Bayesian Reliability Model for Beam Permit System of RHIC at BNL

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MOM3I01 (Systems Engineering & Project Management)

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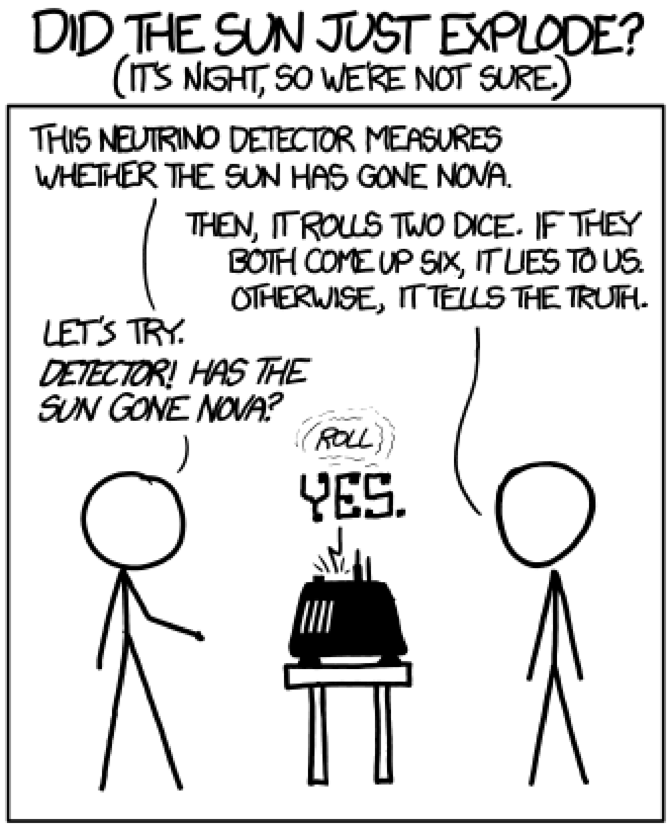

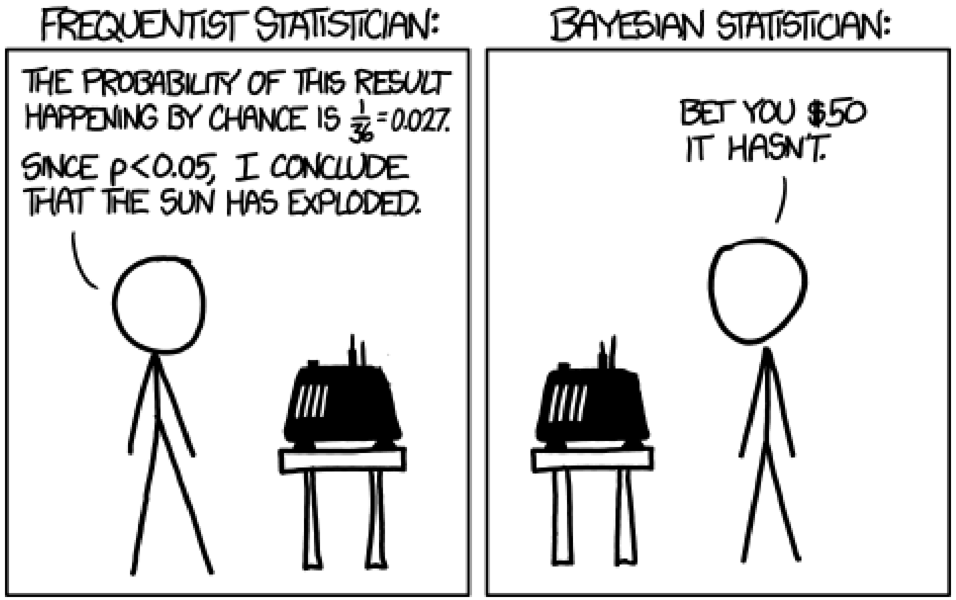

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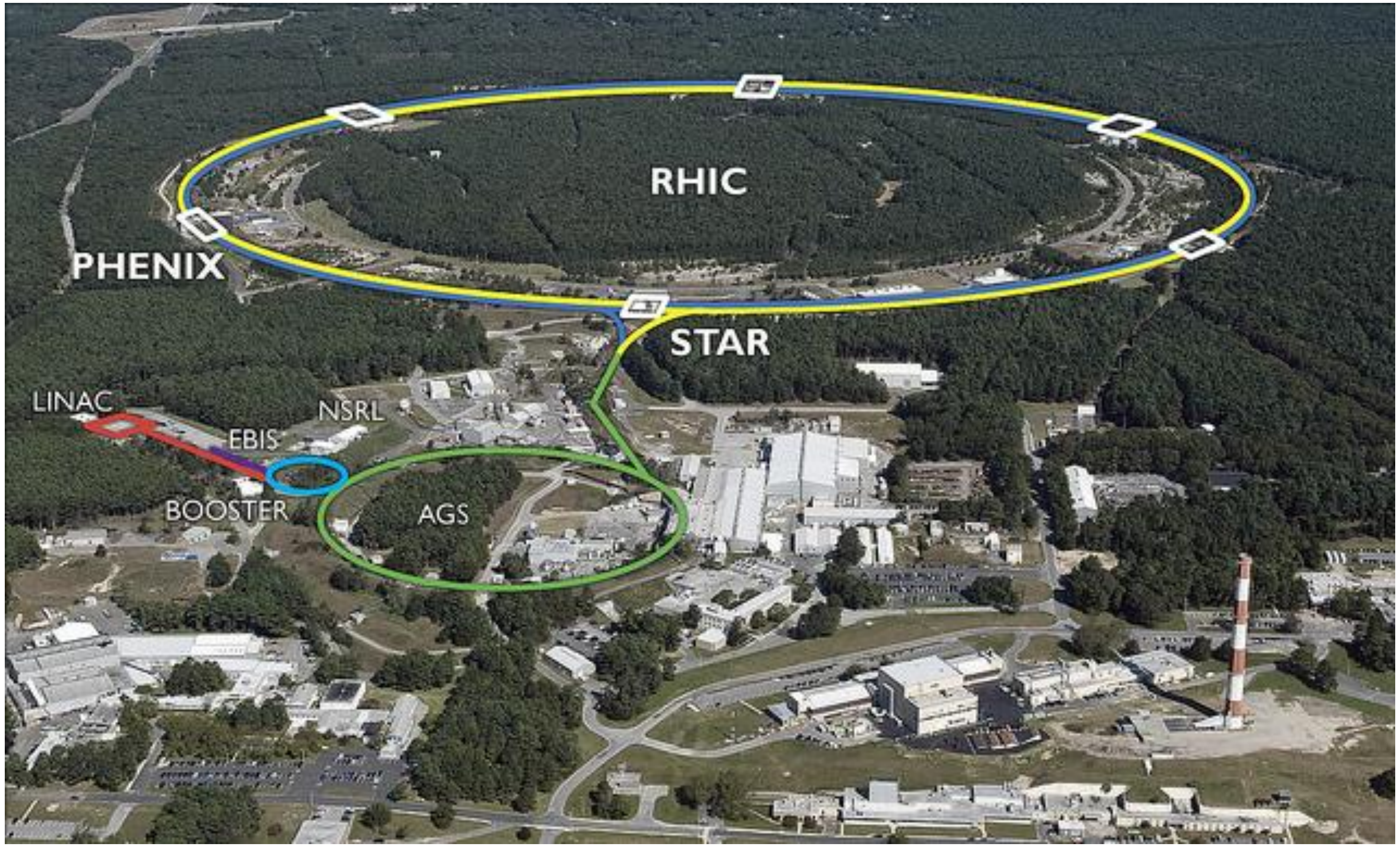
RHIC

