

# A NOVEL ANALYSIS OF TIME EVOLVING BETATRON TUNE

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## Abstract

J-PARC Main Ring (MR) is a high-intensity proton synchrotron and since 2009 delivering beam to the T2K neutrino experiment and hadron experiments. It is essential to measure time variation of betatron tune accurately throughout from beam injection at 3 GeV to extraction at 30 GeV. The tune measurement system of J-PARC MR consist of a stripline-kicker, beam position monitors, and a waveform digitizer. Betatron tune appears as sidebands of harmonics of revolution frequency in the turn-by-turn beam position spectrum. Excellent accuracy of measurement and high immunity against noise were achieved by exploiting a wide-band spectrum covering multiple harmonics.

## INTRODUCTION

J-PARC MR is a high-intensity slow-cycle proton synchrotron started user beam operation in 2009 [1]. Main parameters of the MR are summarized in Table 1. The machine cycle is 2.48 seconds for the T2K neutrino experiment with fast extraction (FX) and 6.0 seconds for hadron experiments with slow extraction (SX). Its beam power is 247 kW for FX mode and 24 kW for SX mode. Typical beam intensity in the SX mode is shown in Fig. 1, overlaid with time variation of kinetic energy of the beam.

Cycle-by-cycle monitoring of various beam properties, such as betatron tune, over entire cycle is indispensable for operation of a high power proton accelerator. Acquiring, analyzing, and visualizing long waveform datum are matters of concern in MR control.

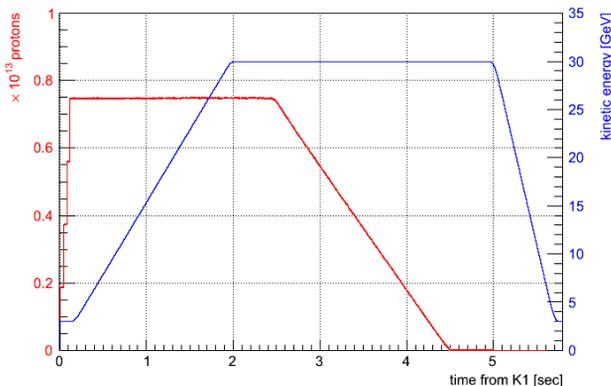


Figure 1: Measured circulating beam intensity in SX mode (red line) overlaid with time variation of kinetic energy of the beam (blue line). Beam is injected 4 times every 40 ms at 3 GeV, accelerated to 30 GeV in 1.9 seconds, and slowly extracted to hadron experiments.

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Table 1: Main Parameters of J-PARC MR

Circumference [m]	1567.5
Typical cycle time for FX [s]	2.48
Typical cycle time for SX [s]	6.0
Typical horizontal betatron tune for FX	22.383
Typical vertical betatron tune for FX	20.755
Typical horizontal betatron tune for SX	22.275
Typical vertical betatron tune for SX	20.770
Injection energy [GeV]	3
Extraction energy [GeV]	30
Harmonic number	9
Number of bunches	8
Number of bending magnets	96
Number of quadrupole magnets	216
Number of sextupole magnets	72
Revolution frequency [kHz]	185.7 – 191.2

## BETATRON TUNE MEASUREMENT

Betatron tune is wavenumber of transverse oscillation per beam revolution in a circular accelerator. It is observed as upper and lower sidebands surrounding a harmonic of revolution frequency, as described in Eq. 1:

$$f = (N \pm \nu) f_{rev}, \quad (1)$$

where  $f$  is sideband frequency,  $N$  harmonic number,  $\nu$  fractional part of betatron tune, and  $f_{rev}$  revolution frequency.

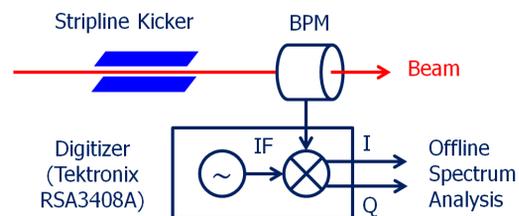


Figure 2: Overview of betatron tune measurement in J-PARC MR.

