Understanding tune spectrum of high intensity beams

IBIC 2013
International Beam Instrumentation Conference

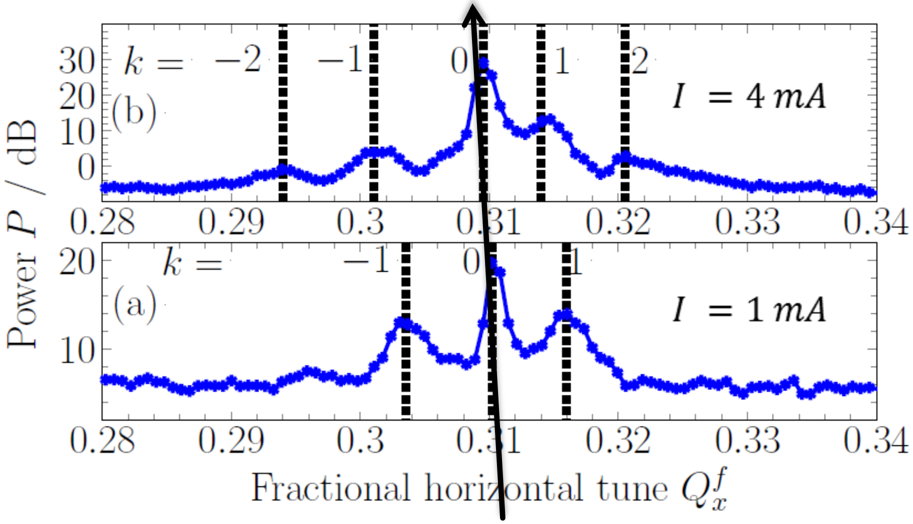
Rahul Singh

GSI, Darmstadt, Germany
TEMF, TU Darmstadt, Germany

IBIC 2013, Oxford

Motivation

Higher current: The „global“ peak moves to the left. The symmetry of the spectrum is broken.



Low current: Tune spectra has symmetric sidebands due to synchrotron motion.

Relevant GSI SIS-18 parameters

Parameter	Value
Circumference	~ 216 m
Injection energy	~ 11.4 MeV/u ($\beta = 0.15$)
Extraction energy	~ 1 GeV/u ($\beta = 0.9$)
Betatron tune (Q_x, Q_y)	~ 4.31, 3.28
Synchrotron tune (Q_s)	~ 0.007 (1 KHz)
Beam current	~$10^6 - 10^9$ U^{73+} particles

1) Why is the tune spectrum modified at increasing beam current?
2) What can we learn from this modification for beam diagnostics ?
3) Why and where are these effects relevent ?

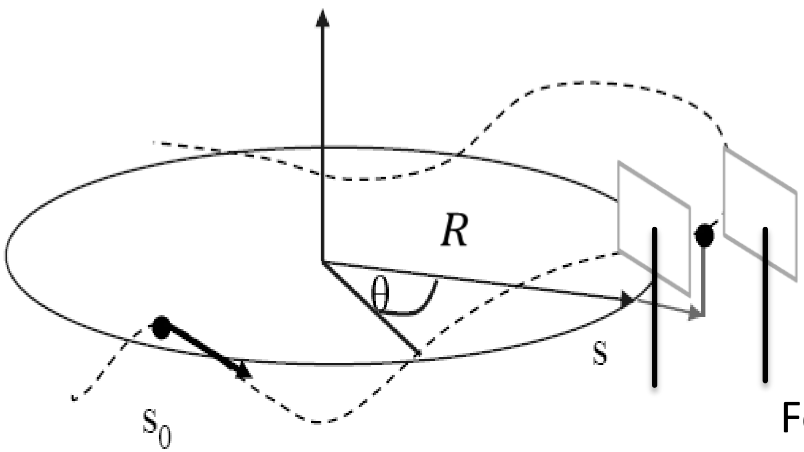
Outline

- Transverse beam spectrum at low intensity
- Head-tail modes
- High intensity effects on tune spectra
- Interpretation of tune spectra
- Beam diagnostics : Measurement of tune, impedances, chromaticity etc.
- Relevance : Why and where?
- Conclusion

Theory

Measurements and results

Introduction: Betatron tune?



Emittance

Lattice beta

$$\mathrm{d}(s) = \sqrt{\varepsilon\beta(s)}\cos(\psi(s) + \phi_0)$$

where $\psi(s) = \int_{s_0}^{s} \frac{1}{\beta(s)} ds$

For one complete revolution

$$2\pi Q_0 = \int_{s_0}^{s_0+2\pi R} \frac{1}{\beta(s)} ds$$

Tune!

At a specific position (BPM) turn by turn (m)

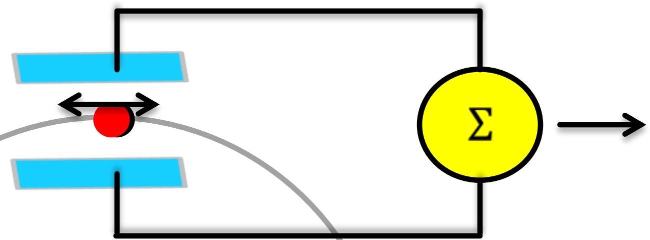
Tune frequency (ω_b)

