Radiation Sources and their Application for Beam Profile Diagnostics

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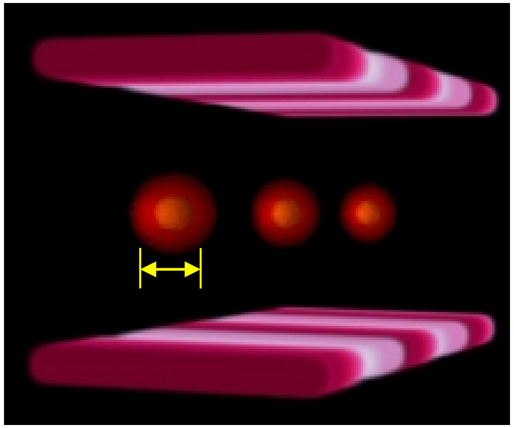
- Introduction to Imaging
- Transverse Profile Measurements based on OTR and ODR
- Parametric X-Ray Radiation
- Coherent Radiation Diagnostics and Smith-Purcell Radiation

Size Measurements

- task
  - determination of beam profile
    → measurement of characteristical size (rms, …)



courtesy:
J. Amundson (FNAL)

- conventional size measurement
  - take object and measure

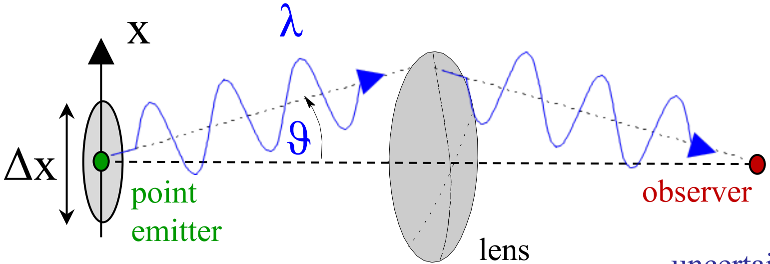
- difficulties
  - object extremely small
  - object not directly accessible
    → inside vacuum beam pipe, accelerator environment, …

- optical imaging
  - generate replica in comfortable environment
  - adjust replica size (image) to size of measuring device (CCD)

Imaging and Resolution

- neglect lens imperfections (aberrations)
  - **diffraction limited** systems → high quality, aberration-free systems

- fundamental resolution limit
  - point observer detecting photons from point emitter → location of emission point ?



Δx

$$\Delta p_x = 2\hbar k \cdot \sin\vartheta \approx 2 \cdot \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} \cdot \sin\vartheta$$

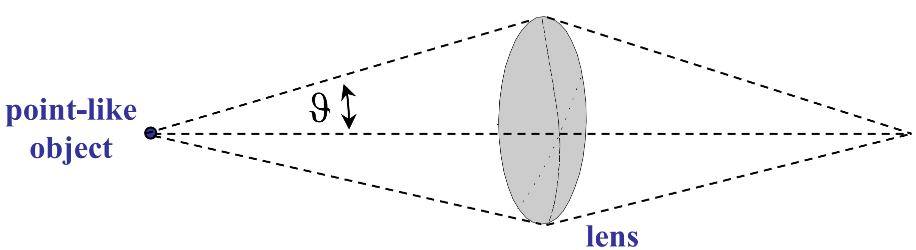
NA = sinϑ: numerical aperture

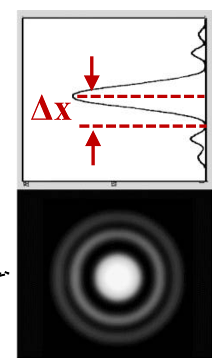
uncertainty principle: $\Delta x \cdot \Delta p_x \approx h \quad \Rightarrow \quad \boxed{\Delta x \approx \frac{\lambda}{2\sin\vartheta}}$

high resolution: (i) **small λ** (ii) **high NA**

- image of point source

http://www.astro.ljmu.ac.uk





image

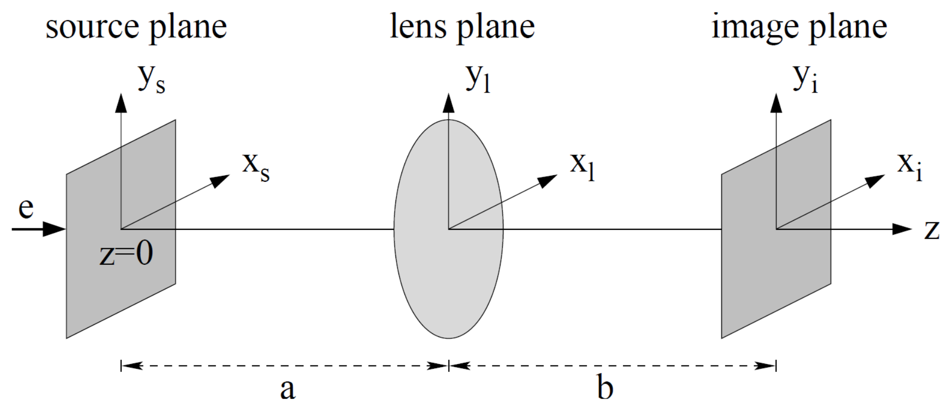
Airy pattern

$$\boxed{\Delta x = 0.61 \frac{M\lambda}{\sin\vartheta}}$$

magnification M

Fundamentals of Image Formation

- detailed resolution information
  - requires basic knowledge of image formation
- simple imaging setup

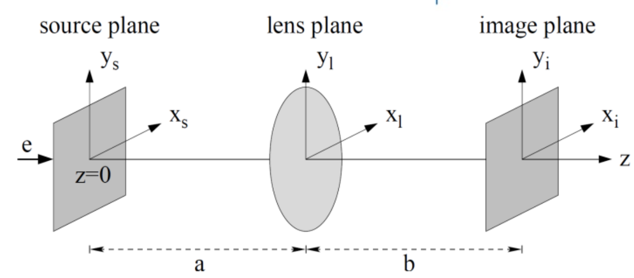


- procedure
  - calculate image of point source (single particle radiation) → **Point Spread Function (PSF)**
  - image of extended object → 2-dim. convolution of source distribution and PSF
  - resolution → difference beween source distribution and image (resp. PSF)
- PSF calculation
  - el. field in source plane
  - field propagation from element to element → in frame of scalar diffraction theory

    (i) source plane – lens input (ii) lens input – lens output (iii) lens output – image plane
  - intensity distribution in the image plane

Fundamentals of Image Formation

- **source field**
  - radiation field → depends on mechanism of radiation generation



- **propagation**
  - scalar diffraction theory

(here: from source to lense plane)

$$E^l_{x_l,y_l}(\vec{r}_l,\omega) = -i\frac{e^{ika}}{\lambda a}\cdot e^{i\frac{k}{2a}\left(x_l^2+y_l^2\right)}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dx_s dy_s\, E^s_{x_s,y_s}(\vec{r}_s,\omega)\cdot e^{i\frac{k}{2a}\left(x_s^2+y_s^2\right)}\cdot e^{-ik\frac{x_s x_l+y_s y_l}{a}}$$

aperture boundaries

  - far field (Fraunhofer) approximation: $\frac{k}{2}\left(x_s^2+y_s^2\right)_{\max} << a$

$$\Rightarrow\quad E^m_{x_l,y_l}(\vec{r}_l,\omega) = -i\frac{e^{ika}}{\lambda a}\cdot e^{i\frac{k}{2a}\left(x_l^2+y_l^2\right)}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dx_s dy_s\, E^s_{x_s,y_s}(\vec{r}_s,\omega)\cdot e^{-i\left(k_x x_s+k_y y_s\right)} \propto \mathcal{F}\left(E^s_{x_s,y_s}\right) \qquad \left(k_{x,y} = k\frac{x_l,y_l}{a}\right)$$

→ basis of **Fourier Optics**

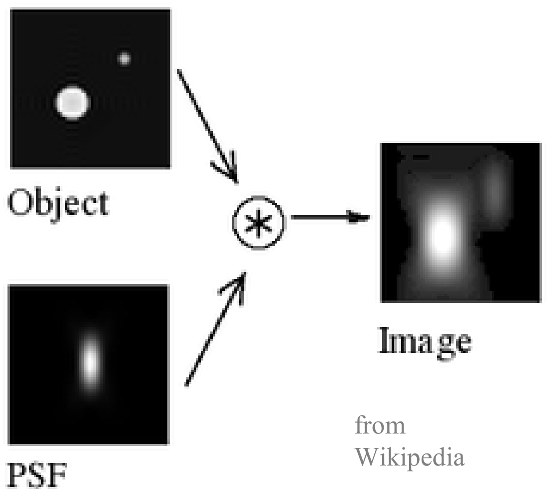
- **thin lens approximation**
  - quadratic phase shift: $E^{l_{out}}_{x_l,y_l}(\vec{r}_l,\omega) = E^{l_{in}}_{x_l,y_l}(\vec{r}_l,\omega)\cdot e^{-i\frac{k}{2f}\left(x_l^2+y_l^2\right)}$ with $\frac{1}{f}=\frac{1}{a}+\frac{1}{b}$

- **intensity** $\frac{\mathrm{d}^2W}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{\mathrm{c}}{4\pi^2}\left(\left|\vec{E}^i_{x_i}(\vec{r}_i,\omega)\right|^2+\left|\vec{E}^i_{y_i}(\vec{r}_i,\omega)\right|^2\right)$

Image Formation: Systems Approach

- image formation

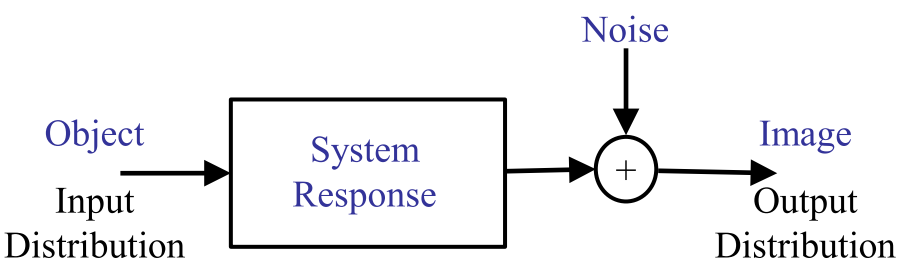
$$\boxed{\text{Image} = \text{PSF} \otimes \text{Object} + \text{Noise}}$$



from Wikipedia

- Point Spread Function (PSF)
  - → image of a point source (single particle)
  - → characteristic of the imaging instrument
  - → deterministic function
- noise
  - → nondeterministic function
  - → described in terms of statistical distributions

- systems approach to imaging (Fourier Optics)



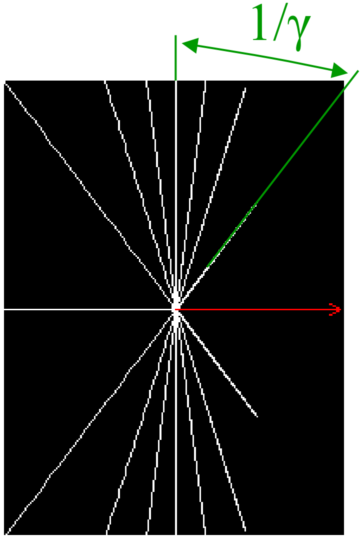
- „standard“ signal theory
  - → 1-dim. signals (in time domain)
  - → system analysis with delta pulse
- imaging
  - → 2-dim. signals (in spatial domain)
  - → system analysis with point source

    system response: PSF

Radiation Generation: Considerations

- ~~radiation generation via particle interaction with matter~~
  - ~~luminescent screen monitors~~

- radiation generation via particle electromagnetic field
  - particle electromagnetic field
  - relativistic contraction characterized by Lorentz factor



electric field lines
in LAB frame

$$\boxed{\gamma = E / m_0 c^2}$$

E : total energy

$m_0 c^2$: rest mass energy

proton: $m_p c^2 = 938.272\ \text{MeV}$

electron: $m_e c^2 = 0.511\ \text{MeV}$

$\gamma \rightarrow \infty$: plane wave

- $mc^2 = 0$ MeV : light $\rightarrow$ „real photon“
- ultra relativistic energies : idealization $\rightarrow$ „virtual photon“

Separation of Particle Field

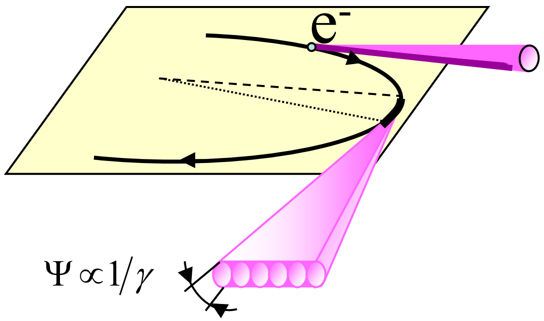
- electromagnetic field bound to particle
  observation in far field (large distances)

  } separate field from particle

- separation mechanisms
  - bending of particle via magnetic field

    synchrotron radiation

    ⟹ $\boxed{\text{circular accelerators}}$



linear accelerators: no particle bending !

