A CONTROL STRATEGY FOR HIGHLY REGULATED MAGNET POWER SUPPLIES USING A LQR APPROACH

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Abstract

A linear quadratic regulator (LQR) based proportional-Integrator-derivative (PID) controller is proposed for the SMPS based magnet power supply of the high current proton injector operational at VECC. The state weighting matrix **Q** of the LQR based controller, for a fixed control weighting matrix **R,** is derived analytically using guaranteed dominant pole placement approach for desired maximum overshoot and rise time. The uniqueness of this scheme is that the controller gives the desired closed loop response with minimum control effort, hence avoiding the actuator saturation by utilizing both optimum approach of LQR technique and simplicity of the conventional PID controller. The perturbation of controller and power supply parameters is studied along with the load disturbance to verify the robustness of proposed control mechanism.

INTRODUCTION

A PID controller is one of the favorite controllers in the industry. The three controlling parameters of PID controller are proportional (K_p), integral (K_i) and derivative (K_d) [1]. The magnet power supply having rating 10V/200A feeds the solenoid coil, which produces desired magnetic field in the plasma chamber. For efficient performance of the ion source we need a highly stable and reliable power supply. In this paper a LQR based PID controller is proposed for the design of the DC-DC converter pulse width modulator (PWM) controller. The PWM controller will be finally used to regulate the magnet power supply. The simulation is performed in MATLAB software. The design methods of PWM controller have been discussed extensively in the literature [2, 3]. We have used LQR approach for obtaining an optimal control which is finally re-arranged to give the parameters of a PID controller [4]. Buck converter is a DC-DC converter that is widely used in the industry today, due to its high efficiency, low cost and more important small size. This small size results due to the use of high frequency switching, which is accomplished by using high frequency IGBT [2]. Figure 1 shows the closed loop optimal controller where **X**(t) is the state variable matrix, **u**(t) is the control vector, **A** is the state transitions matrix, **B** is the control matrix, **C** is the output matrix, **R** is the control weighting matrix and **P** is the Riccati coefficient matrix.

LQR gives the value of control vector **u**(t) in an optimum way using Calculus of Variation approach [5] which minimizes the quadratic performance cost function as given by

$$J(u(t)) = \int_0^\infty \left(\mathbf{X}(t)^T \mathbf{Q} \mathbf{X}(t) + \mathbf{u}(t)^T \mathbf{R} \mathbf{u}(t) \right) dt \quad (1)$$

with the plant dynamics given in the state space form as

$$\dot{\mathbf{X}}(t) = \mathbf{A}(t)\mathbf{X}(t) + \mathbf{B}(t)\mathbf{u}(t) \quad ; \mathbf{X}(t_0) = \mathbf{X}_0 \quad (2)$$

$$\mathbf{Y}(t) = \mathbf{C}(t)\mathbf{X}(t) \quad (3)$$

where $\mathbf{X}_0$ is the initial values of the state variables. **Q** is the state variable weighting matrix. The optimal control **u**(t) is given by [5]

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P}\mathbf{X}(t) = -\mathbf{K}\mathbf{X}(t) \quad (4)$$

The value of matrix **P** can be evaluated by solving the continuous algebraic Riccati equation,

$$\mathbf{A}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P} = \mathbf{0}\,. \quad (5)$$

After getting the optimal gain vector **u**(t), the PID parameters can be easily calculated using the procedure discussed in reference [4]. Since in the present work we have studied only linear time invariant system, the matrices **A**(t), **B**(t) and **C**(t) are therefore, independent of time i.e. **A**(t)=**A**, **B**(t)=**B** and **C**(t)=**C**.