Hardware configuration for betatron tune measurement is shown in Fig. 2. Two devices of the same configuration are installed in the MR, one for horizontal betatron tune measurement the other for vertical. A stripline kicker is used during the measurement to shake the beam horizontally (or vertically) with respect to the beam direction. The horizontal beam oscillation is then monitored by a beam

position monitor (BPM). Signal from the BPM is down converted, separated into in-phase and quadrature phase (I/Q) signals, and digitized by a commercial real-time spectrum analyzer (RSA), Tektronix RSA3408A. I/Q signal is sampled at 1.28 MHz and recorded over 1.2 seconds for off-line spectrum analysis. Center frequency at 5.0 MHz is chosen to avoid noise and bandwidth of 1.28 MHz to cover at least as five times wide as revolution frequency.

## ANALYSIS

### Decomposition to Frames

Analysis of time variation of betatron tune is based on short-time complex FFT. Recorded I/Q data is broken up to frames, which consist of 1024 samples overlapping each other by 512 samples. I/Q pairs in a frame are analyzed by complex FFT with Blackman window. Additional 24576 zero-valued samples were added to each frame to form 25600 samples of data so that frequency resolution is improved.

Measured power spectrum of a frame right after beam injection in SX mode is shown in Fig. 3. Figure 4 shows spectrogram from the beam injection. Harmonics of  $f_{\text{rev}}$  and their sidebands sweeps along time in accordance with beam acceleration.

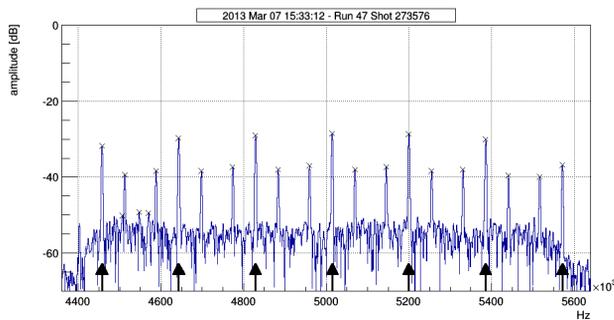


Figure 3: Power spectrum of horizontal oscillation in a frame. Frequencies from 24th to 30th harmonic of  $f_{\text{rev}}$  are shown in arrows. Their sidebands correspond to fractional part of betatron tune.

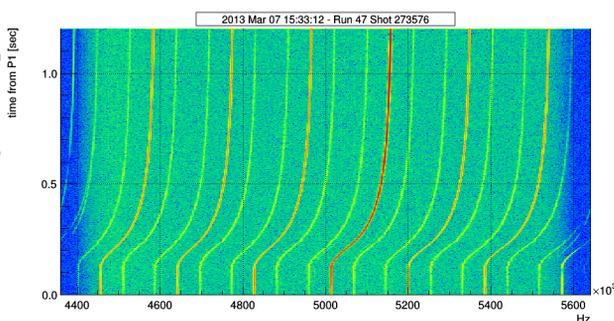


Figure 4: Spectrogram of horizontal oscillation of beam. Horizontal axis is frequency and vertical is time from beam injection.

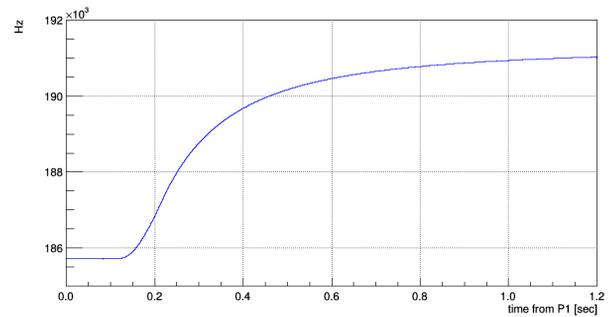


Figure 5: Measured time variation of revolution frequency, which sweeps in accordance with beam acceleration. Horizontal axis is frequency and vertical is time from beam injection.

### Estimation of Revolution Frequency

Estimation of  $f_{\text{rev}}$  became possible thanks to the wide bandwidth. In each frame a combination is sought such that line spectra are simple integer ratio. The common divisor is found to be  $f_{\text{rev}}$  within a constraint of

$$185.7 \text{ kHz} \leq f_{\text{rev}} \leq 191.2 \text{ kHz},$$

which correspond to beam energy between 3 GeV and 30 GeV.