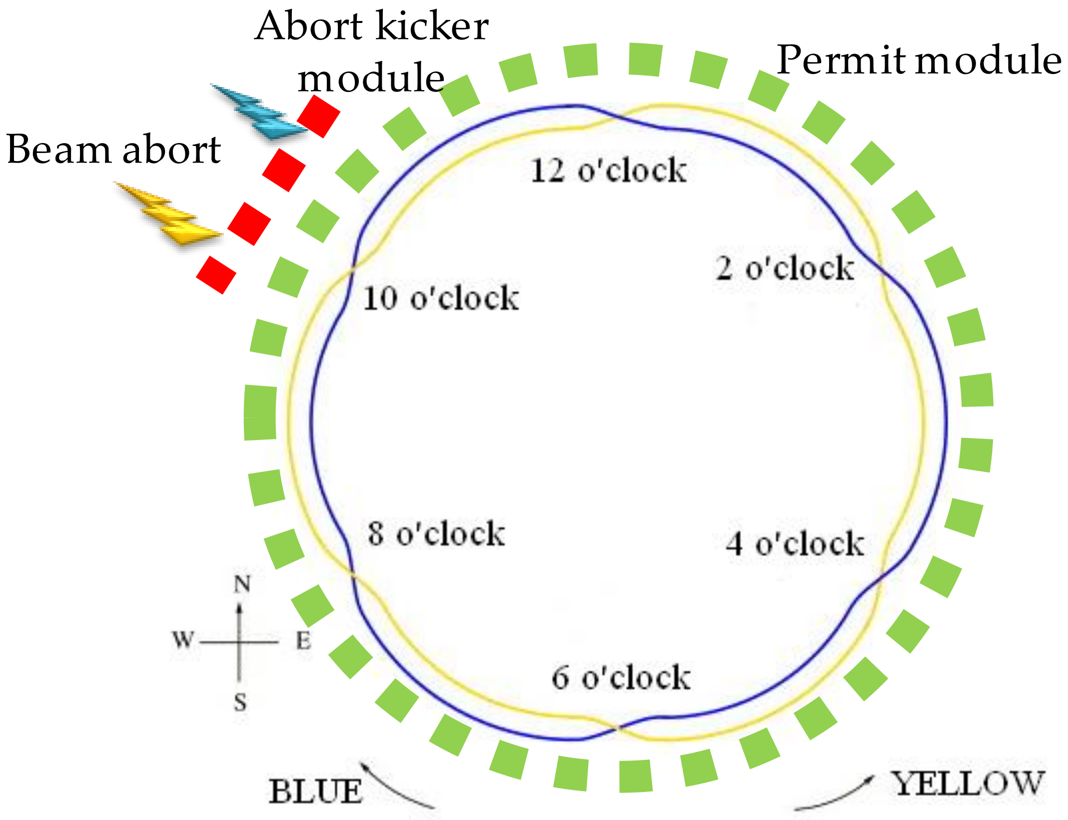
- Relativistic Heavy Ion Collider – 2.4 mile
- Yellow and Blue – two counter circulating beams
- 6 interaction regions



Relativistic Heavy Ion Collider

Beam Permit System

- RHIC peak stored energy 72 MJ
  - 70 MJ in SC magnets
  - 2 MJ in beams
- Beam Permit System (BPS) :
  - Receives subsystems' health inputs
  - Takes decision for disposal of energy
- Impacts reliability and availability of RHIC

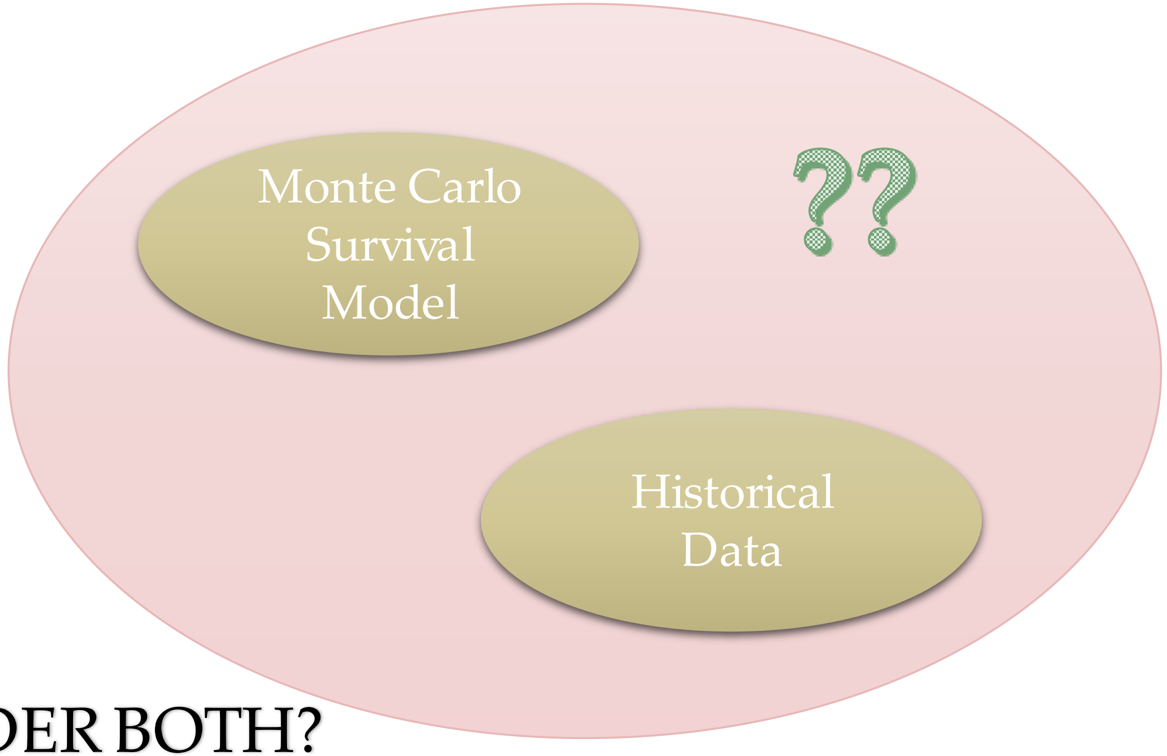


Beam Permit System

Objective

- Estimate Reliability characteristics of BPS - Draw best possible inference
- Reliability Information Sources

1. Monte Carlo model[1]: Profound view
    - Basic component failures[2], Structural bottlenecks
    - Exponential – MIL HDBK[3], Manufacturer's data
2. Historical failure data (15 years): Bird's eye view
    - Real survival characteristics

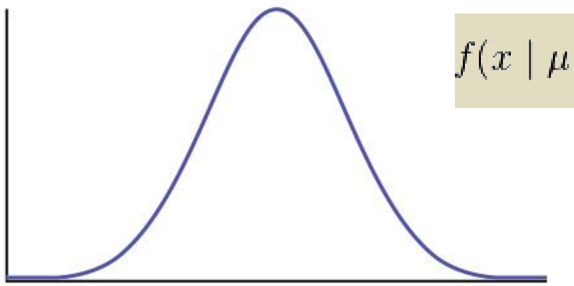


HOW TO CONSIDER BOTH?