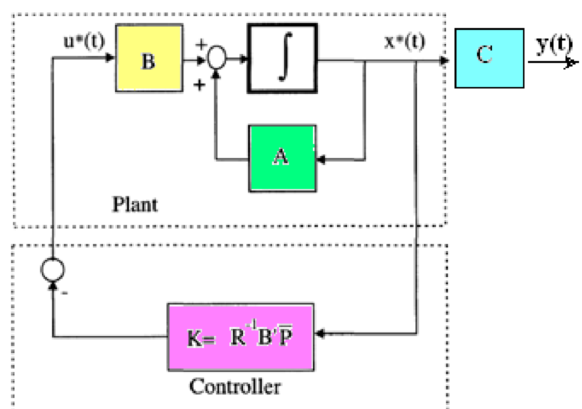

Figure 1: Schematic of a closed loop optimal controller.

MAGNET POWER SUPPLY DESIGN

The buck converter is a switch based DC-DC converter [2]. The output current is regulated based on the control of switch element which is generally IGBT. Figure 2 shows the closed loop block diagram of the buck converter based power supply. The PID controller is used to regulate the duty cycle of the PWM signal. *e(t)* is the error signal, *u(t)* is the control signal, *D*(*t*) is the duty cycle and *Y(t)* is the final output.

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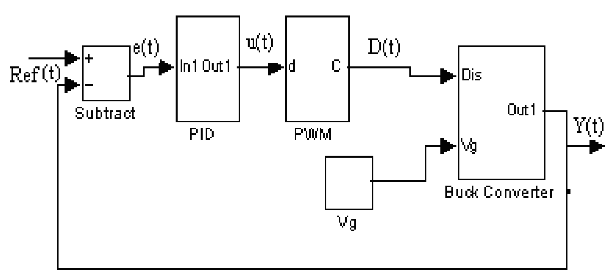

Figure 2: Closed loop block diagram of magnet power supply.

Figure 3 shows that the PWM output that is finally used for the control of power supply. It is clear from Fig. 3 that as the PID output increases, the widening of the PWM also increases. It should be noted that it is the controller which controls the PWM and hence controls the overall stability and robustness of the power supply.

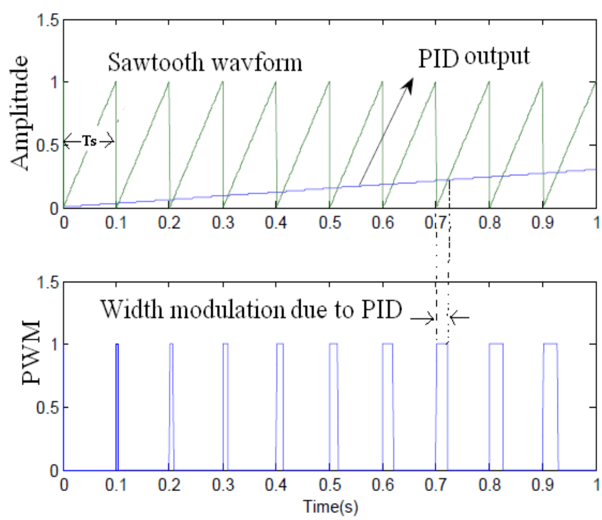

Figure 3: Regulated PWM due to optimal PID controller.

The schematic diagram of buck converter is given in Fig. 4. $V_g(t)$ is the source voltage, $i_L(t)$ is the inductor current that is utilized in magnet power supply. R_L and R_C are the series equivalent resistance of inductor and capacitor respectively and R_{load} is the load resistance. The values of L and C [2] can be obtained using,

$$L = \frac{R_{load}}{2F_s}(1-D) \tag{6}$$

$$C = \frac{V_o}{8\Delta V_0 L F_s^2}(1-D) \tag{7}$$

where D is the duty cycle ratio, F_s is the switching frequency and ΔV_o is the maximum ripple allowed in the output voltage. In the design, choice of L and C should be such that the resonant frequency $f_c = (2\pi\sqrt{LC})^{-1}$ must be sufficiently below the switching frequency F_s.