Revolution frequency

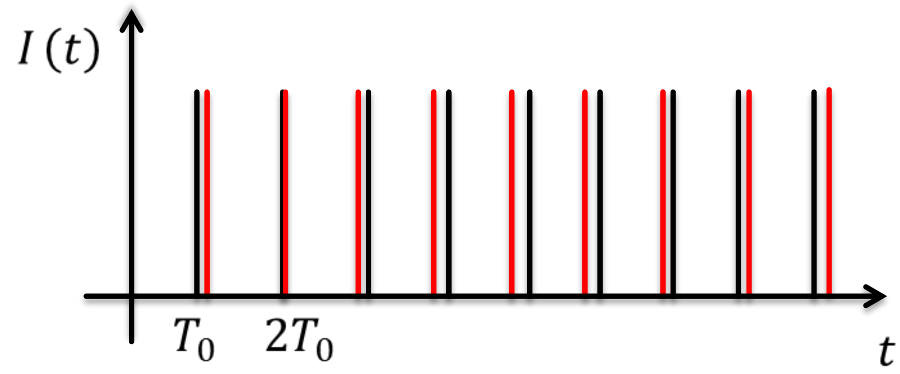
$$\mathrm{d}(m, s_{BPM}) = \sqrt{\epsilon\beta(s_{BPM})}\cos(Q_0\omega_0 mT_0 + \phi_0)$$

- The periodic focussing and de-focussing creates oscillations.
- Number of dipole oscillations per turn is the bare tune or single particle tune.

Longitudinal spectra (single particle)



Sum signal at pick-ups



$a \propto \frac{\Delta p}{p_0}$ (initial)

Longitudinal amplitude

Synchrotron motion!

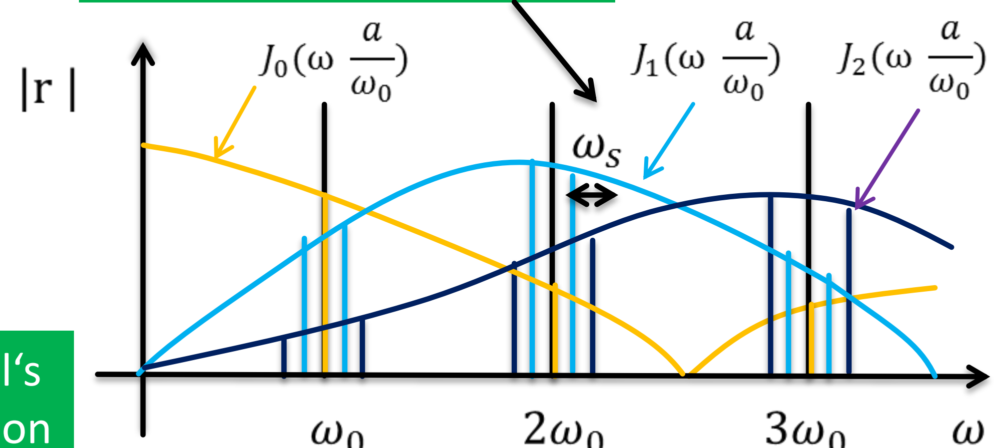
Frequency domain

$\theta(t) = a \sin(Q_s \omega_0 t + \varphi)$

Synchrotron frequency

Satellite number

Revolution harmonic



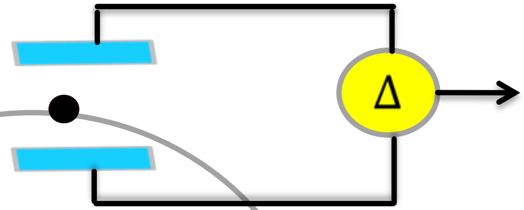
Frequencies $\omega_{m,k} = m\omega_0 + k\omega_s$

Amplitude $r_{m,k} = q\omega_0 J_k(ma)$

Phase $\alpha_{m,k} = -k\varphi$

Bessel's function

Transverse spectra (single particle)



No Chromaticity

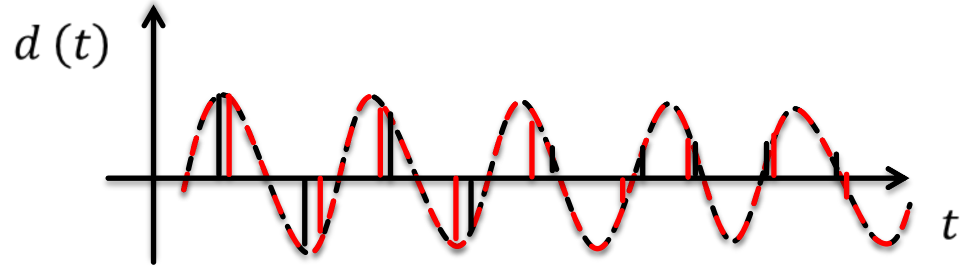
$$\frac{\Delta Q}{Q_0} = \xi \frac{\Delta p}{p_0}$$