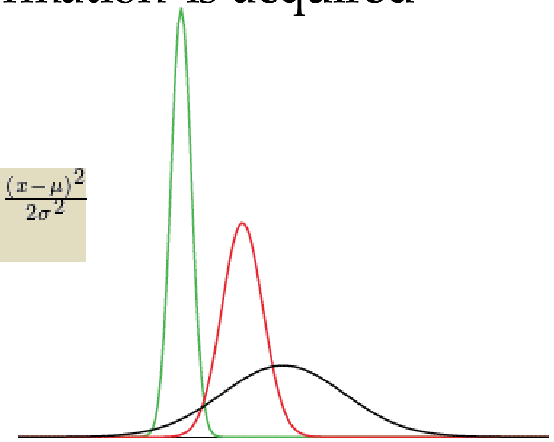
Bayesian Paradigm[4]

Integration of information from multiple sources

Updating current knowledge when new information is acquired



$$f(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Thomas Bayes

Image courtesy: Wikipedia

Frequentist Approach

- Event probability: limit of its long term occurrence rate
- Parameters constant - μ, σ

Bayesian Approach

- Keeps updating probability
- Parameters are random variables - μ, σ

Bayesian Paradigm

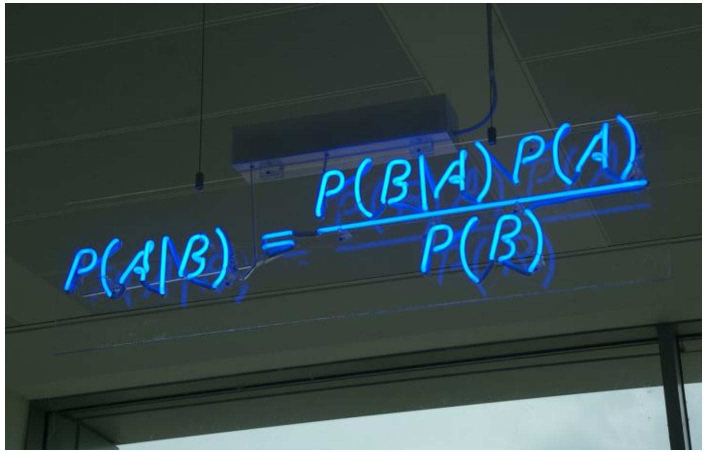


Image courtesy: Wikipedia

Bayes theorem: Continuous form

Unknown parameter of a distribution

New recorded data

Likelihood function

$$\pi(\theta|x) = \frac{L(\theta|x) \times \pi(\theta)}{f(x)}$$

Prior distribution

$\theta \sim N(\upsilon, \omega^2)$

hyperparameters

Posterior distribution

Marginal distribution of x

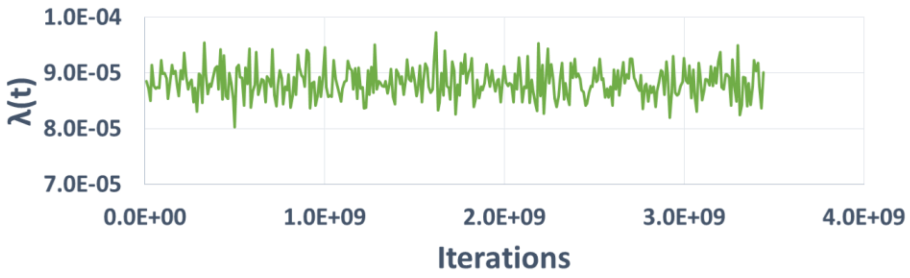
$$\pi(\theta|x) \propto L(\theta|x) \times \pi(\theta)$$

Foundation of Bayesian analysis

Source 1: Monte Carlo model

- Exponential[5] survival distribution of components
- Simulates system behavior – top distribution may be different

STEP 1



Check - Non homogenous Poisson process[6]

λ for 10^7 hours – no trending

STEP 2

Distribution	Parameter	Point estimate	AIC*	BIC**
Exponential	Scale - λ Shape - α	8.831e-5 1	3141125.52	3141136.15
Weibull	Scale - λ Shape - α	8.829e-5 1.00046	3141127.47	3141148.72
Gamma	Scale - λ Shape - α	8.84e-5 1.00106	3141127.44	3141148.66

Check – Distribution with goodness-of-fit

*Akaike Information Criterion[7]
**Bayesian Information Criterion[7]

Source 2: Historical failure data

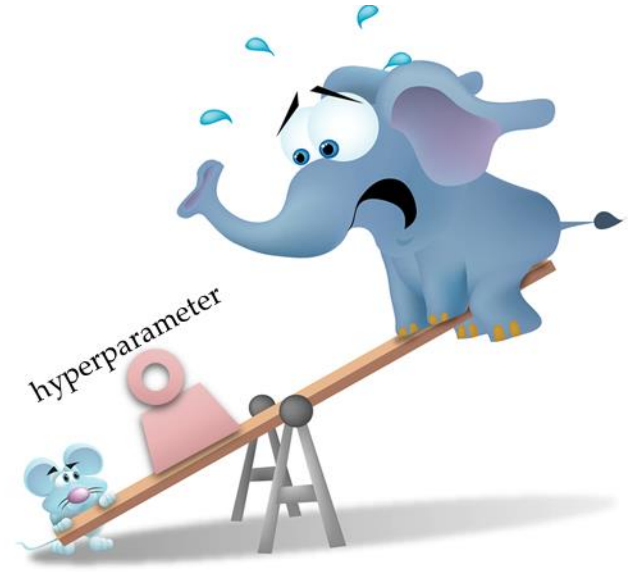
- BPS hardware failure data for 15 years
- 16 data points for the time between failures owing to high reliability of BPS

Check - Distribution of historical data with goodness-of-fit

Distribution	Parameter	Point estimate	AIC*	BIC**
Exponential	Scale - λ Shape – α	1.20E-04 1	322.9	323.7
Weibull	Scale - λ Shape - α	0.000171 0.627457	317.6	319.1
Gamma	Scale - λ Shape - α	6.03E-05 0.503074	318.1	319.7
Lognormal	Scale - λ Shape - α	7.7676 1.99141	319.0	320.5

