_{L_s}}$ does not become the only internal state variable algorithmically linked with the circuit's output voltage ***y(t)***, based on the first derivative of its current. Finally, we expect from a practical perspective that a model-based state estimator's capacity to remove noise from its estimates will be enhanced if the two state variables are influenced to be mutually out of phase based on parasitic component values in our selected current transformer sensor.

## UNKNOWN INPUT ESTIMATOR DESIGN

To obtain a model of the unknown input system we modify state-space equations (1) and (2), eliminating ***u(t)*** leaving additive disturbance ***d(t)*** as the sole system input, to obtain equations (17) and (18).

$$\dot{x}(t) = Ax(t) + Ed(t) \tag{17}$$

$$y(t) = Cx(t) + E_y d(t) \tag{18}$$

The state-space equations for the auxiliary UIE system with only disturbance inputs are shown in equations (19) and (20). In this estimator ***z(t)*** maintains the state estimate internally, ***y(t)*** is the output from the target system, and the $\boldsymbol{\hat{x}(t)}$ provides the output estimated variables. The time-invariant auxiliary estimator matrices ***F***, ***K***, and ***H*** need to be designed.

$$\dot{z}(t) = Fz(t) + Ky(t) \tag{19}$$

$$\hat{x}(t) = z(t) + Hy(t) \tag{20}$$

A fundamental assertion of UIE design is that the error **e(t)** in equation (5) is *defined* to approach zero asymptotically independent of the presence of an unknown input **d(t)**, and that the estimation process can be decoupled from the disturbance. The estimation error can be solved for algebraically using (5), (17), (18), (19), and (20). Based on the dual conditions that the error approaches zero asymptotically, and the estimator structure requires $\dot{e}(t) = Fe(t)$, then a solution can be derived for an estimator. This class of estimator is described theoretically in the literature [7][8][9][10][11], in a textbook chapter [12], and as stepwise design in [13]. The tuple $\langle C, A_1\rangle$ must be observable in equation (7) as a necessary condition for UIE existence. In the beam current transformer context this observability criteria was satisfied without additional effort, but there are additional options decomposing the tuple $\langle C, A_1\rangle$ into observer canonical form obtaining observability whenever a stable UIE exists. Next some of the UIE time-invariant matrices can be calculated in equations (21) and (22).

$$H = E[CE]^{-1} \tag{21}$$

$$A_1 = (I - HC)A \tag{22}$$

Feedback stability of the UIE requires stable eigenvalues of ***F*** in equation (23). This is easily accomplished using an Octave control system package library functions such as **place** or **lqr** to calculate feedback gain **K1** passing $\boldsymbol{A_1}^{\boldsymbol{T}}$ and $\boldsymbol{C^T}$ for the function's A and B matrix parameters. Both methods were tried with the **lqr** based pole placement appearing to be more efficient for arriving at an estimator with good performance removing noise while also faithfully tracking the sensor voltage out, but this of course will be highly dependent on the skills of the system designer. With either approach we can adjust the estimator performance for various metrics, but in contrast to the famous Kalman algorithm, we expect that mathematical optimal error reduction for noise and or system errors with known Guassian statistics is no-longer guaranteed.

Finally, we can finish the UIE design calculating matrix ***K*** in equation (24).

$$F = A_1 - K_1 C \tag{23}$$

$$K = K1 + K2 = K1 + FH \tag{24}$$

Once we have created a stable feedback UIE then it is useful also to calculate the disturbance value, in this situation the beam-current, given the UIE's estimate of the sensor's internal states. This must be approached carefully so that we obtain a meaningful and accurate result. The proper approach can be derived by taking $\frac{d}{dt}$ of the output in equation (18), and next substituting (17) to obtain equation (25) [10].

$$\dot{y} = CAx(t) + CEd(t) + E_y\dot{d}(t) \tag{25}$$

From (25) an estimate for ***d(t)*** can then be obtained algebraically in (26). Initially there are concerns about increased noise coupling due to summation with $\boldsymbol{\dot{\hat{y}}}$ a derivate term. However in practice positive results are obtained, perhaps because the derivative of the state estimate $\boldsymbol{\hat{y}}$ and not the derivative of the sensor output voltage ***y*** is the basis for our result.

$$\hat{d}(t) = [CE]^{-1}\left[\dot{\hat{y}} - CA\hat{x}(t)\right] \tag{26}$$

To obtain simulated estimates for $\boldsymbol{\hat{y}}$ and $\boldsymbol{\dot{\hat{y}}}$ it is necessary to add some additional estimator states to the UIO in Octave.