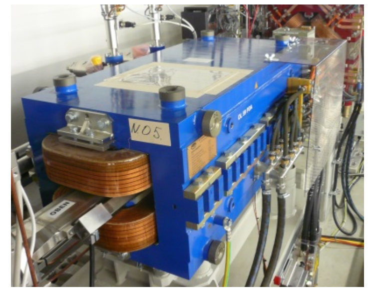
  - diffraction/reflection of particle electromagnetic field via material structures

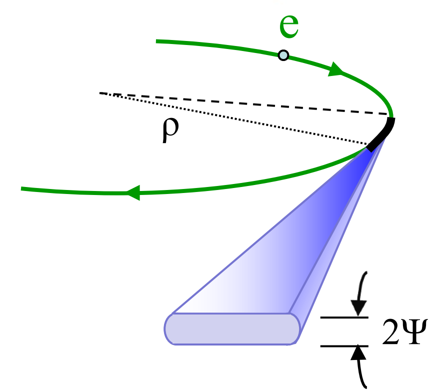
    exploit analogy between real/virtual photons:

    - light reflection/refraction at surface ↔ backward/forward transition radiation (TR)
    - light diffraction at edges ↔ diffraction radiation (DR)
    - light diffraction at grating ↔ Smith-Purcell radiation
    - light (X-ray) diffraction in crystal ↔ parametric X-ray radiation (PXR) …

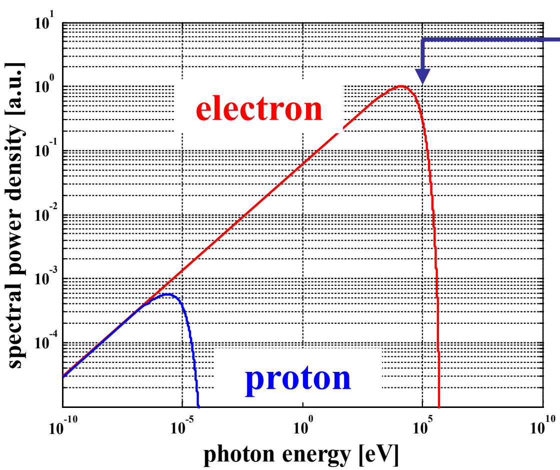
Synchrotron Radiation

- circular accelerator: radiation source available for free
  - bending magnet (wiggler, undulator)
- non-invasive
- strong collimation (vertical)
  - opening angle: $\boxed{\Psi \propto 1/\gamma}$





- emission over wide spectral range
  - choice of operational range



$$\boxed{\hbar\omega_c = \frac{3}{2}\hbar c\frac{\gamma^3}{\rho}}$$

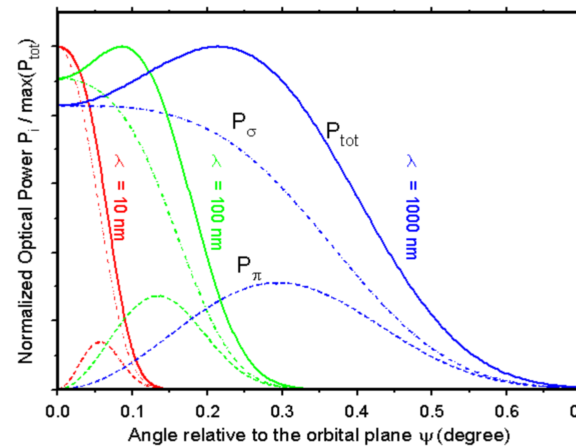
g: Lorentz factor

r : bending radius

$E_{kin} = 20$ GeV

$\rho = 370$ m

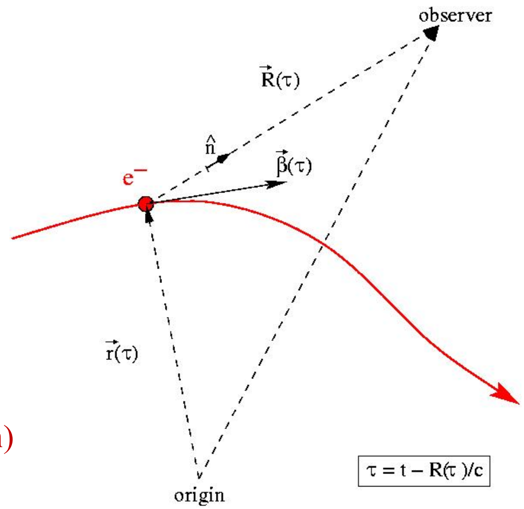
- polarized
  - define vertical angular divergence



- particle beam diagnostics: resolution
  - electric field propagation through optical elements → radiation field

SR Field: Standard Text Book

- **source field:** particle field described by **Liénard-Wiechert potentials:**



$$\varphi(t)=\left(\frac{-e}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau , \qquad \vec{A}(t)=\left(\frac{-e\,\vec{\beta}}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau$$

- field derivation: $E(t)=-\vec{\nabla}\varphi(t)-\frac{1}{c}\dot{\vec{A}}(t), \quad \vec{H}(t)=\vec{\nabla}\times\vec{A}(t)$

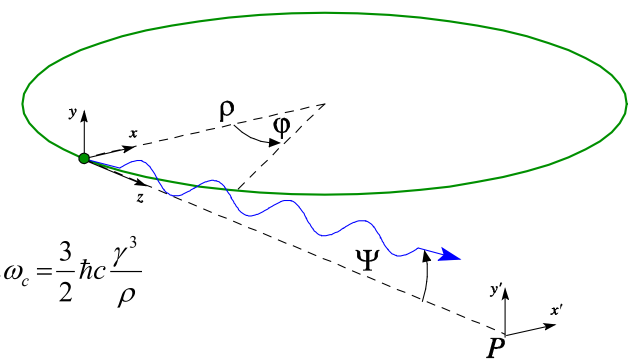
$$\Rightarrow \quad \vec{E}(t)=-e\left(\cancel{\frac{\left(1-\beta^2\right)\left(\hat{n}-\vec{\beta}\right)}{R^2\left(1-\hat{n}\cdot\vec{\beta}\right)^3}}+\frac{\hat{n}\times\left[\left(\hat{n}-\vec{\beta}\right)\times\dot{\vec{\beta}}\right]}{cR\left(1-\hat{n}\cdot\vec{\beta}\right)^3}\right)_\tau , \qquad \vec{H}(t)=\hat{n}\times\vec{E}(t)$$

neglect velocity term (far field **approximation**)

- Fourier transform:

$$\vec{E}(\omega)\approx-\frac{i\omega e}{cR}\int_{-\infty}^{+\infty}d\tau\left[\hat{n}\times\left[\hat{n}\times\vec{\beta}\right]\right]e^{i\omega(\tau+R(\tau)/c)}$$

- **special case:** charged particle moving on circular orbit



$$E_x(\omega)=E_\sigma=A_\sigma\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)\cdot K_{2/3}\left[\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)^{3/2}\right]$$

$$E_y(\omega)=E_\pi=A_\pi\frac{\hbar\omega}{2\hbar\omega_c}\gamma\Psi\sqrt{1+\gamma^2\Psi^2}\cdot K_{1/3}\left[\frac{\hbar\omega}{2\hbar\omega_c}\left(1+\gamma^2\Psi^2\right)^{3/2}\right]$$

with $\hbar\omega_c=\frac{3}{2}\hbar c\frac{\gamma^3}{\rho}$

$\Rightarrow$ analytical field description

- **comments:** (i) approximative field description $\rightarrow$ far field approximation
  (ii) emission from single point on orbit $\rightarrow$ additional contributions: depth of field, orbit curvature

Synchrotron Radiation Field

- **second representation:** starting point again **Liénard-Wiechert potentials**

O.Chubar and P.Elleaume, Proc. EPAC96, Stockholm (1996) 1177

$$\varphi(t)=\left(\frac{-e}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau , \quad \vec{A}(t)=\left(\frac{-e\,\vec{\beta}}{R\left(1-\hat{n}\cdot\vec{\beta}\right)}\right)_\tau$$

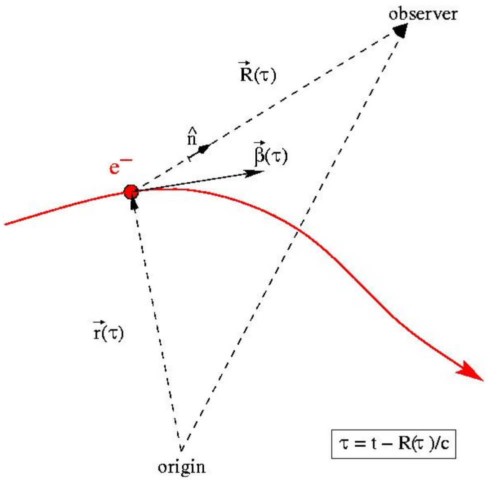
- Fourier transform of potentials:

$$\varphi(\omega)=-e\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\frac{1}{R(\tau)}e^{i\omega(\tau+R(\tau)/c)} , \quad \vec{A}(\omega)=-e\int_{-\infty}^{+\infty}\mathrm{d}\tau\,\frac{\vec{\beta}(\tau)}{R(\tau)}e^{i\omega(\tau+R(\tau)/c)}$$

- field derivation:

$$\boxed{\vec{E}(\omega)=-\frac{i\omega e}{c}\int_{-\infty}^{+\infty}\mathrm{d}\tau\left[\frac{\left(\vec{\beta}-\hat{n}\right)}{R(\tau)}-\frac{ic}{\omega}\frac{\hat{n}}{R^2(\tau)}\right]\mathrm{e}^{i\omega(\tau+R(\tau)/c)}}$$

with $\tau=\int_0^z\frac{\mathrm{d}z}{c\beta_z(z)}=\frac{1}{c}\int_0^z\mathrm{d}z\left[1+\frac{1+(\gamma\beta_x)^2+(\gamma\beta_y)^2}{2\gamma^2}\right]$



➡ knowledge of arbitrary particle orbit: $\vec{E}(\omega)$ determined

➡ arbitrary magnetic field configuration: determines orbit and $\vec{E}(\omega)$

- **comments:**
  (i) exact field description → numerical near field calculation
  (ii) includes depth of field & curvature → no additional contributions, only field propagation
  (iii) free codes available → easy field calculation, even field propagation!