0

$$\omega_{m,k} = m\omega_0 \pm \omega_b + k\omega_s$$

$$r_{m,k} \sim q\omega_0 A J_k\left(\omega_{m,k} \frac{a}{\omega_0}\right)$$

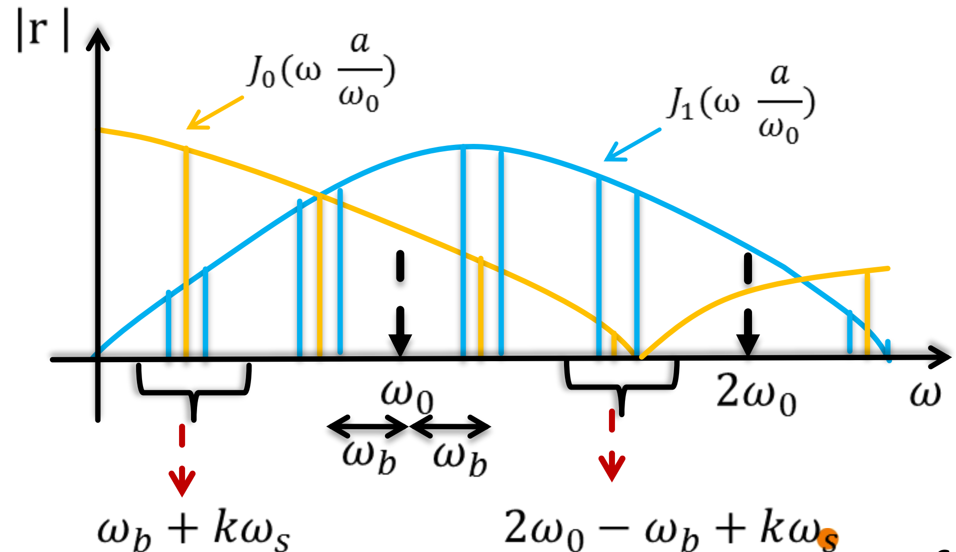
$$\alpha_{m,k} = \phi + k\varphi$$



$$d\,(t) = x\,(t) \cdot I\,(t)$$

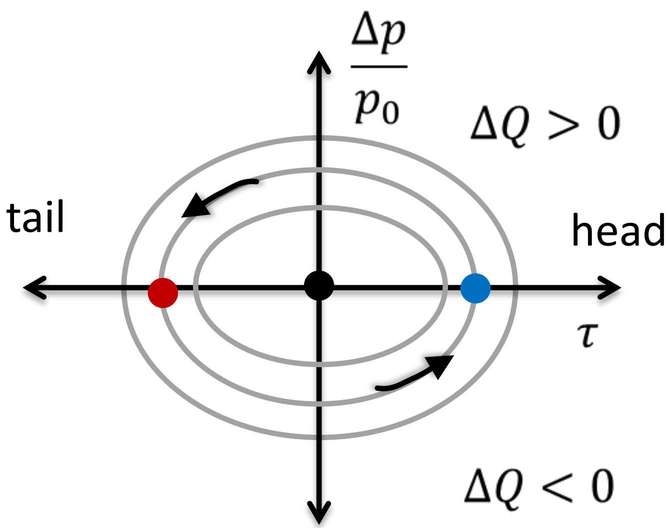
$$= A\,\cos(\omega_b t + \phi) \cdot I\,(t)$$

For $\xi_y = 0$, Amplitude modulation

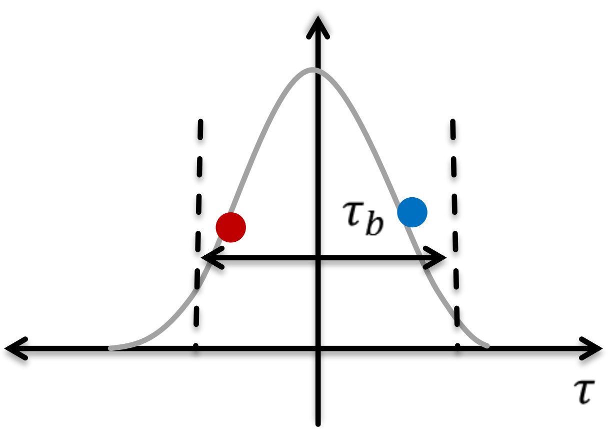


Effect of chromaticity

Longitudinal phase space



Half synchrotron period



$$\frac{\Delta Q}{Q_0} = \xi \frac{\Delta p}{p_0} \qquad \xi > 0$$

Depending on the position within the bunch the betatron phase is different

Between any two positions τ

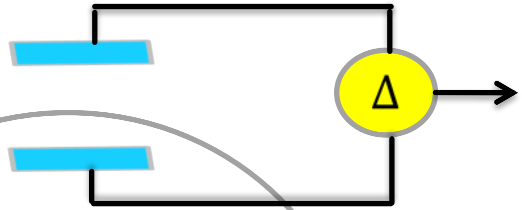
$$\chi = \frac{\xi}{\eta} Q_0 \omega_0 \tau = \omega_\xi \tau$$

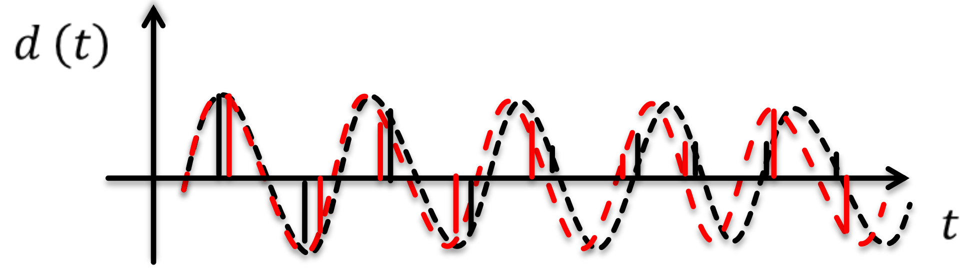
Chromatic frequency (ω_ξ)

Maximum phase difference
(Head-tail phase shift)

$$\chi_b = \frac{\xi}{\eta} Q_0 \omega_0 \tau_b$$

Transverse spectra (with chromaticity)