*Akaike Information Criterion
**Bayesian Information Criterion

Bayesian reliability model



- **Posterior**
  - Tradeoff between Prior and Data distribution
  - Tradeoff level: depends on relative strength, can be changed by altering hyperparameters
- Conjugate prior: Yields a posterior of the same form as the data distribution
- Model choice :

Bayesian – Weibull
(Scale and shape unknown)

Data - Weibull

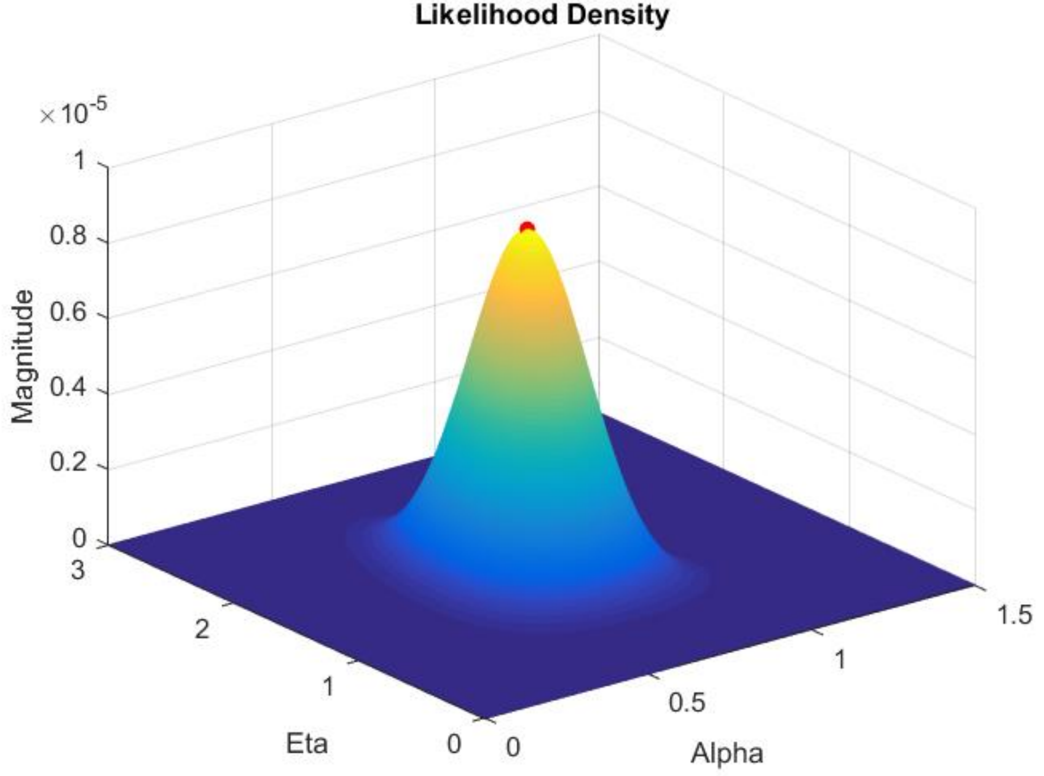
Prior – Exponential
(Weibull with shape 1)

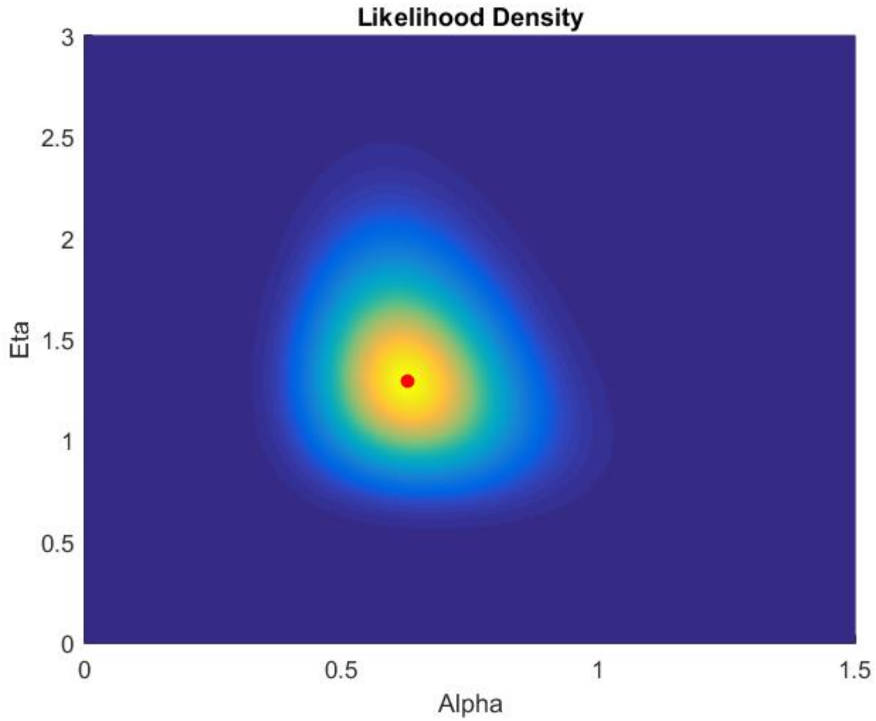
Data Distribution [8]