SRW: http://www.esrf.eu/Accelerators/Groups/InsertionDevices/Software/SRW (Chubar & Elleaume, ESRF)

Spectra: http://radiant.harima.riken.go.jp//spectra/index.html (Tanaka & Kitamura, SPring8)

Light Sources: Emittance Diagnostics

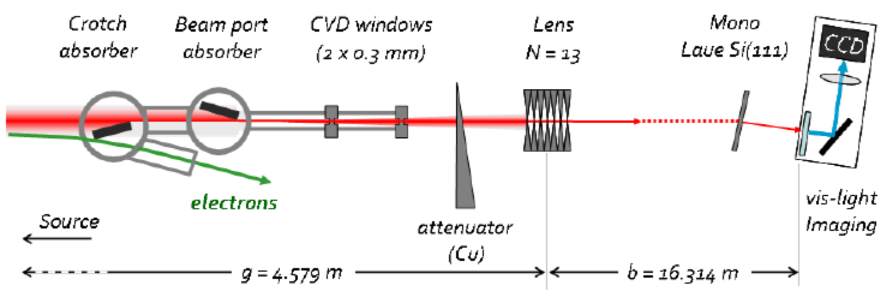
- emittance typical values ε_x = **1-5 nm.rad and 1% emittance coupling**
  - example: $\sigma_{hor} = 40\ \mu m$, $\sigma_{vert} = 20\ \mu m$ (PETRA III @ DESY)

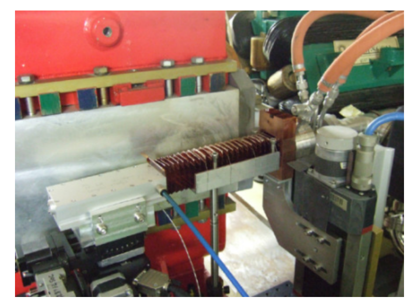
    fundamental resolution limit (uncertainty principle) $\boxed{\Delta\sigma \approx \dfrac{\lambda}{2\Delta\Psi}}$

  optical imaging: $\lambda = 500$ nm and $\Delta\Psi \approx 1.7$ mrad $\Rightarrow$ $\Delta\sigma_{vert} = 145\ \mu m$ diffraction limited

- X-ray imaging: focusing optics
  - reflective optics: Kirkpatrick-Baez mirrors,…
  - diffractive optics: Fresnel zone plates,…
  - refractive optics: Compound Refractive Lenses (CRL)

CRL monitor @ ESRF (32 keV)





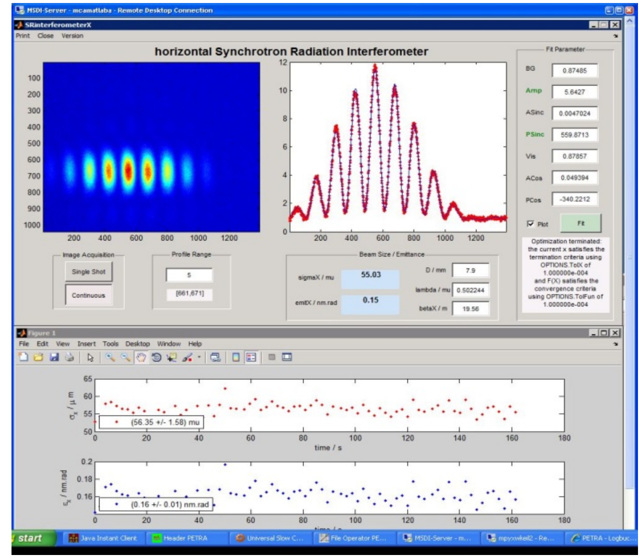
F. Ewald et al., Proc. IBIC 2013, Oxford, UK (2013) 833

- X-ray imaging: non-focusing optics
  - pinhole camera example: Diamond Light Source C. Thomas et al., Phys. Rev. ST Accel. Beams **13** (2010) 022805

Light Sources: Emittance Diagnostics

- SR interferometer

T. Mitsuhashi, Proc. of BIW 2004 Knoxville, Tennessee, p.3

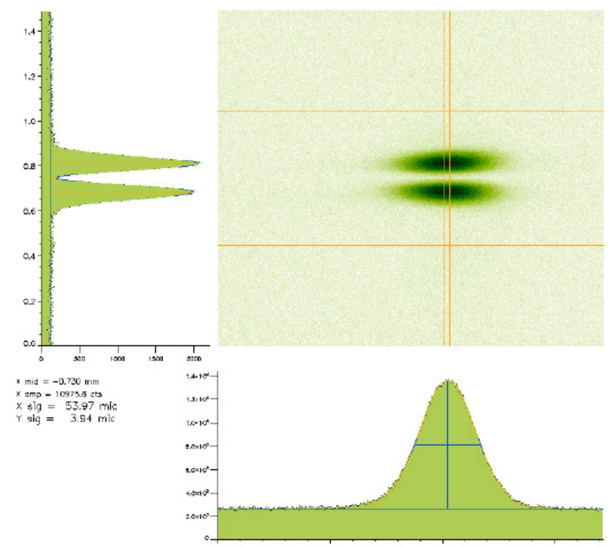


USR studies at PETRA III (DESY):

ε_x = 160 pm.rad @ 3 GeV

- π-polarisation imaging

V. Schlott et al., Proc. IBIC 2013, Oxford, UK (2013) 519



widely applied @ SLS

- coded aperture imaging

R.H. Dicke, Astrophys. Journal 153, L101, (1968)

J.W. Flanagan et al., Proc. DIPAC 2011, Hamburg, Germany (2011) 561

C. Bloomer, „Coded Aperture @ DLS", TUCZB2

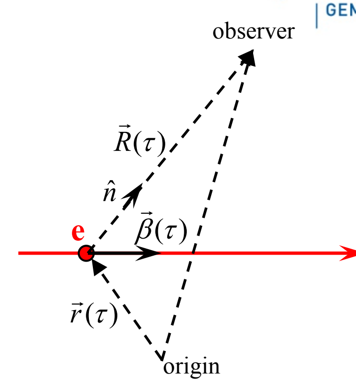
Constant Linear Motion

- source field

  - point charge with **constant** velocity v → Liénard-Wiechert fields

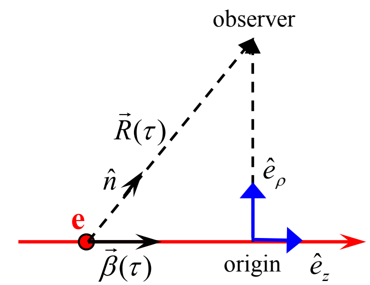
$$\Rightarrow \quad \vec{E}(t) = -e\left(\frac{(1-\beta^2)(\hat{n}-\vec{\beta})}{R^2(1-\hat{n}\cdot\vec{\beta})^3} + \cancel{\frac{\hat{n}\times\left[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{cR(1-\hat{n}\cdot\vec{\beta})^3}}\right)_\tau , \qquad \vec{H}(t) = \hat{n}\times\vec{E}(t)$$

no acceleration term



  - common representation → cylindrical coordinate system

$$\Rightarrow \quad \vec{E}(\rho,\varphi,z,\omega) = \frac{e\alpha}{\pi\,\mathrm{v}} e^{i\frac{\omega}{\mathrm{v}}z}\left(\mathrm{K}_1(\alpha\rho)\hat{e}_\rho - \frac{i}{\gamma}\mathrm{K}_0(\alpha\rho)\hat{e}_z\right) \qquad \text{with} \qquad \alpha = \frac{\omega}{\gamma\,\mathrm{v}} = \frac{2\pi}{\lambda\beta\gamma}$$

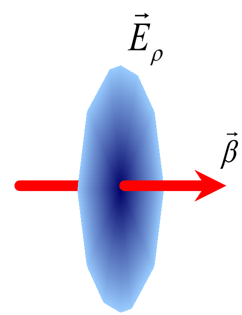


  - ultra-relativistic particles (γ >> 1)

    → neglect longitudinal field component

    → pure transverse „pancake“ structure

    → radial extension: $\alpha\rho \approx 1$



$$\boxed{\rho = \frac{\lambda\beta\gamma}{2\pi} \approx \gamma\lambda}$$

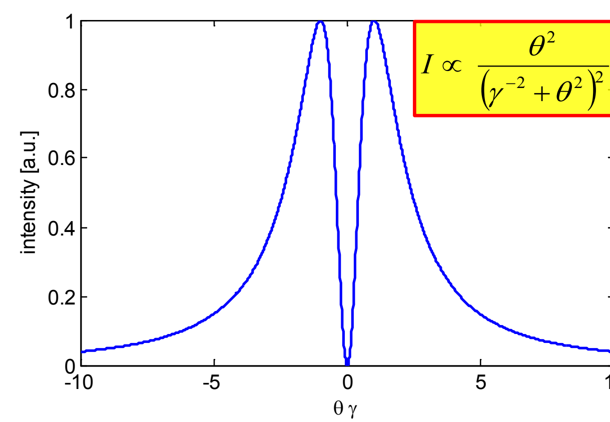
virtual photon range

3-dim. theories

A.G. Shkvarunets and R.B. Fiorito, Phys. Rev. ST Accel. Beams **11** (2008) 012801

D.V. Karlovets and A.P. Potylitsyn, Nucl. Instr. and Meth. **B266** (2008) 3738 ……..