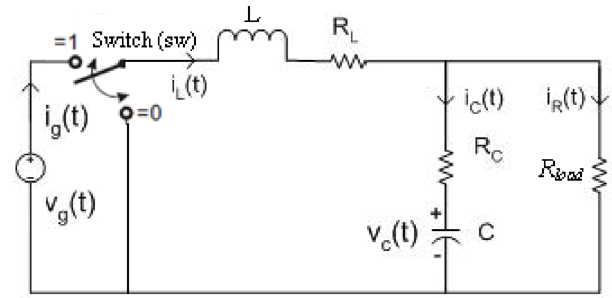

Figure 4: Magnet power supply based on buck converter.

The value of R_{load} for 10V/200A power supply rating will be 0.05Ω. With duty cycle of 0.75 and switching frequency of 20 kHz the values of L and C are 31.25µH and 3.3mF respectively. The corresponding resonant frequency f_c=5kHz is thus much less than the switching frequency of 20kHz. In the calculation we have assumed the series equivalent resistances $R_c = R_L$ = 10mΩ.

The transfer function of the buck converter calculated during on condition of switch (*sw*) can be written in standard form as

$$G(s) = \frac{K}{s^2 + as + b} \tag{8}$$

where $K = R_{load}/LC(R_{load} + Rc)$

$a = (L + R_L C R_{load} + R_L C R_c + R_{load} R_c C)/LC(R_{load} + Rc)$

and $b = (R_{load} + R_L)/LC(R_{load} + Rc)$.

OPTIMAL PID TUNING

PID tuning comprises the selection of best values of K_p, K_i and K_d of the PID controller so that the system performance can be improved. In this section, we describe an optimal PID tuning method using LQR. Let the state vector **X**(t) is given by

$$\mathbf{X}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T$$

$$\text{where } x_1 = \int e(t)dt \; ; x_2 = e(t) \;\&\; x_3 = \frac{de(t)}{dt} \tag{9}$$

Matrices **P** and **Q** are given as

$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{bmatrix} \tag{10}$$

From Ref. [4] utilizing the dominant pole placement approach, the values of matrix elements of **Q** and **P** can be obtain by equating the characteristic equation to the desired dominant poles as

$$\left| s\mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P}) \right|$$
$$= (s + m\zeta_{cl}\omega_{cl})(s^2 + 2s\zeta_{cl}\omega_{cl} + \omega_{cl}^2) \tag{11}$$

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where m is known as relative dominance. It affects the location of third pole on the left half of complex plane. ζ_{cl} and ω_{cl} are the desired closed loop damping ratio and natural frequency. Equating the coefficients of the powers of s in Eq. (11) after some algebraic manipulations, gives three elements p_{11}, p_{12}, p_{13} of matrix **P**. Using these three values and utilizing Eq. (5) the other elements of matrices **P** and **Q** can be obtain as

$$p_{11} = \frac{m\zeta_{cl}\omega_{cl}^5(1+2m\zeta_{cl}^2)}{r^{-1}K^2}, \quad p_{12} = \frac{(2+m)m\zeta_{cl}^2\omega_{cl}^4}{r^{-1}K^2}$$

$$p_{13} = \frac{m\zeta_{cl}\omega_{cl}^3}{R^{-1}K^2}, \quad p_{23} = \frac{\omega_{cl}^2 + 2m\zeta_{cl}^2\omega_{cl}^2 - b}{R^{-1}K^2}$$

$$p_{33} = \frac{(2+m)\zeta_{cl}\omega_{cl} - a}{R^{-1}K^2},$$

$$p_{22} = \frac{(2\omega_{cl}^3\zeta_{cl} + 4m\zeta_{cl}^3\omega_{cl}^3 + 2m^2\zeta_{cl}^3\omega_{cl}^3 - ab)}{r^{-1}K^2}$$

$$q_1 = \frac{m^2\zeta_{cl}^2\omega_{cl}^6}{r^{-1}K^2},$$

$$q_2 = \frac{\omega_{cl}^4 + 4m^2\zeta_{cl}^4\omega_{cl}^4 - b^2 - 2m^2\zeta_{cl}^2\omega_{cl}^4}{r^{-1}K^2}$$

$$q_3 = \frac{4\zeta_{cl}^2\omega_{cl}^2 + m^2\zeta_{cl}^2\omega_{cl}^2 + 2b - a^2 - 2\omega_{cl}^2}{r^{-1}K^2} \qquad (12)$$