Chromaticity

$$\frac{\Delta Q}{Q_0} = \xi \, \frac{\Delta p}{p_0}$$

$$d\,(t) = x\,(t) \cdot I\,(t)$$

$$= A\,\cos\left(\omega_b(1 + \xi\,\frac{\Delta p}{p_0})t + \phi\right) \cdot I\,(t)$$

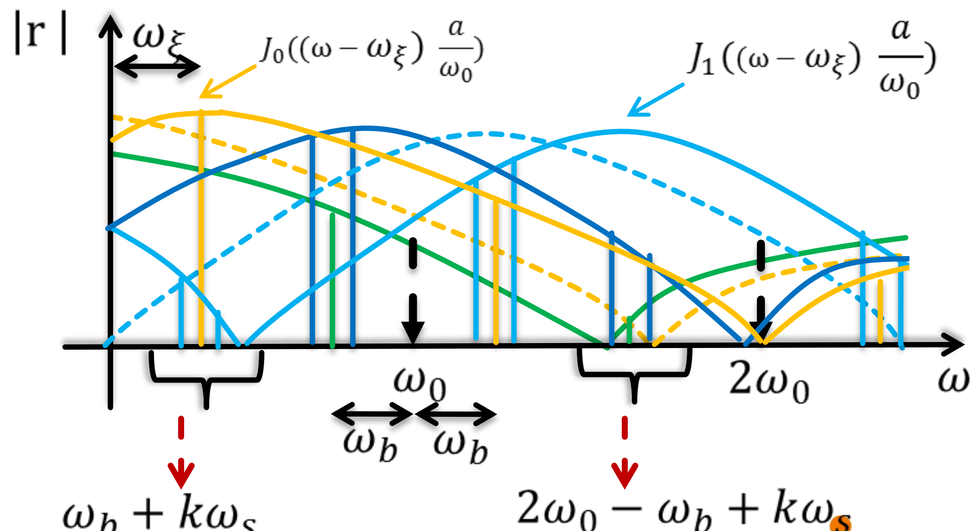
For $\xi_y \neq 0$, Envelope shifted

Chromatic frequency

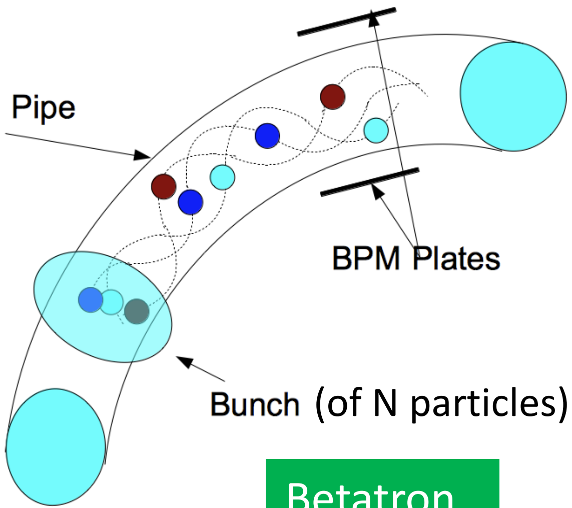
$$\omega_{m,k} = m\omega_0 \pm \omega_b + k\omega_s$$

$$r_{m,k} \propto q\omega_0 A\, J_k((\omega_{m,k} - \omega_\xi)\,\frac{a}{\omega_0})$$

$$\alpha_{m,k} = \phi + k\varphi$$



What do we measure?



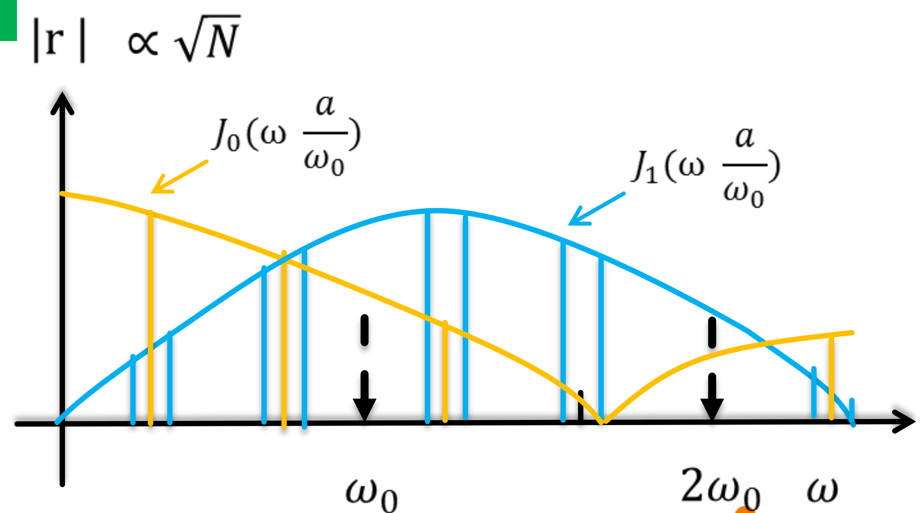
Betatron phase

Synchrotron phase

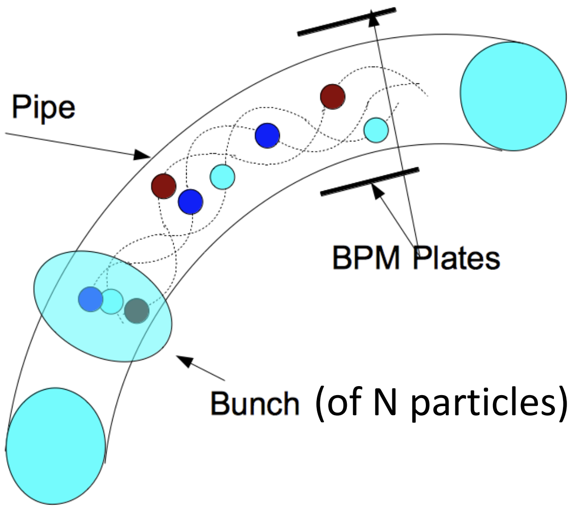
For n^{th} particle, the phases of k^{th} satelites are $\alpha_{m,k}(n) = \phi_n + k\varphi_n$