  - angular distribution



  - separation of field → different radiation sources

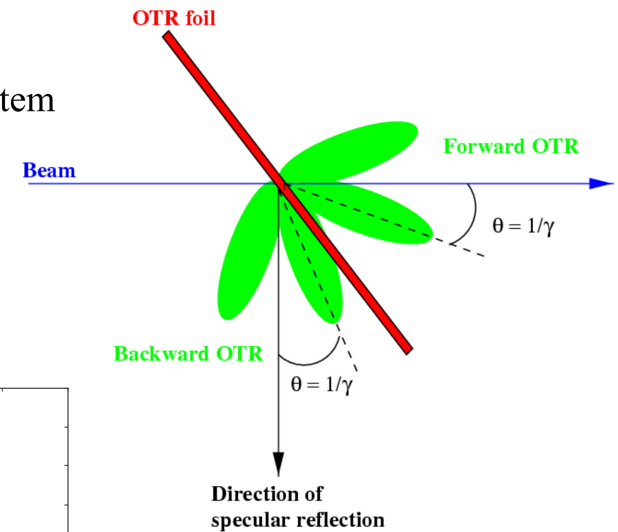
Transition Radiation

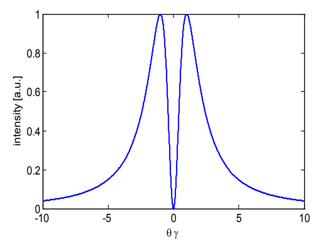
- **transition radiation:** electromagnetic radiation emitted when a charged particle crosses boundary between two media with different optical properties
- **visible part:** Optical Transition Radiation (OTR)
- **beam diagnostics:** backward OTR

  typical setup: image beam profile with optical system
- **advantage:** fast single shot measurement

  linear response (neglect coherence !)
- **disadvantage:** high charge densities may destroy radiator

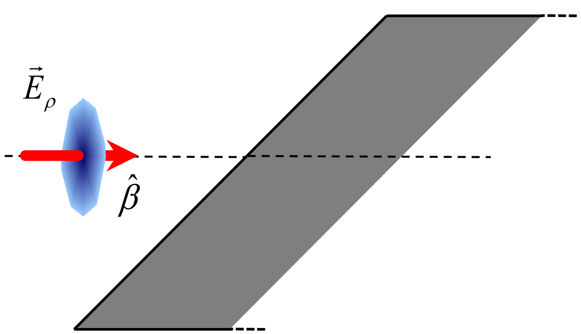
  → limitation on bunch number

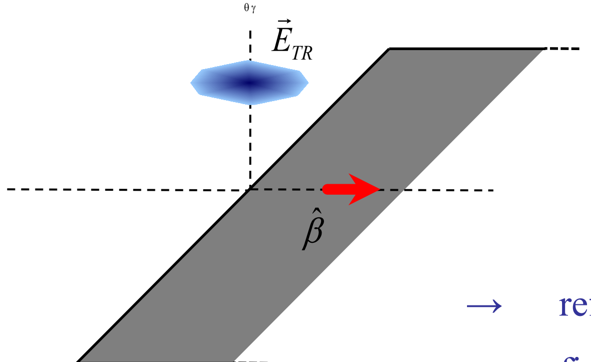




- **field separation mechanism**

  → reflection at boundary (perfect conductivity)





→ reflected and incident field are the same

OTR Monitor Resolution

- **PSF calculation in image plane**
  - field propagation in frame of scalar diffraction theory

M. Castellano and V.A. Verzilov, Phys. Rev. STAB 1 (1998) 062801

A.P. Potylitsyn, in "Advanced Radiation Sources and Applications", p.149

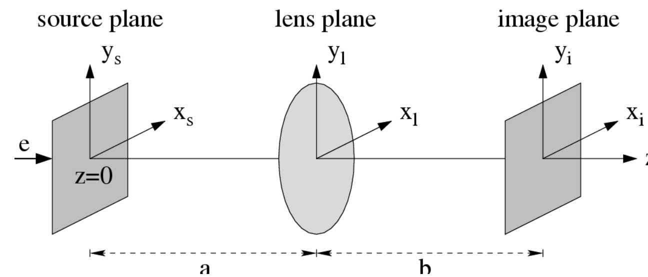
D. Xiang, W.-H. Huang, Nucl. Instr. Meth. A570 (2007) 357

G. Kube, TESLA-FEL Report 2008-01

G. Stupakov, SLAC-PUB-14758 (2011) …….

$$E^{l}_{x_l,y_l}(\vec{r}_l,\omega) = -i\,\frac{e^{ika}}{\lambda a}\cdot e^{i\frac{k}{2a}\left(x_l^{\,2}+y_l^{\,2}\right)} \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dx_s\,dy_s\,E^{s}_{x_s,y_s}(\vec{r}_s,\omega)\cdot e^{i\frac{k}{2a}\left(x_s^{\,2}+y_s^{\,2}\right)}\cdot e^{-ik\frac{x_s x_l+y_s y_l}{a}}$$

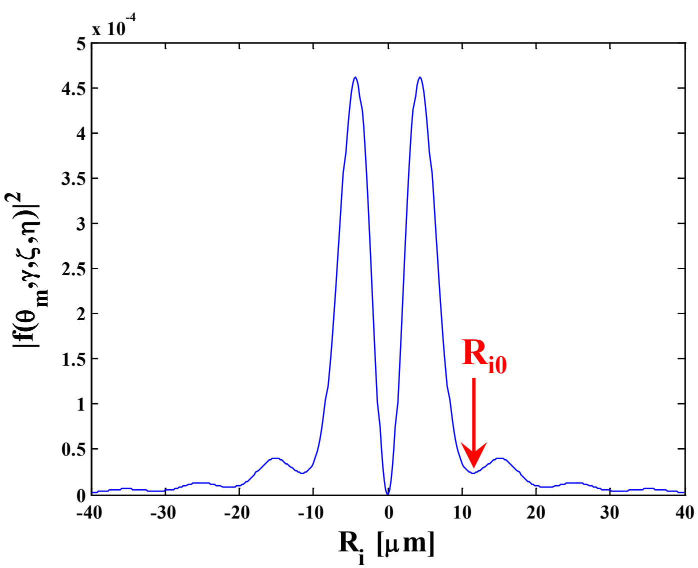
integration limits



  - **care:** screen dimension $\leftrightarrow$ field extension $\gamma\lambda$

    $\rightarrow$ might modify radiation properties

- **OTR resolution for beam imaging** (far field)



  - resolution definition according to classical optics:

    $\Longrightarrow$ first minimum of PSF

    ($\rightarrow$ diameter of Airy disk)

$$\boxed{R_{i0} \approx 1.12\,\frac{M\lambda}{\theta_m}}$$

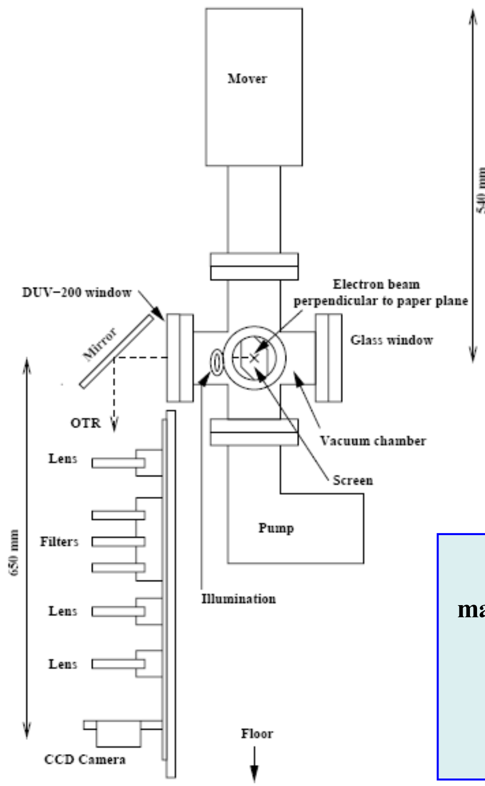
M: magnification

θ_m: lens acceptance angle (NA)

$\Rightarrow$ θ_m determined by optics, **not** by radiation properties !

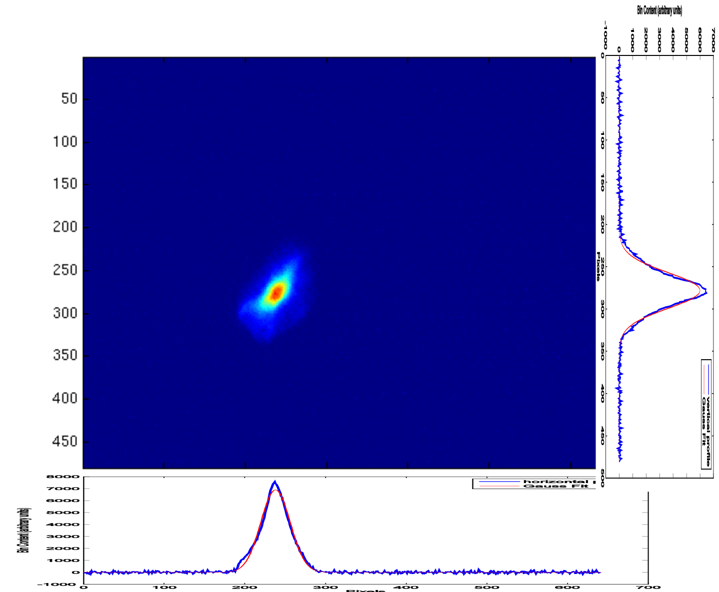
OTR Monitors

- example: FLASH @ DESY



optical system			
magnification	f / mm	a / mm	b / mm
1	250	500	500
0.382	200	724	276
0.25	160	800	200

- screen shot: 6 DBC2



K. Honkavaara et al., Proc. PAC 2003, p.2476

- standard monitors @ e-Linacs
  - 10 keV: R.B. Fiorito et al., Proc. PAC 2007, p.4006
  - 30 GeV: P. Catravas et al., Proc. PAC 1999, p.2111

- OTR @ hadron accelerators
  - protons: O.V. Afanasyev et al., Proc. BIW 2006, p.534
    V.E. Scarpine et al., Proc. BIW 2006, p.473
  - heavy ions: B. Walasek-Höhne et al., Proc. HB 2012, p.580

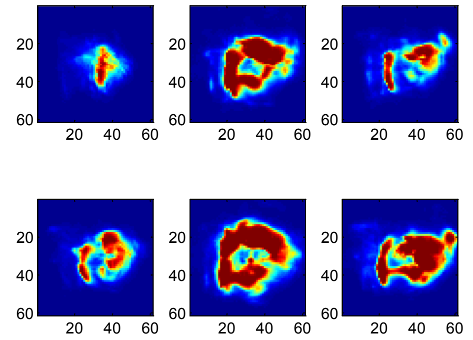
COTR and possible Mitigation

- unexpected Coherent OTR observation during LCLS commissioning

R. Akre et al., Phys. Rev. ST Accel. Beams **11** (2008) 030703 H. Loos et al., Proc. FEL 2008, Gyeongju, Korea, p.485.

  - strong shot-to-shot fluctuations
  - doughnut structure
  - change of spectral contents

→ measured spot is no beam image!



courtesy:

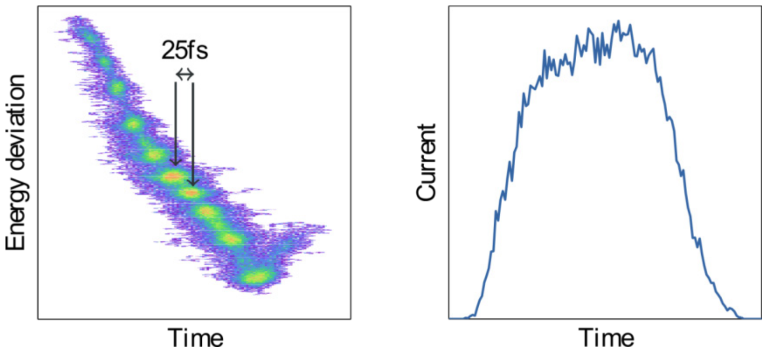
H. Loos (SLAC)

- interpretation of coherent formation in terms of "Microbunching Instability"

E.L. Saldin et al., NIM **A483** (2002) 516

Z. Huang and K. Kim, Phys. Rev. ST Accel. Beams **5** (2002) 074401

G. Stupakov, Proc. IPAC 2014, Dresden, Germany (2014), p.2789.



- alternative schemes for transverse profile diagnostics

  - short term perspective: scintillating screen monitors
  - long term perspective: TR imaging at smaller λ

proof of principle experiment @ λ = 19.6 nm: L.G. Sukhikh et al., Proc. IPAC 2012, New Orleans (USA), p. 819 and submitted to PRST-AB