The value of $a = 2\zeta_{ol}\,\omega_{ol}$ and $b = \omega_{ol}^2$, where ζ_{ol} and ω_{ol} are the damping ratio and natural frequency of the open loop system given in Eq. (8) respectively. The control signal due to PID parameters are given as

$$u(t) = K_p x_2(t) + K_i x_1(t) + K_d x_3(t) \qquad (13)$$

Finally, the PID parameters can be obtained by equating the coefficient of state variables in Eqs. (4) and (13) as

$$\begin{bmatrix} K_p & K_i & K_d \end{bmatrix} = \begin{bmatrix} r^{-1}Kp_{23} & r^{-1}Kp_{13} & r^{-1}Kp_{33} \end{bmatrix} \qquad (14)$$

SIMULATION RESULTS

Using the design values for 10V/200A as discussed in magnet power supply design section, the values of PID parameters obtain utilizing Eqs. (14) and (12) by considering $\zeta_{cl} = 0.98$, $\omega_{cl} = 10\times10^3$ ($\geq \sqrt{b}$ [6]), with $m = 3$ is

$$\begin{bmatrix} K_p & K_i & K_d \end{bmatrix} = \begin{bmatrix} 5.7624 & 2.8\times10^4 & 5\times10^{-4} \end{bmatrix} \qquad (15)$$

Figure 5 shows that when load disturbance of 20% are applied at $t = 1.01$s then the inductor current regulate it to the desired 100A (operating current range). The widening of PWM pulse confirms the load disturbance at $t = 1.01$s and regulation of PWM controller.

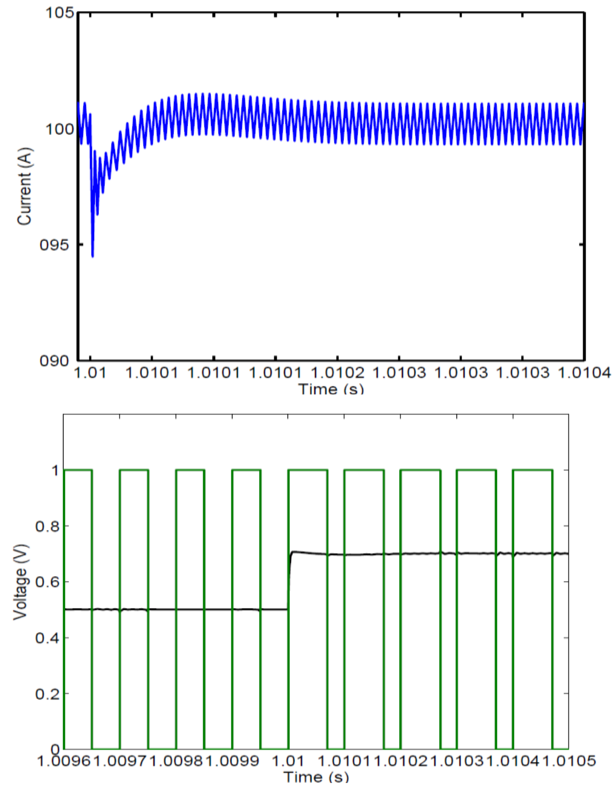

Figure 5: Load disturbance introduce at $t = 1.01$s and its corresponding PWM output (widening of PWM is observed).

CONCLUSION

A LQR based PID controller is proposed for the SMPS based magnet power supply of the high current proton injector operational at VECC. The state weighting matrix **Q** of the LQR and PID parameters is obtain analytically using guaranteed dominant pole placement approach with desired closed loop damping ratio and natural frequency. Simulation results confirm the utility of the proposed scheme.

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