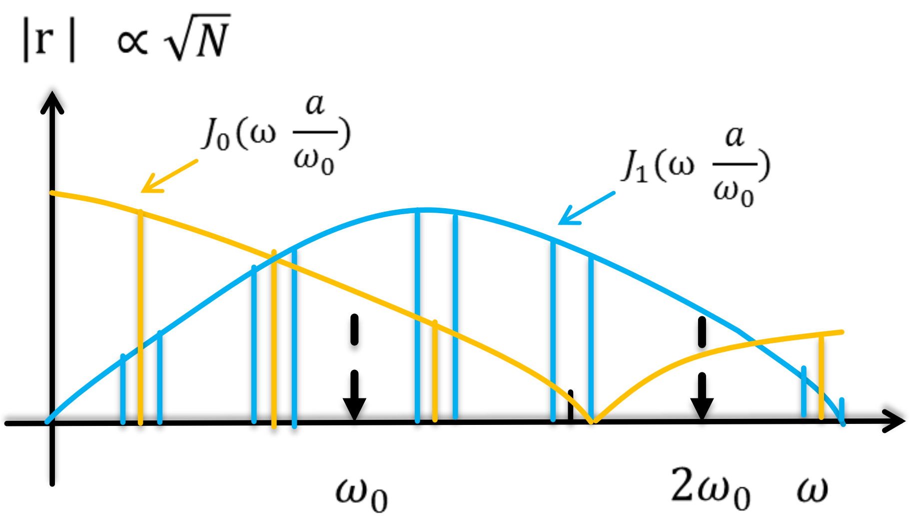
Phases are uniformly distributed between $(0, 2\pi)$,spectral lines incoherently add

Not visible to BPMs, difficult to measure, modified significantly at high intensities

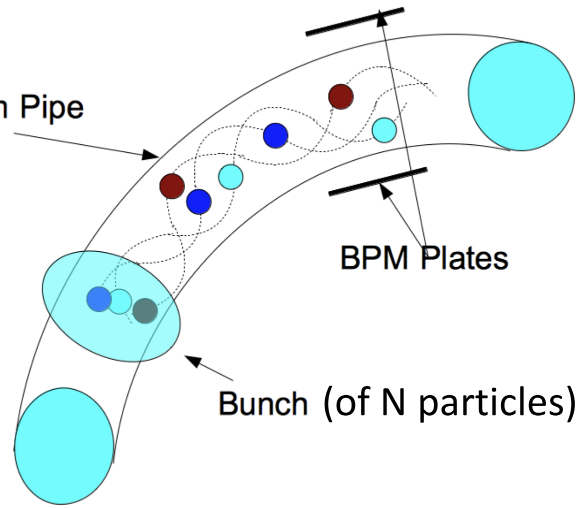


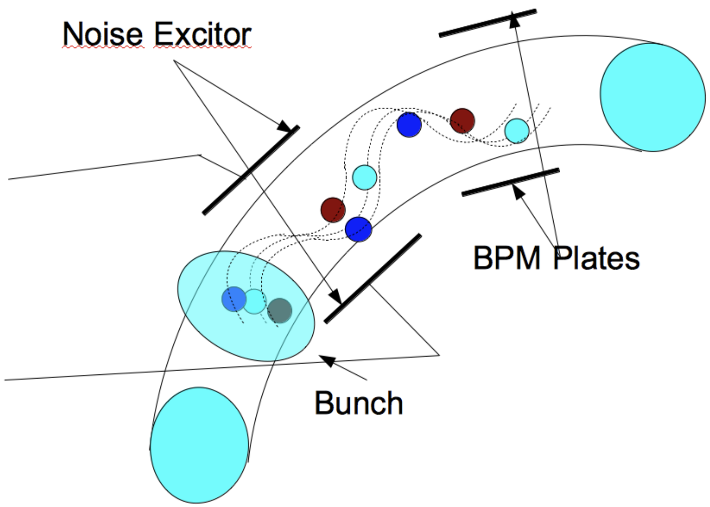
What do we measure?