Weibull: $f(x|\alpha,\eta) = \alpha\eta x^{\alpha-1}e^{-\eta x^{\alpha}}$

Shape = α

Scale = $\eta^{-1/\alpha}$



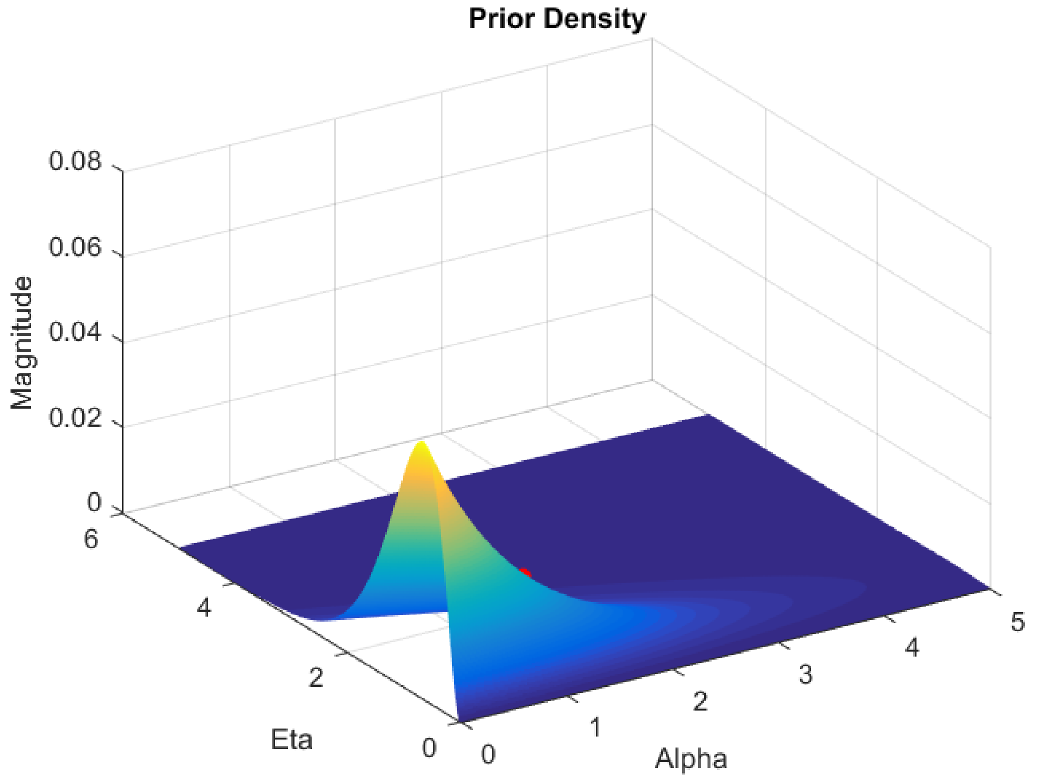


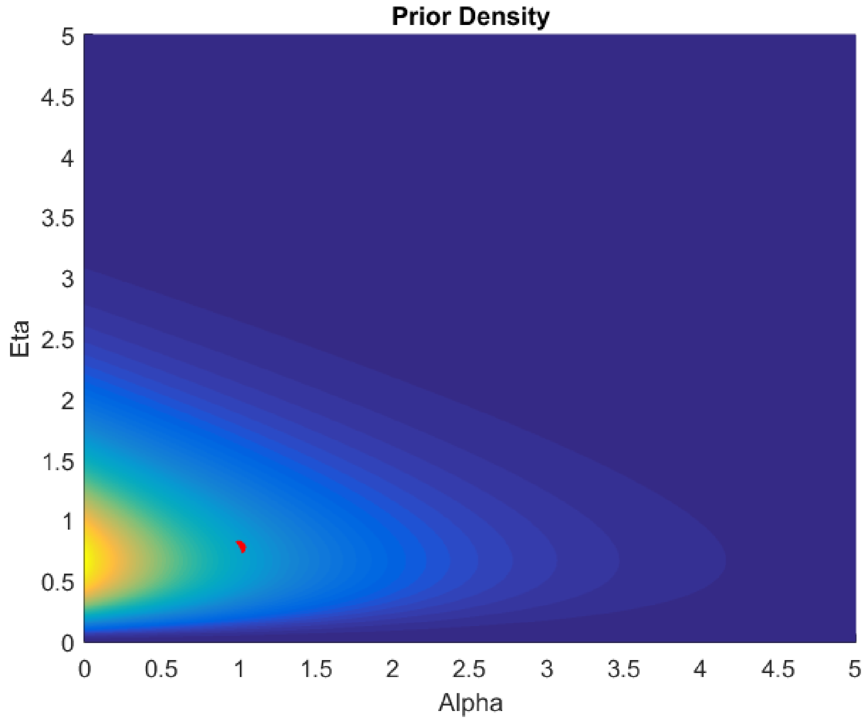
Likelihood function: $L(\alpha,\eta|x) = \prod_i^k \alpha\eta x_i^{\alpha-1}e^{-\eta x_i^{\alpha}}$

$\alpha = 0.6275, \eta = 1.2904$

$\alpha,\eta \sim [0,\infty)$

Conjugate Prior Distribution [8]