$\Longrightarrow$ additional advantage of better resolution

PSF dominated Imaging

$$\text{Image} = \text{PSF} \otimes \text{Object} \ (+\text{Noise})$$

- **image formation**
  - standard imaging:

    minimize PSF contribution → image is true replica of object

- **PSF dominated imaging**
  - object size << PSF

    image dominated by PSF properties

  - non-zero object size

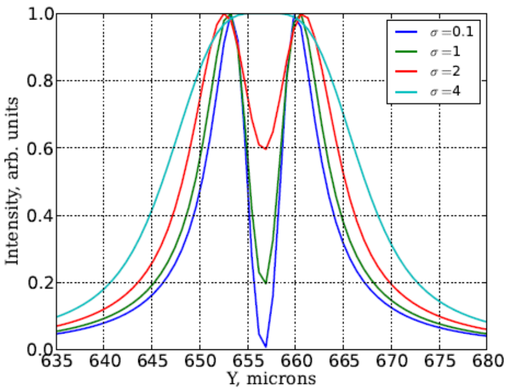
    → smearing out of PSF

    → beam size determination

    via image contrast

  - resolution below diffraction limit

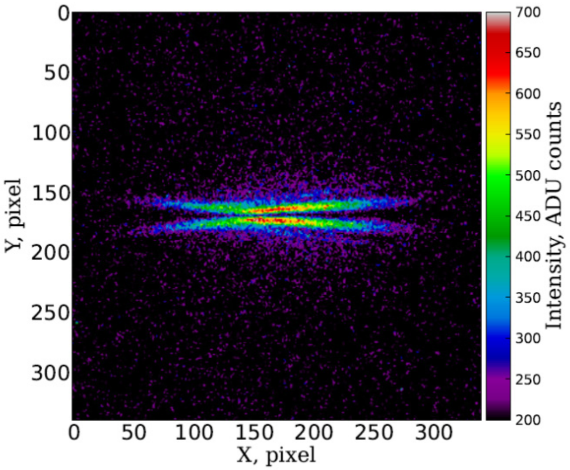
    → resolve sub-micron beam sizes with optical methods



K. Kruchinin et al.,
Proc. IBIC 2013, Oxford, UK, 615
J. Phys.: Conf. Series 517 (2014) 012011

- **experimental verification**
  - synchrotron radiation → π-polarisation imaging
  - OTR → test experiment @ ATF2

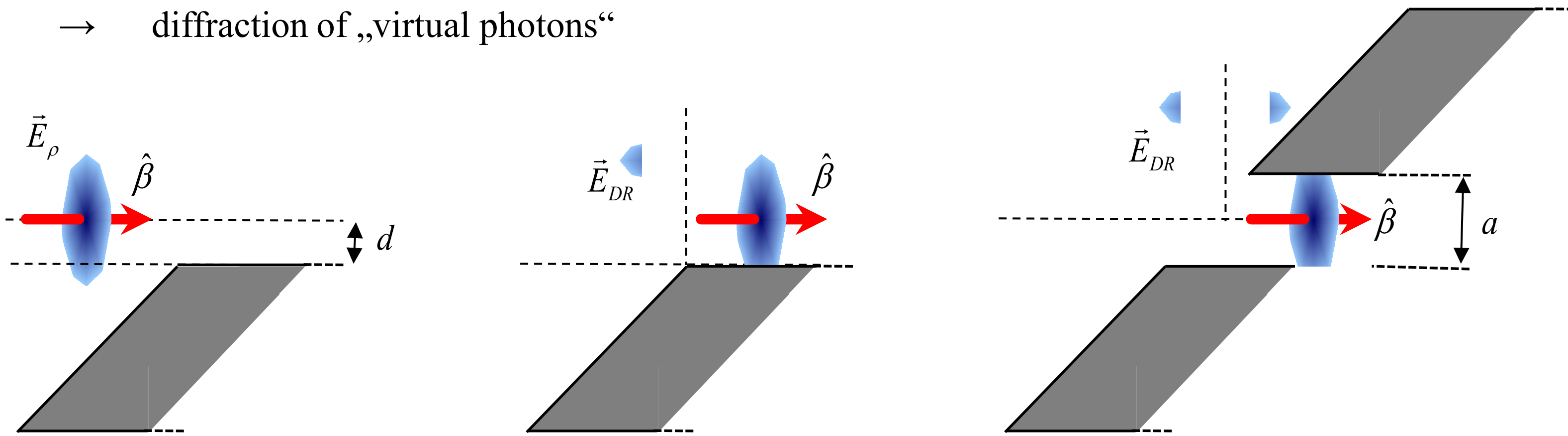
⇒ minimum measured beam size (0.754 ± 0.034) μm



P. Karataev et al., PRL 107 (2011) 174801

Diffraction Radiation

- **problem OTR:** screen degradation/damage
  - → limited to only few bunch operation, no permanent observation
- **Optical Diffraction Radiation (ODR):** non-intercepting beam diagnostics
  - DR generation via interaction between particle EM field and conducting screen
    - → diffraction of „virtual photons“



screen: half-plane

slit screen

- radial field extension
  - → radius $\lambda\beta\gamma / 2\pi \approx \lambda\gamma$
- limiting cases
  - $a \gg \lambda\gamma$: no radiation
  - $a \cong \lambda\gamma$: DR
  - $a \ll \lambda\gamma$: TR

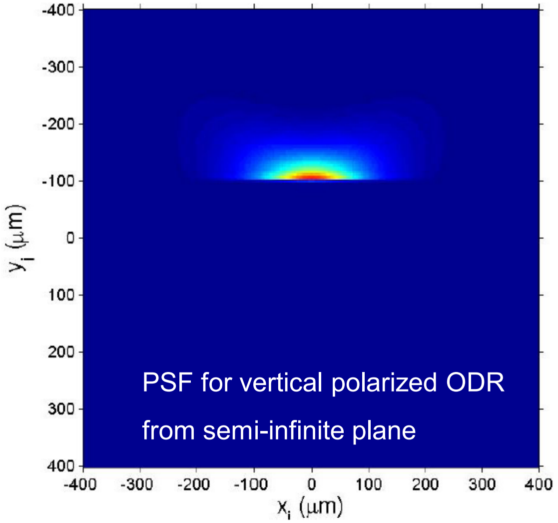
- **comment:** ODR in circular accelerator (CesrTA, Cornell)

L. Bobb et al., Proc. IBIC 2013, Oxford, UK, p. 619

ODR Imaging

- **PSF calculation in image plane**
  - field propagation in frame of scalar diffraction theory
    - → no beam image, illuminated edge of half-plane

D. Xiang et al., Phys. Rev. ST Accel. Beams **10** (2007) 062801



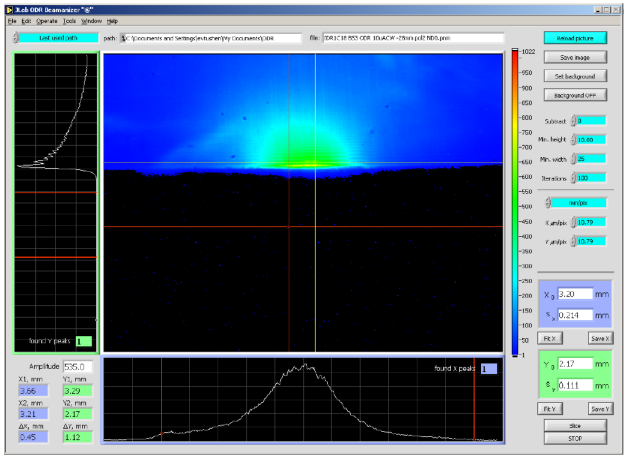
- **ODR imaging for beam diagnostics**

A. Lumpkin et al., Phys. Rev. ST Accel. Beams **10** (2007) 022802

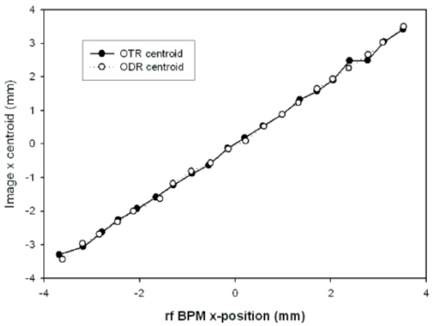
P. Evtushenko et al., Proc. BIW08, WECOTC01 (2008), p.332

  - (relative) 1D beam size monitor: σ_x
    - → (i) Gaussian beam profile
    - (ii) known distance between slit edge and beam center
    - → pre-defined ROI: projected 1D intensity profile
    - → fit profile with Gaussian distribution (σ_x)
    - → cross-calibrate σ_x with OTR beam profiles

courtesy: P. Evtushenko (JLab)

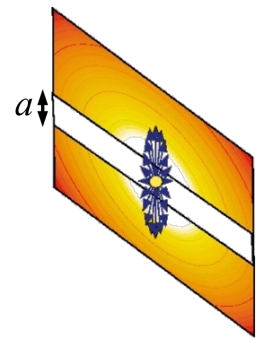


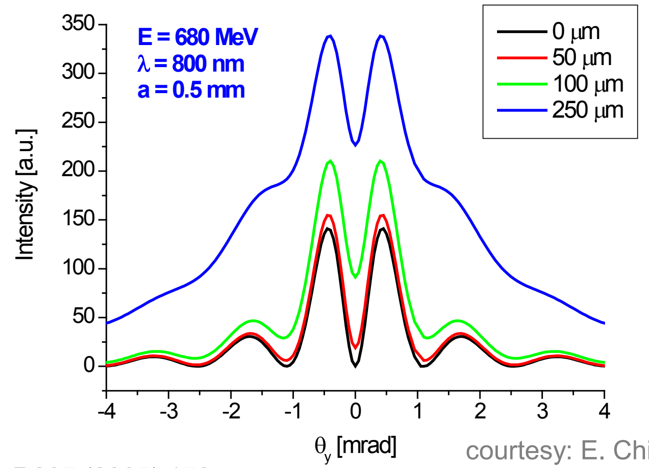
  - 1D beam position monitor
    - → ODR centroid
    - → achieved sensitivity: 50-100 µm
    - (σ_x = 1.3 mm, depends on beam size)



ODR Angular Distribution

- angular distribution dependence
  - on beam size
  - on beam offset
    - → beam centered in slit aperture
  - on beam divergence x´
    - → interferometric methods

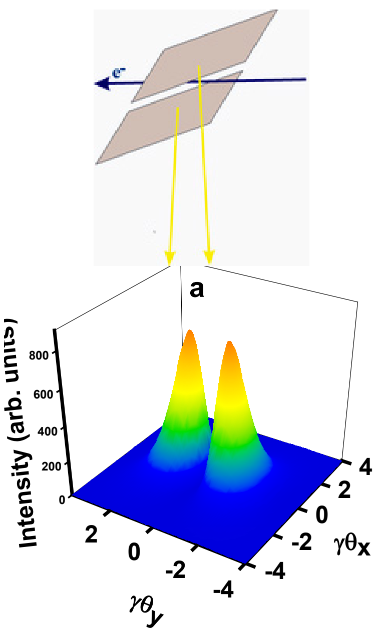




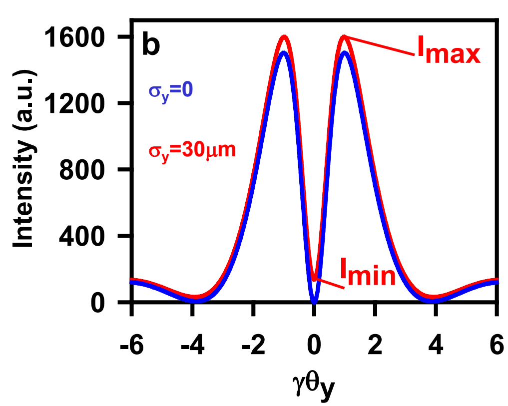
courtesy: E. Chiadroni (INFN)