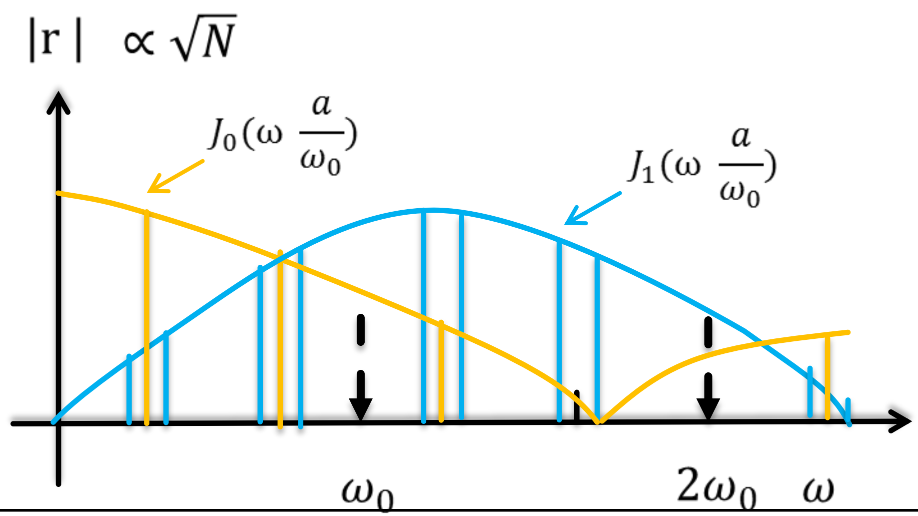




What do we measure?





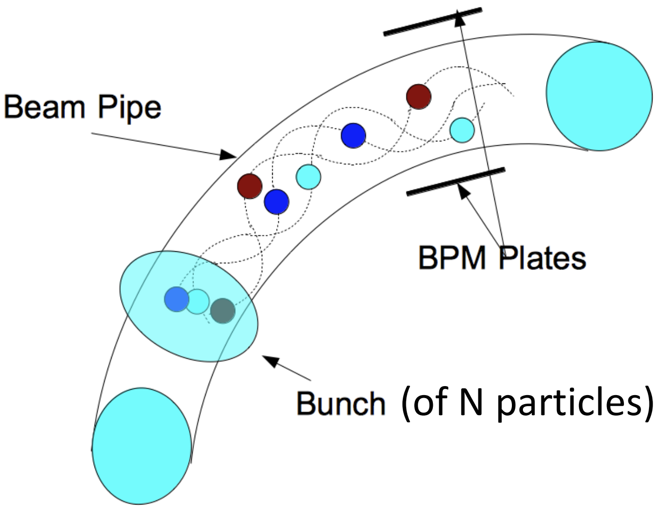


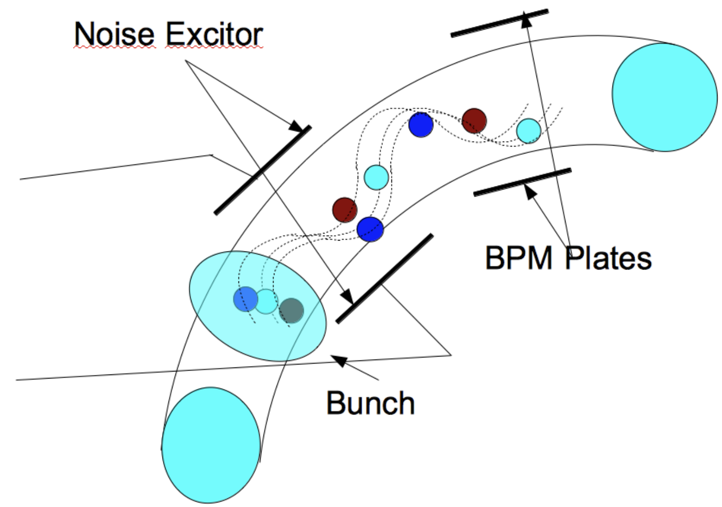
The phases of particles are aligned due to excitation.

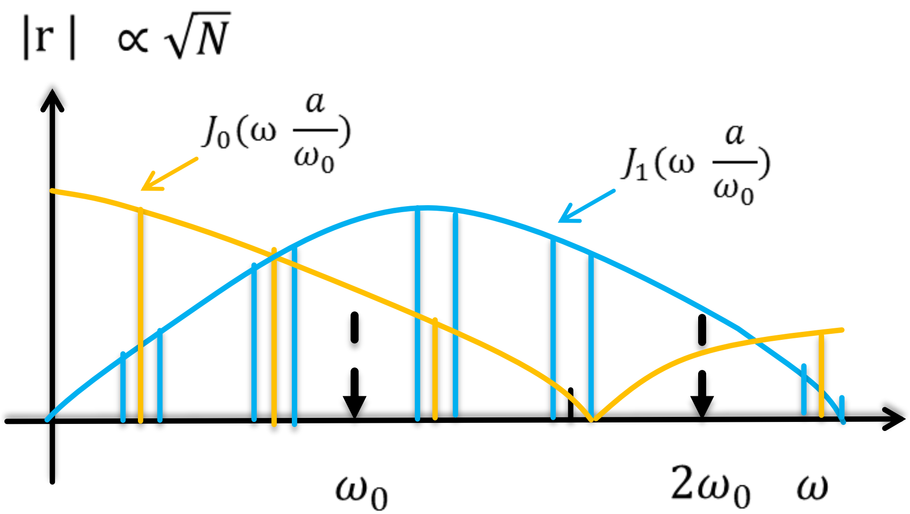
$$\alpha_{m,k}(n) = \phi_n + k\varphi_n = const.$$

This results in coherent signals

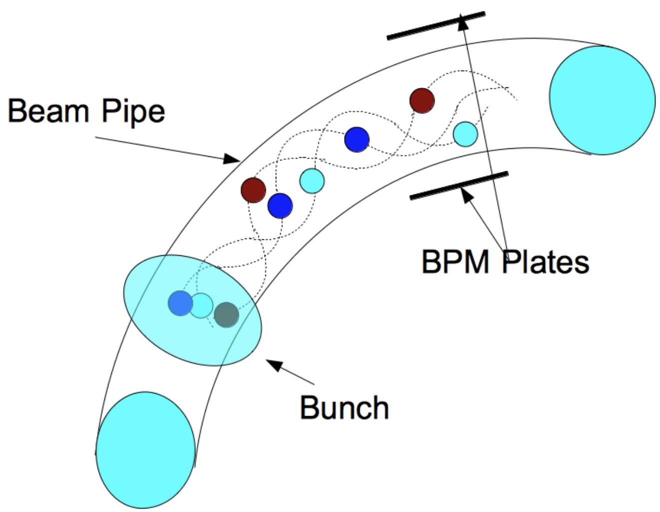
What do we measure?