Figure 5 shows reconstructed  $f_{\text{rev}}$  in each frame laid along time. Its time variation is consistent with kinetic energy shown in Fig. 1, apart from difference in their time range.

### Determination of Tune

Betatron tune appears as sidebands surrounding a harmonic of revolution frequency as mentioned above. Fractional part of betatron tune  $\nu_i$  is obtained as following in each frame:

$$\nu_i = \left| \frac{f_i}{f_{\text{rev}}} - N \right|, \quad (2)$$

where  $f_i$  is frequency of  $i$ -th line spectrum observed in the frame, and  $N$  is the harmonic number nearest to  $f_i/f_{\text{rev}}$ . Distribution of  $\nu_i$  is shown in Fig. 6. Its mean value is evaluated as betatron tune and thus frequency separation is improved from FFT. Noises and outliers were rejected beforehand for further improvement by applying Bayesian Information Criterion [2].

Figure 7 shows reconstructed betatron tune in each frame laid along time. Tune fluctuation observed around 0.15 – 0.25 seconds is caused by tracking error of main magnet power supplies. Ripple observed over the entire time range is caused by current ripple of the power supplies [3, 4]. An FFT power spectrum of time variation of the tune is shown in Fig. 8. Ripples of 100 – 150 Hz and 600 Hz caused by the power supplies are clearly seen.

## SUMMARY

Betatron tune in J-PARC MR is analyzed by short-time FFT of BPM data. Revolution frequency is extracted from

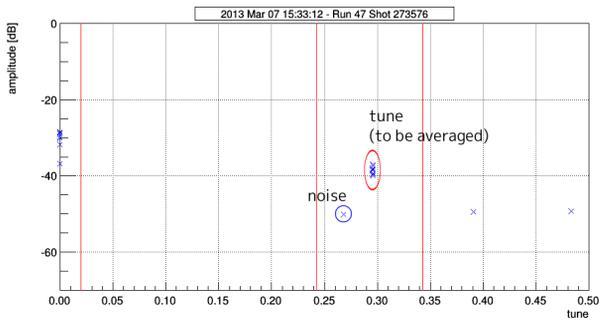


Figure 6: Distribution of fractional part of betatron tune in a frame.

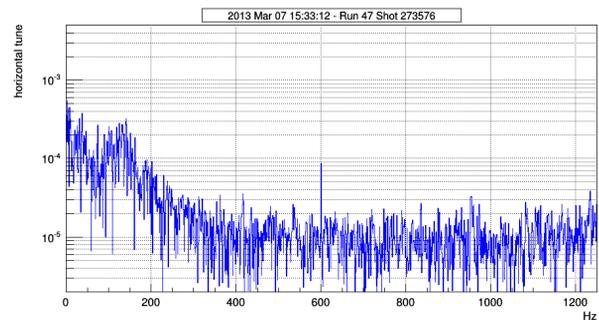


Figure 8: FFT power spectrum of time variation of horizontal betatron tune.

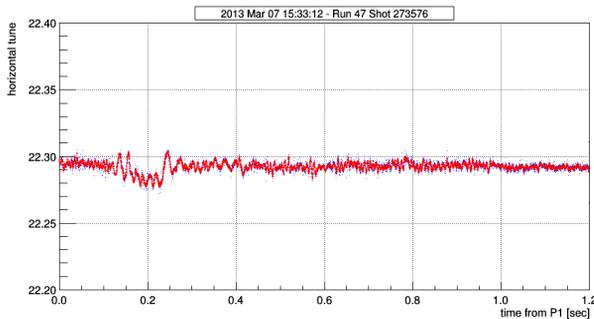


Figure 7: Measured time variation of horizontal betatron tune in J-PARC MR. Horizontal axis is tune and vertical is time from beam injection.

FFT power spectrum in each frame, thanks to the wide bandwidth covering multiple harmonics. Betatron tune is evaluated as mean value of the distribution of sidebands in the frame. Frequency separation is improved from FFT by statistical method as well as by applying Bayesian Information Criterion to noise and outlier rejection. Effect of tracking error and ripples of main magnet power supplies are clearly observed in the time variation of betatron tune.

## REFERENCES

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