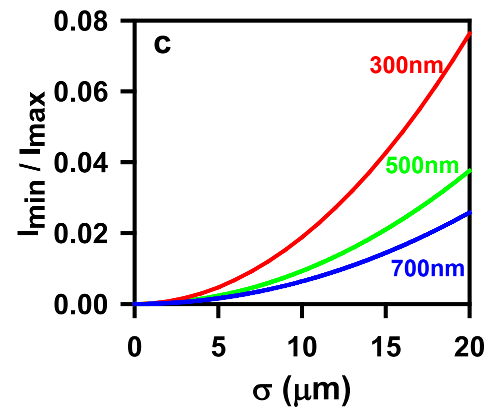
- 1D beam size determination (σ_y)

P. Karataev et al., Phys. Rev. Lett. **93** (2004) 244802 and Nucl. Instrum. Meth. **B207** (2005) 158



- very low emittance beam ($\varepsilon_y = 1.5 \times 10^{-11}$ m.rad) @ KEK-ATF, centered in slit
- exploit visibility I_{min} / I_{max} of projected vertical polarization component





courtesy:
P. Karataev (RHUL)
A. Potylitsyn (TPU)

⟹ sensitive on beam size $\sigma_y \approx 10$ μm

ODR Interferometry

- beam divergence: DR / ODTR interferometer

  R.B. Fiorito and D.W. Rule, NIM **B173** (2001) 167

  R.B. Fiorito et al., Phys. Rev. ST Accel. Beams **9** (2006) 052802

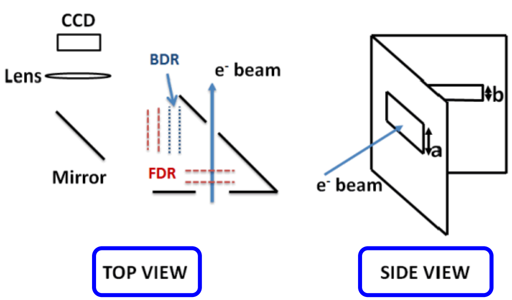
  M.A. Holloway et al., Phys. Rev. ST Accel. Beams **11** (2008) 082801

- ODRI: 1D beam size determination @ FLASH (DESY) (separation of beam size, divergence and offset)

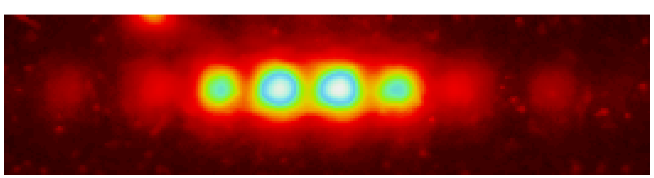
  A. Cianchi et al., Phys. Rev. ST Accel. Beams **14** (2011) 102803 and Proc. IPAC 2012, New Orleans, USA, p. 831

  - compact double slit arrangement
  - both slits with different sizes

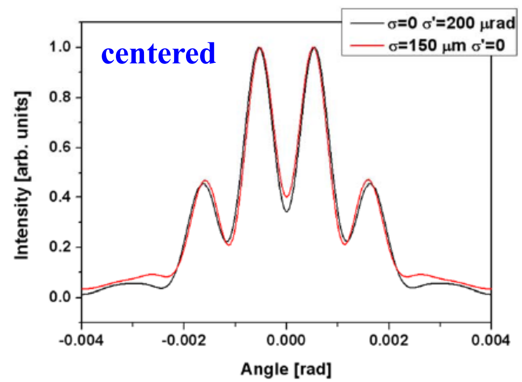
    second slit within radiation formation length of first one

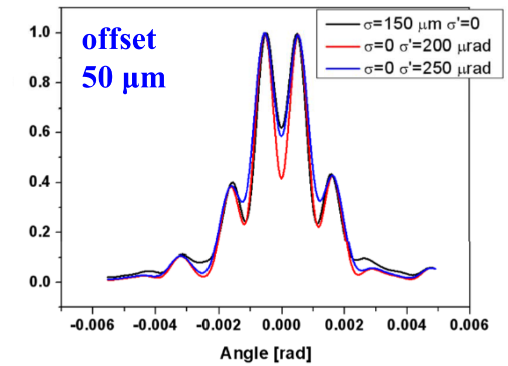


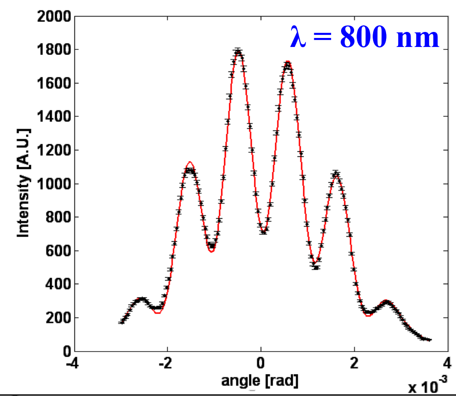
  - σ_y , σ_y' and offset by complex fit routine

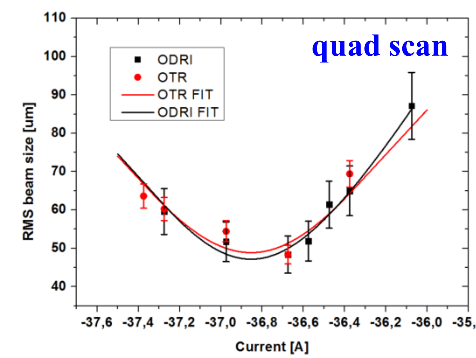


  - slit centers slightly off-centered









⇒ excellent agreement

Parametric X-Ray Radiation (PXR)

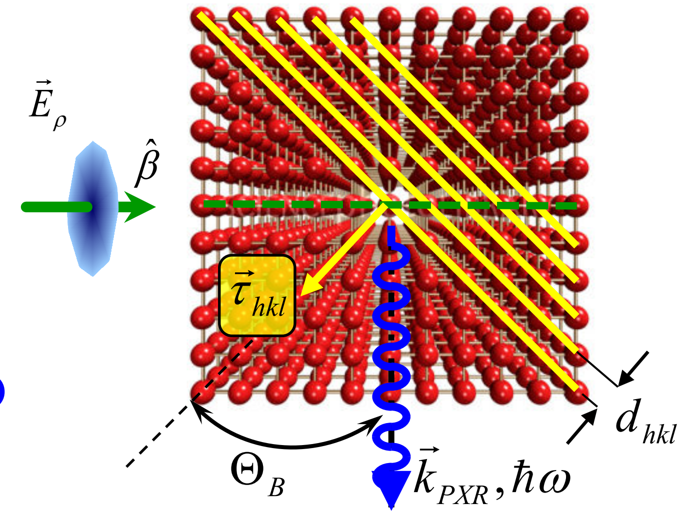
- idea: higher photon energies ħω
  - better resolution
  - insensitive on coherent effects
- real photons
  - X-rays ↔ Bragg reflection, crystals
- virtual photons
  - field separation by Bragg reflection at crystal lattice

    → radiation field: **Parametric X-Ray Radiation (PXR)**
- crystal periodicity (3D)
  - discrete momentum transfer (reciprocal lattice vector $\vec{\tau}_{hkl}$)

    → emission of line spectrum

courtesy: M.J. Winter

(Science Photo Library)



$$\boxed{\sin\Theta_B = \frac{\lambda}{2\,d_{hkl}}}$$

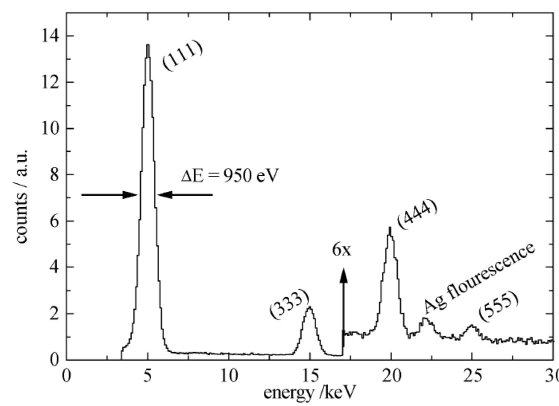
$$\vec{p}_i = \vec{p}_f + \hbar\vec{k} + \hbar\vec{\tau}_{hkl}$$

$$\delta E = \left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{\mathrm{v}} = \hbar\vec{k}\cdot\vec{\mathrm{v}} + \hbar\vec{\tau}_{hkl}\cdot\vec{\mathrm{v}} = \hbar\omega$$

$$\boxed{\hbar\omega_{hkl} = \hbar c\,\frac{\left|\vec{\beta}\cdot\vec{\tau}_{hkl}\right|}{1-\sqrt{\varepsilon}\,\vec{\beta}\cdot\hat{k}}}$$

$$\varepsilon = 1 - |\chi_0|$$

dielectric constant (≈ 1)



Si crystal

E = 855 MeV

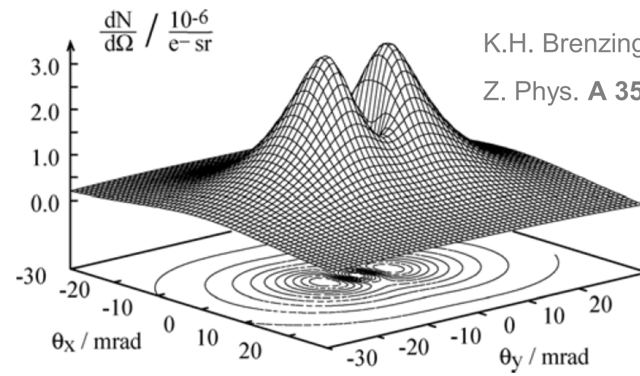
$\Theta_B = 22.5°$

K.H. Brenzinger et al.,

Z. Phys. **A 358** (1997) 107

Parametric X-Ray Radiation (PXR)

- **PXR:** Bragg scattering of virtual photons
  - virtual photon properties retained
    - → double lobe angular distribution



K.H. Brenzinger et al., Z. Phys. **A 358** (1997) 107

Si crystal
E = 855 MeV
$\Theta_B = 22.5°$

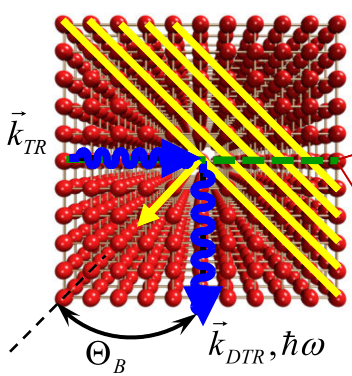
- radiation generation inside crystal
  - material properties influence radiation characteristics
    - → angular width: $$\Delta\theta = \sqrt{\left(\frac{1}{\gamma}\right)^2 + \left(\frac{\hbar\omega_p}{\hbar\omega}\right)^2}$$

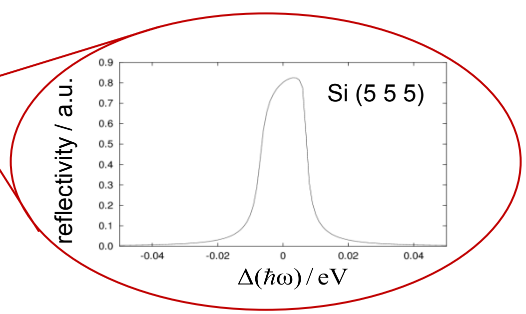
plasma energy $\hbar\omega_p$ (Si: 31 eV)