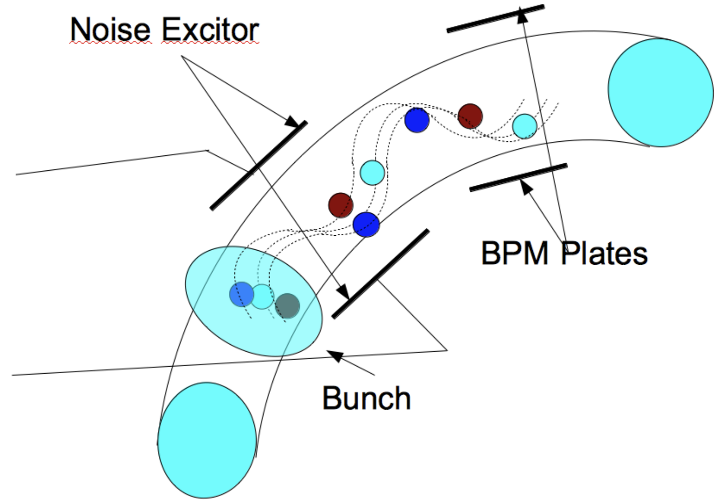




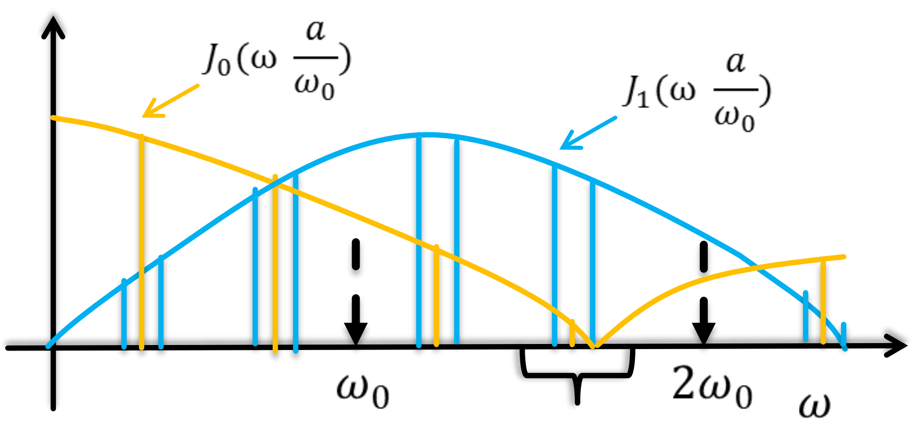


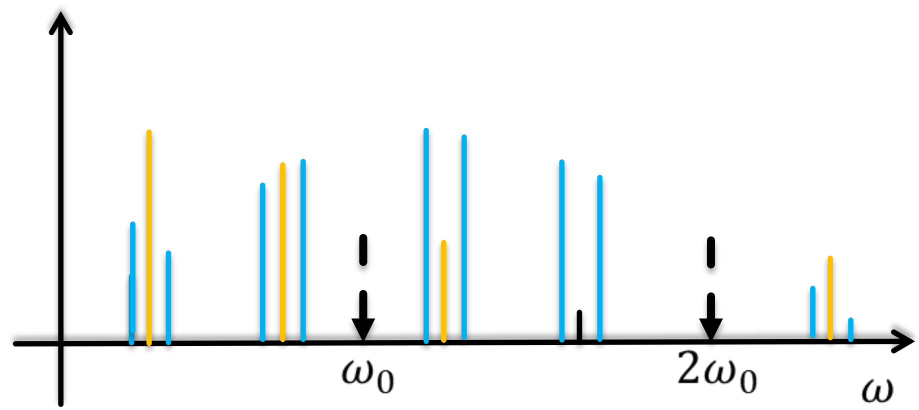
What do we measure? Coherent signals