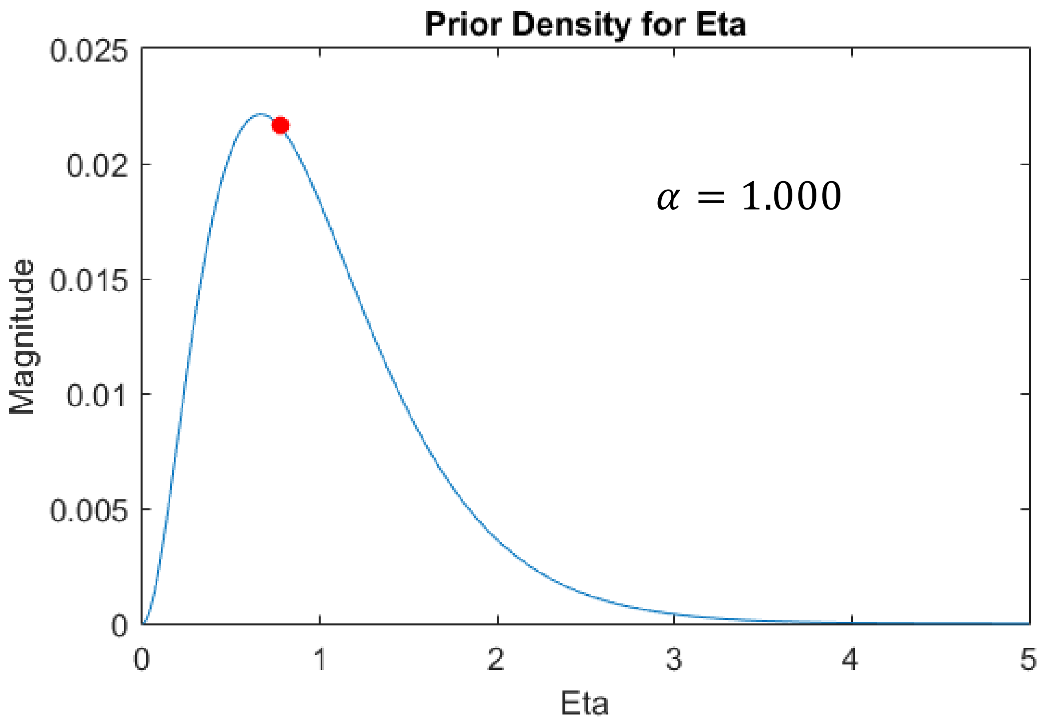


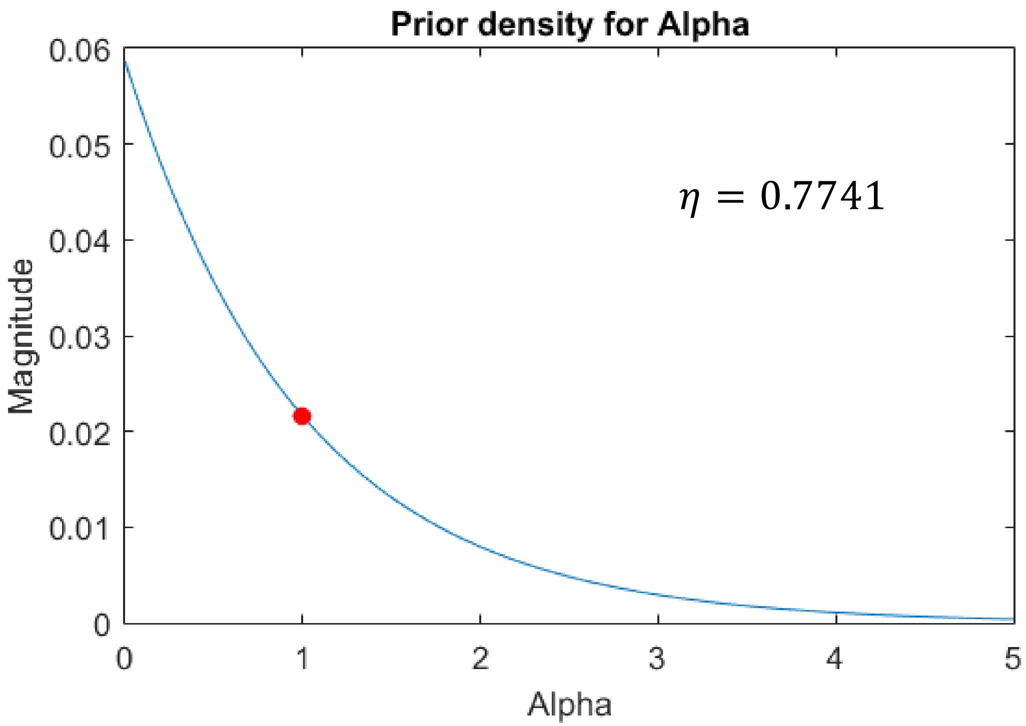


Joint prior distribution for α, η $\quad p(\alpha,\eta) \propto e^{-\alpha}\eta^{\beta-1}e^{-\eta\beta}$ $\quad \alpha = 1, \eta = 0.7741$ $\quad \alpha,\eta \sim [0,\infty)$

Hyperparameter $\beta = 3$

Hyperparameter choice





Hyperparameter $\beta = 3$

$\eta = 0.7741, \alpha = 1.000$

Posterior parameter samples [4,8]

- Posterior kernel: Contains all the information of α and η

$$p(\alpha,\eta|x) \propto \alpha^{k}\eta^{k+\beta-1}\left(\prod_{i}^{k} x_i\right)^{\alpha-1} e^{\left(-\eta\sum x_i^{\alpha}-\alpha-\eta\beta\right)}$$

- Proposal density

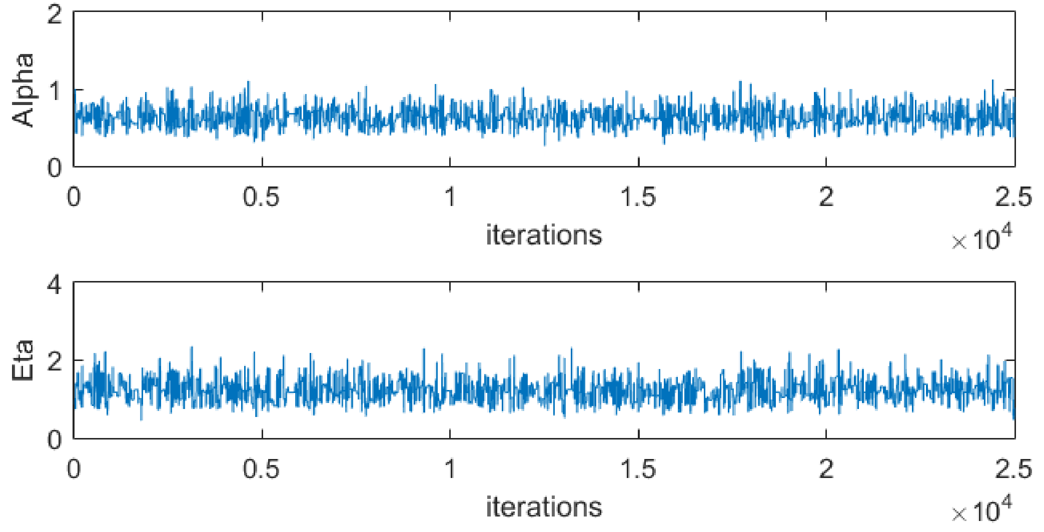
$$q(\alpha',\eta'|\alpha,\eta) = \frac{1}{\alpha\eta} e^{\left(\frac{\alpha'}{\alpha}-\frac{\eta'}{\eta}\right)}$$

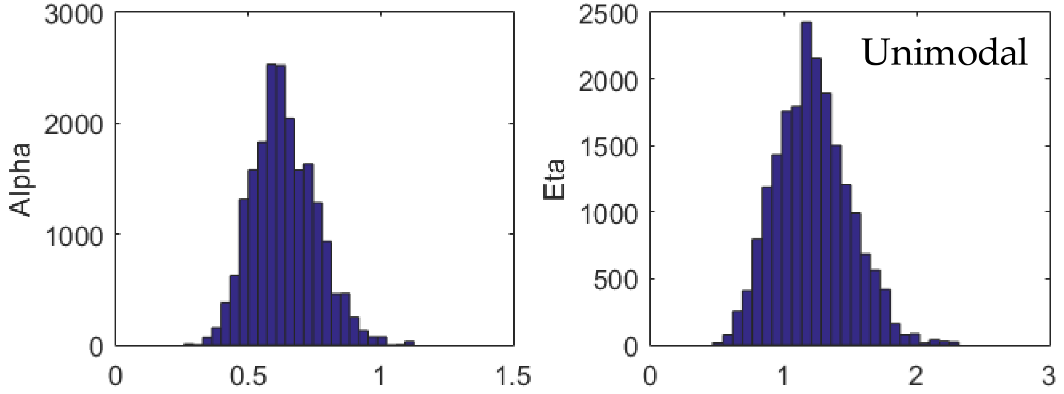
- Metropolis Hastings Algorithm

$$a\big((\alpha',\eta'),(\alpha,\eta)\big) = min\left\{1, \frac{p(\alpha',\eta')/q(\alpha',\eta'|\alpha,\eta)}{p(\alpha,\eta)/q(\alpha,\eta|\alpha',\eta')}\right\}$$