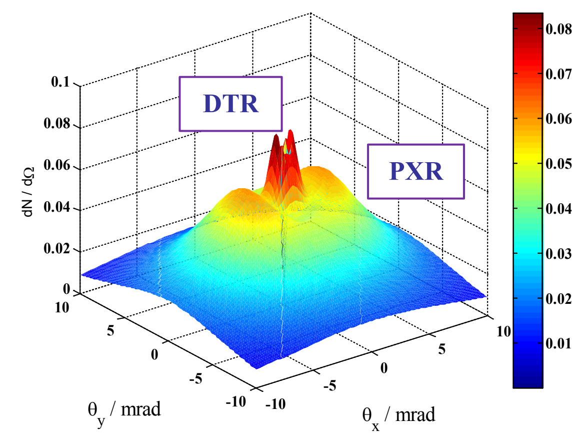
- additional contribution to angular distribution
  - → DTR: smaller angular width

- background contribution: real photon diffraction
  - transition radiation from crystal entrance surface
    - → diffracted at crystal planes under same Bragg angle

$$\hbar\omega_{PXR} \approx \hbar\omega_{DTR}$$







A. Caticha, Phys. Rev. **A 40** (1989) 4322

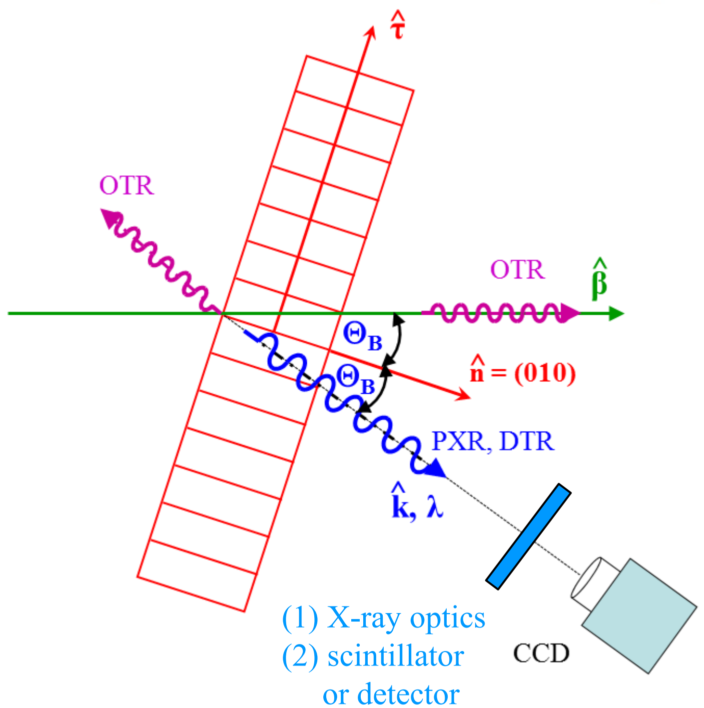
radiation amplitude: $A_r = A_{c\tau} + R_A\left(A_v - A_{c0}\right)$

PXR for Beam Profile Diagnostics

- advantage of PXR diagnostics
  - spatial separation from COTR background
    - → OTR reflection wrt. surface normal $\hat{n}$
    - → PXR reflection wrt. reciprocal lattice $\hat{\tau}$
- proposals

  A. Gogolev et al., J. Phys.: Conf. Series **357** (2012) 012018

  Y. Takabayashi, Phys. Lett. **A 376** (2012) 2408
- detection scheme (1)
  - imaging with X-ray optics
    - → sensitivity ?
- detection scheme (2)
  - X-ray scintillator/detector close to emission point
    - → conversion of X-rays to visible light
      allows usage of standard optics and CCD
    - → parallel object and image plane
    - → sensitivity, background ?



- detection scheme (3)
  - exploit angular distribution
    - → requires exact knowledge of shape
      PXR / DTR interference, …
    - → additional background contributions ?

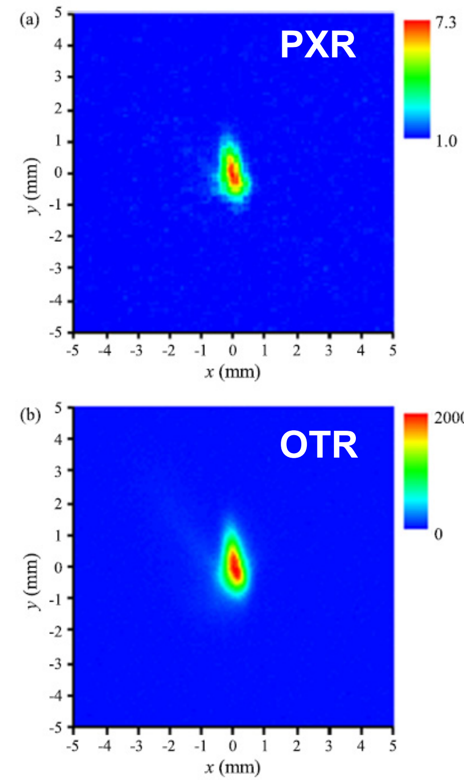
PXR for Beam Profile Diagnostics

- direct imaging with pinhole camera

  Y. Takabayashi and K. Sumitani, Phys. Lett. **A 377** (2013) 2577

  - test experiment at SAGA Light Source (Japan)
    - → 255 MeV linac beam, $f_{rep} = 1$ Hz, $I_{avg} = 7$ nA
    - → Si crystal, t = 20 µm, (220) reflection @ 11.6 keV
  - OTR beam profile
    - → single shot
  - PXR beam profile
    - → 12600 shots
    - → 3.5 h exposure time

  ➡ image plate as detector



- detector close to emission point
  - test experiment @ SAGA

    Y. Takabayashi, Phys. Lett. **A 376** (2012) 2408

    - → image plate 55.6 mm from target crystal
    - → 1 sec exposure time
    - → image plate inside vacuum chamber
    - → large background contribution
  - test experiment @ MAMI (Mainz, Germany)

    G. Kube et al., Proc. IPAC 2013, Shanghai, China, p.491

    - → scintillator close to target + CCD
    - → sensitivity to low, no beam image

PXR Angular Distribution

- angular distribution measurements

  G. Kube et al., Proc. IPAC 2013, Shanghai, China, p.491

  - test experiment @ MAMI (Mainz, Germany)
    - → 855 MeV, I_{avg} = 500 nA
    - → use of low-cost X-ray CCD
    - → (100)-cut Si-crystal, t = 50 μm

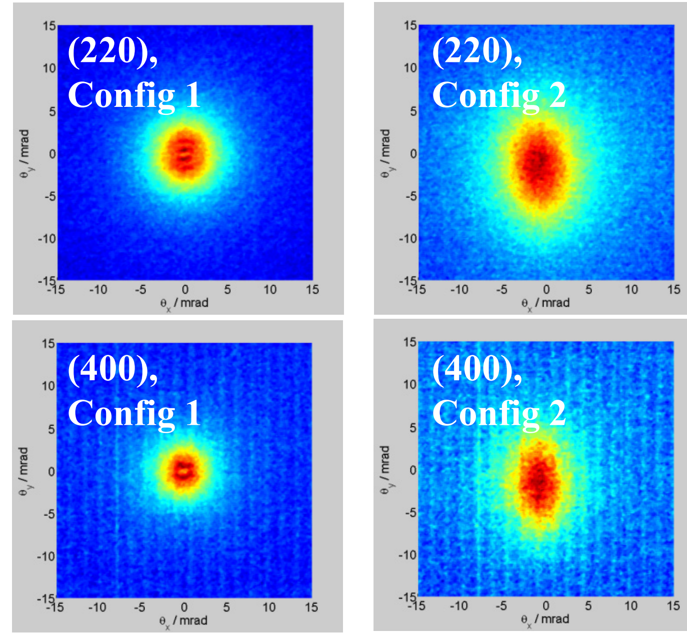
    ħω(220) = 16.55 keV   ħω(400) = 23.40 keV

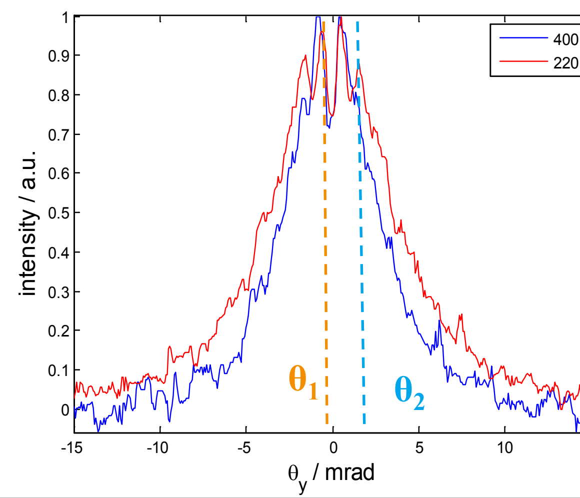
    - → two (out of 6) beam configurations

    Config 1: σ_x = 45.7 μm, σ_y = 42.9 μm   Config 2: σ_x = 44.7 μm, σ_y = 796 μm

  ➡ angular distribution sensitive on beam size

  - observation
    - → θ_1 independent on photon energy: $\theta_1 = 0.6\,\text{mrad} \approx 1/\gamma$
    - → additional lobes at θ_2 ~ 1.8 mrad
  - interpretation
    - → significant DTR contribution
    - → additional contribution from diffracted bremsstrahlung ???



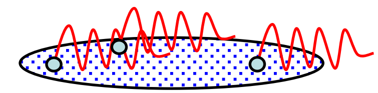


Longitudinal Profile Diagnostics

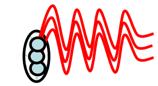
- **Coherent Radiation Diagnostics (CRD)**
  - standard method for radiation based bunch length diagnostics

O. Grimm, Proc. PAC 2007, Albuquerque, USA, p.2653

long bunch ($\lambda<\sigma_z$)



short bunch ($\lambda>\sigma_z$)



- **basic procedure**
  - **principle:** bunch length/shape dependent emission spectrum of coherent radiation

single particle spectrum — bunch form factor

$$\frac{\mathrm{d}U}{\mathrm{d}\lambda} = \left(\frac{\mathrm{d}U}{\mathrm{d}\lambda}\right)_1 \left(N + N(N-1)\left|F(\lambda)\right|^2\right) \qquad \text{with} \qquad F(\lambda) = \int_{-\infty}^{+\infty} \mathrm{d}z\, S(z)\, \mathrm{e}^{i\frac{2\pi z}{\lambda}}$$

no. of particles per bunch — bunch profile

  - measure radiation intensity as function of wavelength in spectral region of interest

    → bunch length determination requires spectral decomposition of intensity

    → intensity-interferometer in THz region (Michelson or Martin-Puplett interferometer)

  - Fourier transform

    → **bunch profile and bunch length**

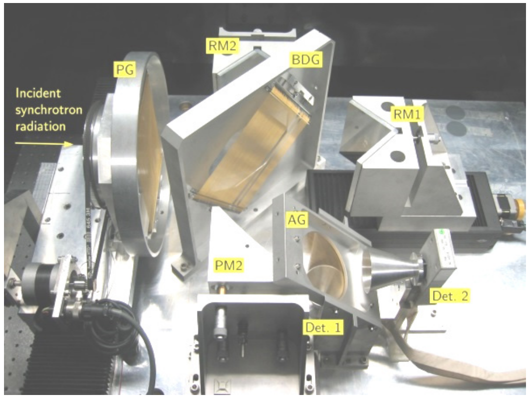
  - radiation generation

    → <u>coherent radiation source:</u> synchrotron radiation, transition radiation, diffraction radiation, Smith-Purcell radiation, Cherenkov radiation, …