## SIMULATION RESULTS

In figure 3 results from simulating the target beam current transformer sensor system along with the auxiliary unknown input estimator can be seen. *First*, a simulated beam current source was created by removing the negative excursions of a sine function, next imposing a LANSCE mini-pulse time structure, and finally adding Gaussian distribution noise. *Second*, the target beam current transformer sensor system was simulated, using the simulated beam current source as its input, and capturing its output. *Third*, Gaussian distribution noise was added also to the target beam current transformer sensor system simulated output. *Fourth*, the UIE system was simulated,

ISBN: 978-3-95450-193-9 doi:10.18429/JACoW-ICALEPCS2017-THPHA128

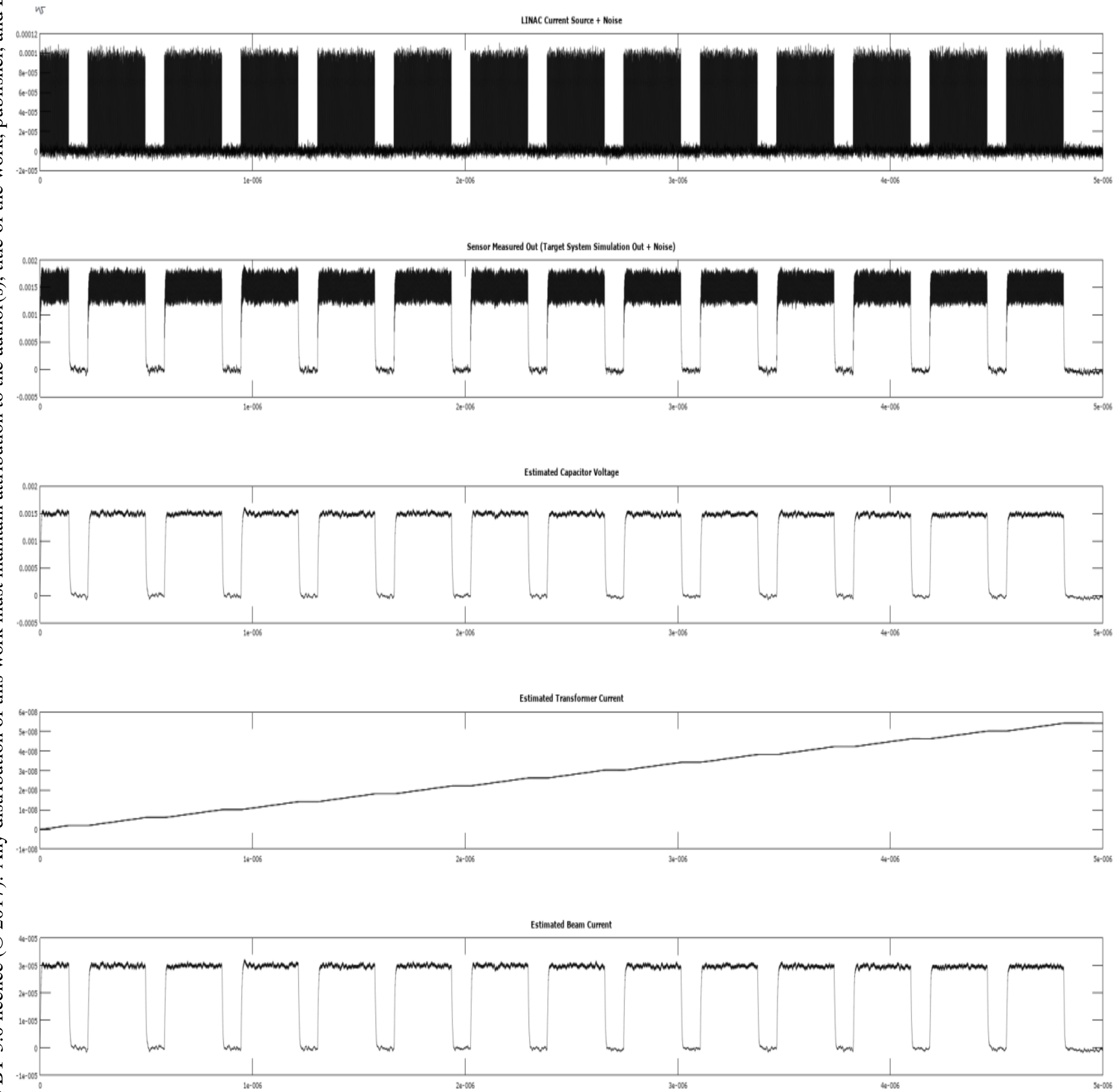


Figure 3: Octave Simulation Results

using the simulated sensor system output plus added noise as its input, and capturing its output.