- Posterior parameters
  - $\alpha = 0.6327$
  - $\eta = 1.2225$

Convergence

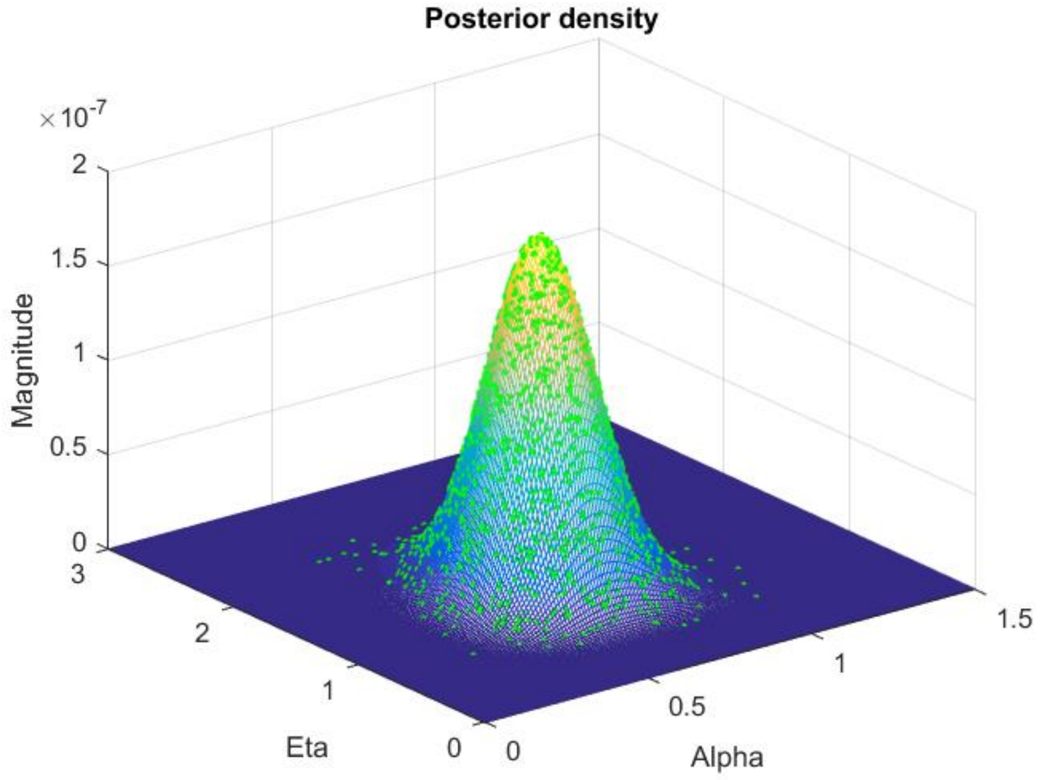


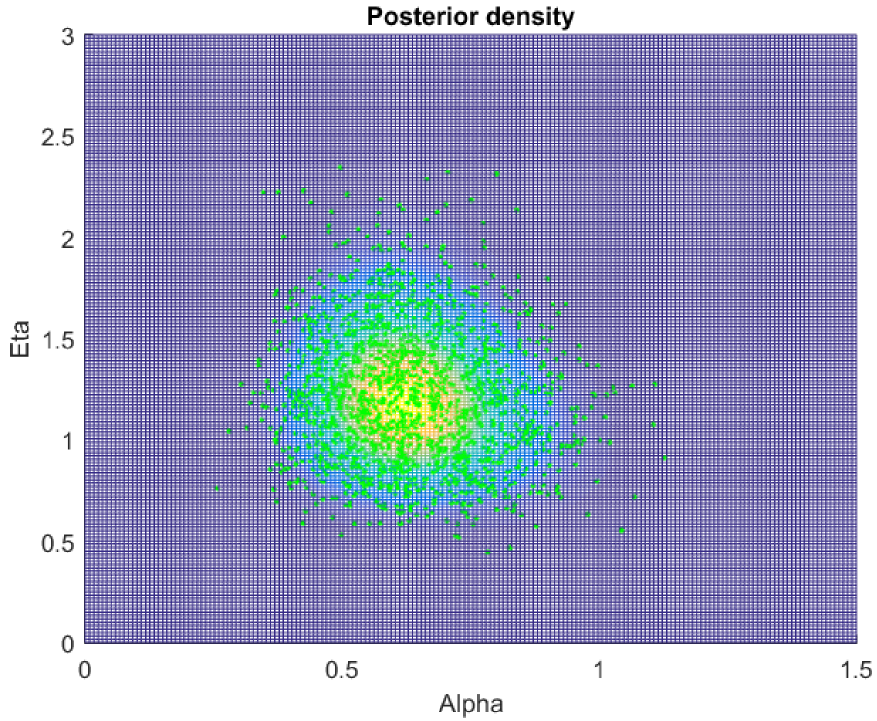


Posterior Distribution

Prior: $\alpha = 1.0000, \eta = 0.7741$

Data: $\alpha = 0.6275, \eta = 1.2904$



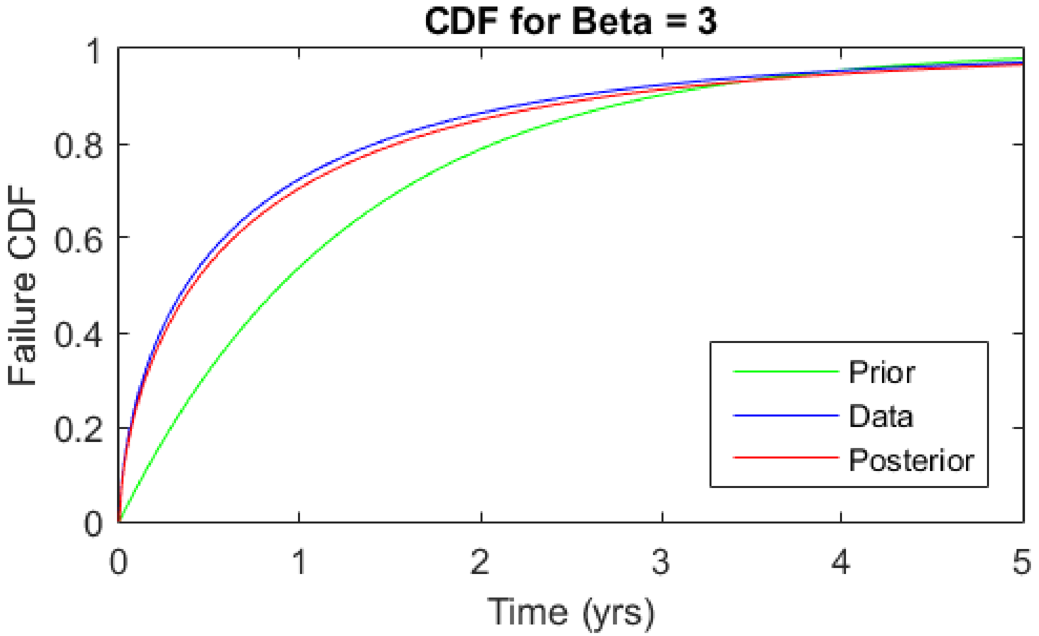


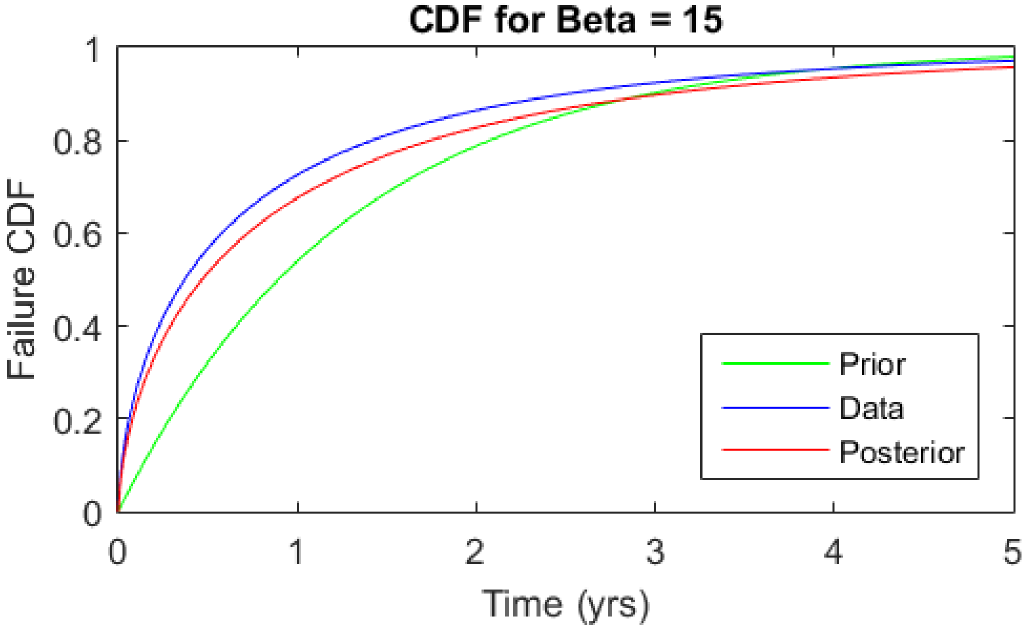
Posterior: $\alpha = 0.6327, \eta = 1.2225$