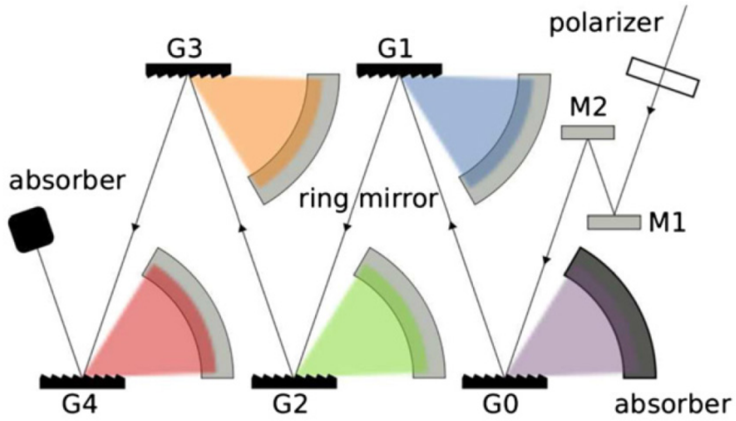
Coherent Radiation Diagnostics

- TR, DR or SR based CRD
  - polychromatic emission spectrum
    → spectrometer required for spectral decomposition
  - Michelson / Murtin-Puplett interferometers: scanning devices
    → no single-shot capability
- single-shot CRD
  - extension to multi-stage single-shot grating spectrometer

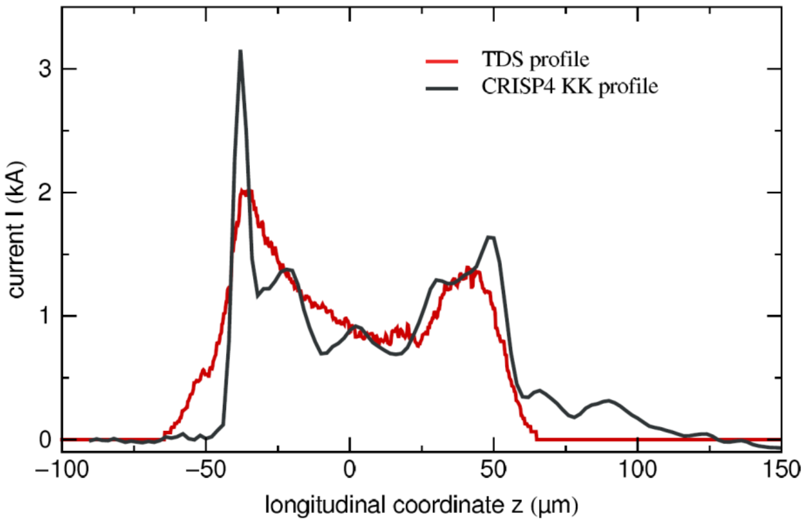
Martin-Puplett interferometer @ FLASH



S. Wesch et al., Nucl. Instrum. Meth. **A665** (2011) 40



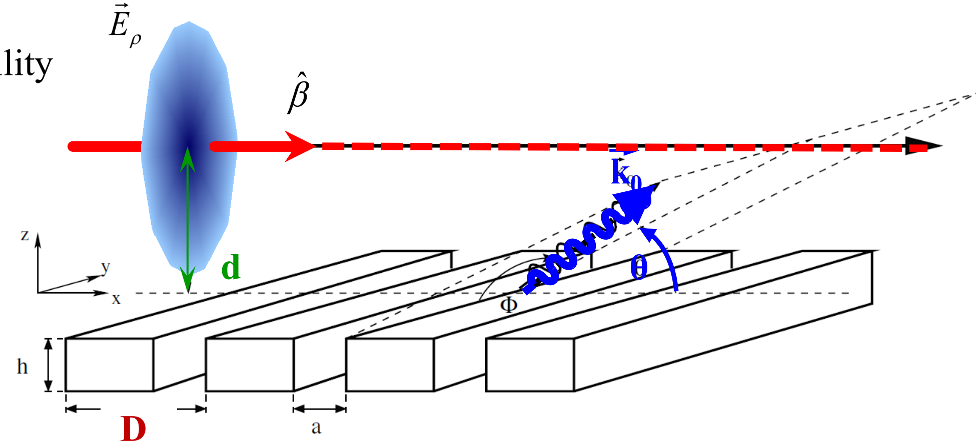
pyro-electric line detector
30 channels @ room temperature
no window, works in vacuum
fast read out
sensitivity ~ 300 pJ (S/N=5)



S. Wesch et al., Proc. BIW'12, Newport News (VA), USA, p.256

Smith-Purcell Radiation

- **idea:** dispersive radiation generation
  - radiation generation and analysis with one device
    - → compact setup, option for single-shot capability
- **Smith-Purcell radiation (SPR)**
  - field separation
    - → virtual photon diffraction at 1D Bravais-structure (grating)
    - → grating provides 1D discrete momentum



momentum conservation:

$$\vec{p}_i = \vec{p}_f + \hbar\vec{k} + \hbar n \frac{2\pi}{D}\hat{\mathrm{v}}$$

$$\left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{\mathrm{v}} = \hbar\omega = \hbar\vec{k}\cdot\vec{\mathrm{v}} + \hbar n\frac{2\pi}{D}\hat{\mathrm{v}}\cdot\vec{\mathrm{v}}$$

$$2\pi\frac{c}{\lambda} = \frac{2\pi}{\lambda}\mathrm{v}\cos\theta + n\frac{2\pi}{D}\mathrm{v}$$

$$\boxed{n\lambda = D\left(\frac{1}{\beta} - \cos\theta\right)}$$

→ **SPR dispersion relation**

- distance dependence
  - → range of el. field: $\lambda\beta\gamma / 2\pi$
  - 2D field description: $\vec{E} \propto \mathrm{e}^{-\frac{2\pi}{\lambda\beta\gamma}d}$
  - intensity scaling: $\boxed{I \propto \left|\vec{E}\right|^2 \propto \mathrm{e}^{-\frac{4\pi}{\lambda\beta\gamma}d}}$
- SPR identification
  - → dispersion relation: necessary condition
  - → distance dependence: sufficient condition

SPR for Bunch Length Diagnostics

- proposals

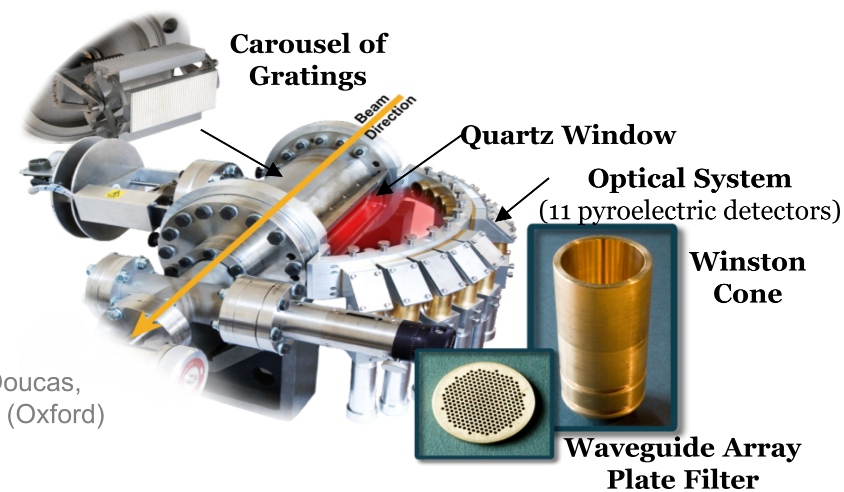
M.C. Lampel, Nucl. Instrum. Meth. **A 385** (1997) 19
D. Nguyen, Nucl. Instrum. Meth **A 393** (1997) 514

} dispersion relation: → convert angle θ to wavelength λ

- bunch length monitor based on SPR

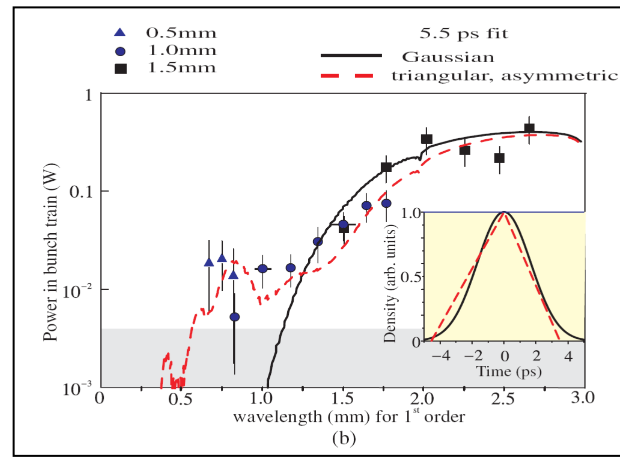
G. Doucas et al., Phys. Rev. ST Accel. Beams **9** (2006) 092801 **@ 45 MeV (FELIX)**
V. Blackmore et al., Phys. Rev. ST Accel. Beams **12** (2009) 032803 **@ 28.5 GeV (SLAC, ESA)**
H.L. Andrews et al., Phys. Rev. ST Accel. Beams **17** (2014) 052802 **@ 20.35 GeV (SLAC, FACET)**

similar setup



courtesy G. Doucas, V. Blackmore (Oxford)

measurement at 45 MeV, FELIX



- critical items
  - limited number of points for reconstruction → interferometer: typically about 200 points
  - single photon emission spectrum → different model predictions, especially at high γ

D.V. Karlovets and A.P. Potylitsyn, Phys. Rev. ST Accel. Beams **9** (2006) 080701

Summary

- radiation physics widely used for beam diagnostics
  - longitudinal and transverse beam profiles
  - beam divergence, beam energy, …
- circular accelerators: synchrotron radiation
  - new 3rd generation light sources with ultra-small beam sizes

    → X-ray imaging: possibility to measure beam sizes down to μm level
- linear accelerators: working horse OTR (+ screens), ODR in experimental stage
  - OTR: invasive measurement, usually resolution of about 10 μm
  - better resolution → smaller wavelengths (EUV), PSF-dominated imaging
  - new 4th generation light sources → coherent emission compromises use of OTR as reliable diagnostics
  - ODR: high resolution measurements via angular distribution → ODRI offers possibility to resolve ambiguities
- PXR: interesting for X-ray region
  - still in early experimental stage → first experiments in view of beam diagnostics
- CRD: bunch length/shape measurements
  - CTR, CDR, CSR → spectral decomposition with interferometers
  - CSPR → dispersive emission characteristic, but still some open questions…

Outlook

- commercial codes applied to radiation physics
  - TR generation with CST Particle Studio®

    K. Lekomtsev et al., Journal of Physics: Conference Series **517** (2014) 012016
  - OTR/ODR generation and propagation with ZEMAX®

    T. Aumeyr et al., Proc. IBIC 2013, Oxford, UK, p.852

    T. Aumeyr et al., Journal of Physics: Conference Series **517** (2014) 012026

    T. Aumeyr et al., Proc. IPAC 2014, Dresden,Germany, p.3722
- (Surface) Cherenkov Radiation
  - growing interest → as radiation source

    → but also for beam diagnostics

    A.S. Konkov et al., Journal of Physics: Conference Series **517** (2014) 012003 …

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  - for your attention