Initial simulation results are positive with the estimator appearing to exhibit good performance removing sensor and process noise as anticipated. When running the simulation it is observed that the UIE performance *is* sensitive to the RLC bandpass high frequency cut-off of the sensor. If we open up the high frequency bandwidth of the sensor sufficiently so that the LANSCE RF micro pulses pass through the sensor then the UIE does not perform as well removing noise. Otherwise, the UIE appears to perform well for a range of different sensor component value selections that were tried.

## FUTURE WORK

At this time, mismatching the UIE's knowledge of the target sensor components and the target sensor simulation's knowledge of sensor components has not been tried. This would require some substantial modifications to the simulation script.

Modern tools exist for converting Octave (MATLAB) models into a physical FPGA-based signal processing system, and the next step would be to test the algorithm on a physical system.

For steady-state beam droop distortion perhaps isn't noticed because the sensor's transformer coupled output and the UIE's estimate of it will settle to a quiescent offset. In contrast, for pulsed beam structure droop distortion might become problematic depending on sensor component values. However, it is expected that, with an FPGA implementation, additional logic could be added resetting UIE initial conditions whenever measured beam current drops below a certain threshold, and consequently droop distortion might be significantly reduced, at least for

ISBN: 978-3-95450-193-9 doi:10.18429/JACoW-ICALEPCS2017-THPHA128

some modes of beam pulse time structure at LANSCE. Also, potential for eliminating this type of distortion with enhanced estimations of **d(t)** needs further investigation.

## CONCLUSION

The current transformer sensor was characterized in SPICE considering additional parasitic component values. A novel approach for beam current transformer data acquisition was designed and simulated. The specific Kalman algorithm was found to be incompatible with this context and instead a related estimator algorithm, the so-called the Unknown Input Estimator, was evaluated, for beam current transformer sensor systems. Initial Octave simulation results are positive with this estimator appearing to exhibit good performance reducing sensor and process noise. We hope to test the UIE based data acquisition on a physical system in the near future. The UIE implementation also appears to have potential for reducing droop distortion.

## REFERENCES

[1] ”State-space representation”, https://en.wikipedia.org/wiki/State-space_representation
[2] “Kalman Filter”, https://en.wikipedia.org/wiki/Kalman_filter
[3] ”Algebraic Riccati Equation”, https://en.wikipedia.org/wiki/Algebraic_Riccati_equation
[4] P. Forck, “Lecture Notes on Beam Instrumentation and Diagnostics”, http://www-bd.gsi.de/conf/juas/juas_script.pdf
[5] Martin O’ Hara, Modelling Non-Ideal Inductors in SPICE, www.intusoft.com/articles/inductor.pdf
[6] R Webber, “Tutorial on Beam Current Monitoring”, http://lss.fnal.gov/archive/2000/conf/Conf-00-119.pdf
[7] Ron J. Patton Jie Chen, Robust model-based fault diagnosis for dynamic systems, Springer, New York, 1999.
[8] R. Clark R. Patton, P. Frank, Fault diagnosis in dynamic systems: Theory and applications, Prentice Hall, New York, 1989.
[9] Thoralf A. Schwarz, Uncertainty Analysis of a Fault Detection and Isolation Scheme for Multi-Agent Systems, Master's thesis, KTH, Sweden, 2012. State and Input Estimation for a Class of Uncertain Systems
[10] Martin Corless, Jay Tu, “State and Input Estimation for a Class of Uncertain Systems”, http://www.sciencedirect.com/science/article/pii/S0005109898000132
[11] Didier Maquin, Besma Gaddouna and Jose Ragot, “Estimation of Unknown Inputs in Linear Systems”, http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=751939
[12] Bahram Shafai and Mehrdad Saif, “Systems Engineering A Report on Four Decades of Contribtions”, https://link.springer.com/content/pdf/10.1007%2F978-3-319-14636-2.pdf
[13] S. Nazari, “A Review Of the Unknown Input Observer With Examples”, arXiv:1504.07300v2 [cs.SY] 29 Apr 2015