Discussion

Prior: $\alpha = 1.0000, \eta = 0.7741$

Data: $\alpha = 0.6275, \eta = 1.2904$





Hyperparameter β=3

$\alpha = 0.6327$
$\eta = 1.2225$

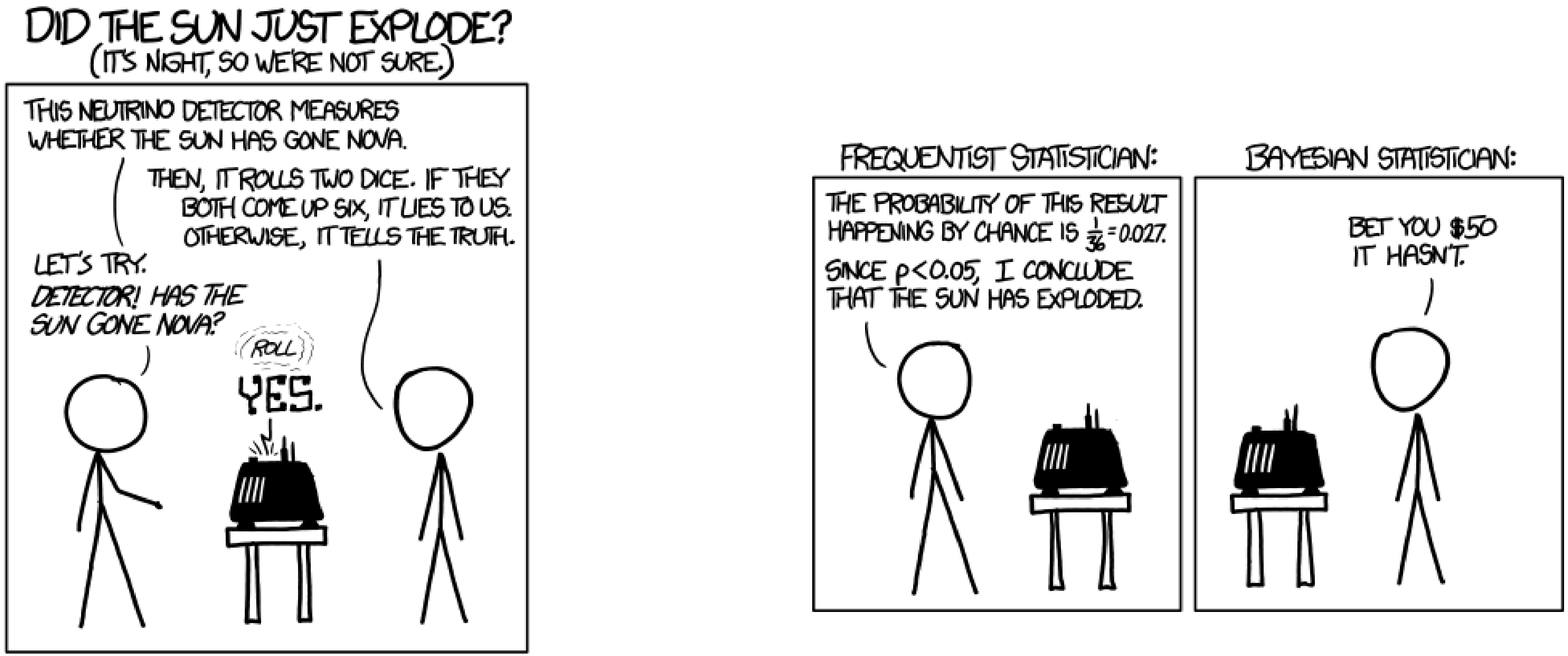
Hyperparameter β=15

$\alpha = 0.6404$
$\eta = 1.1249$

Conclusion

- Bayesian analysis furnishes the most informed inference for BPS
- Emphasize the importance of both the information sources
  - MC Model - Fine failure characteristics
  - Historical data - Real survival behavior
- Advocate the value of $\beta = 3$
  - High confidence in the actual machine failure data
  - Mild influence of the MC model results
- Ability to regulate the influence of either information sources

Why?



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References

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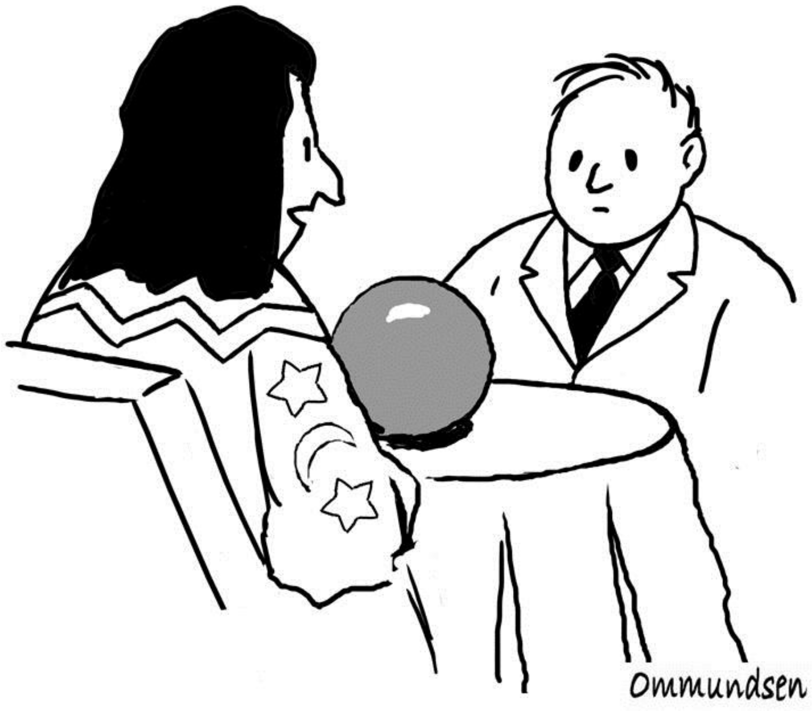

"Is this needed for a Bayesian analysis?"

Image courtesy: http://capewest.ca/cartoons.html

Thank You!

Questions?