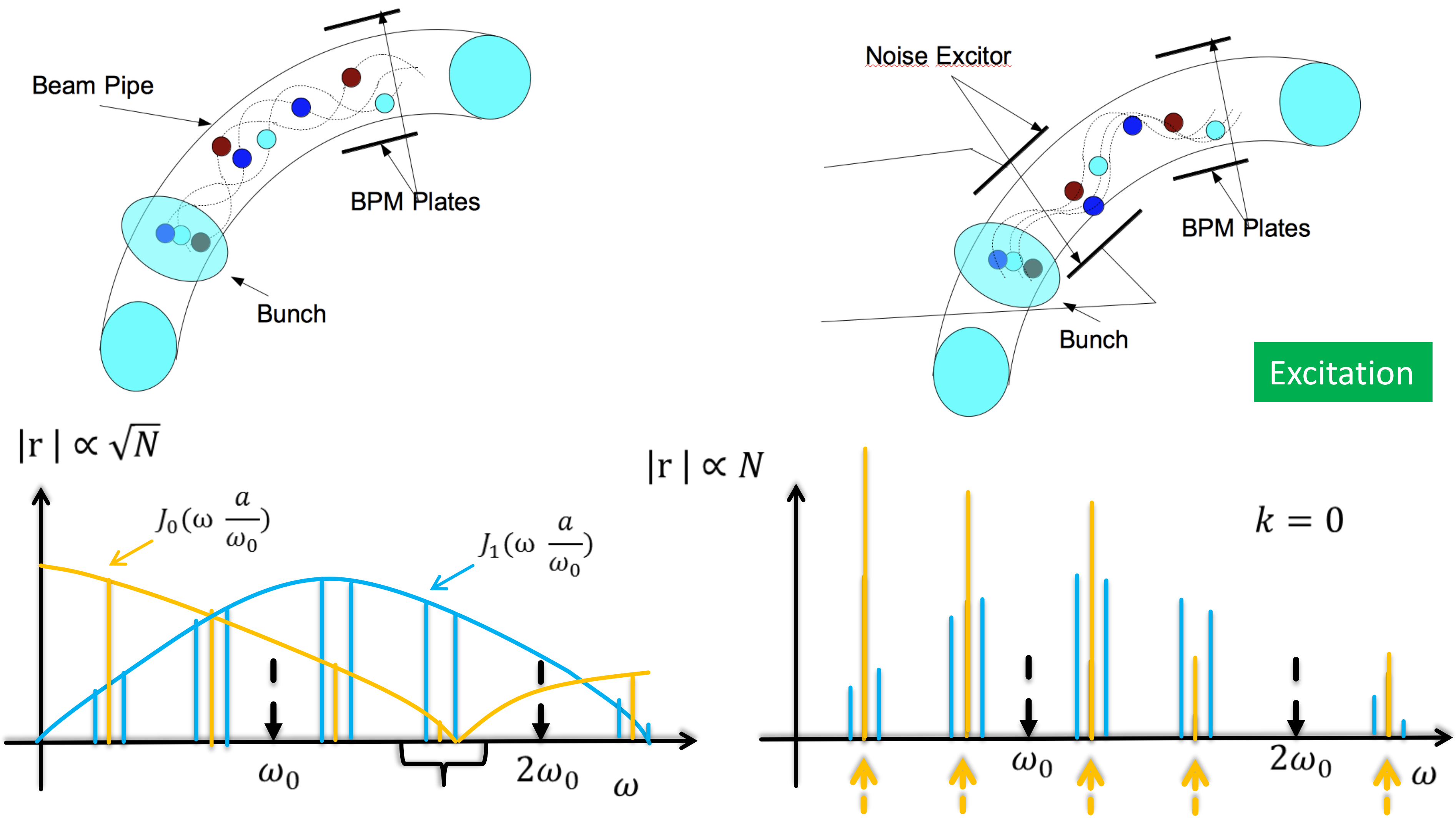


$|r| \propto \sqrt{N}$

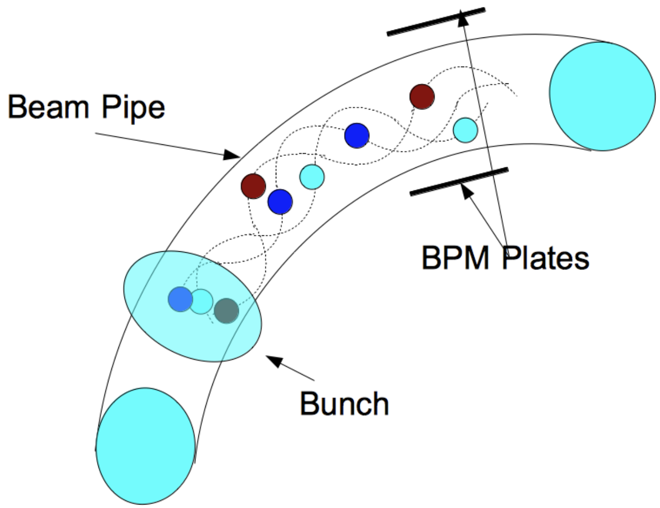


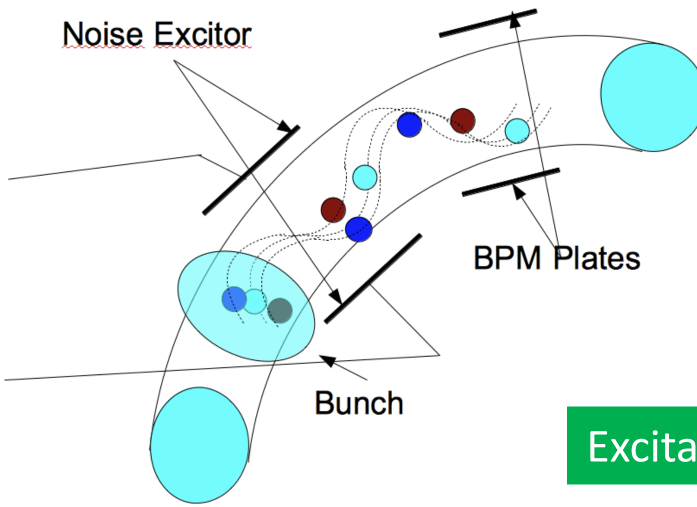


What do we measure? Coherent signals



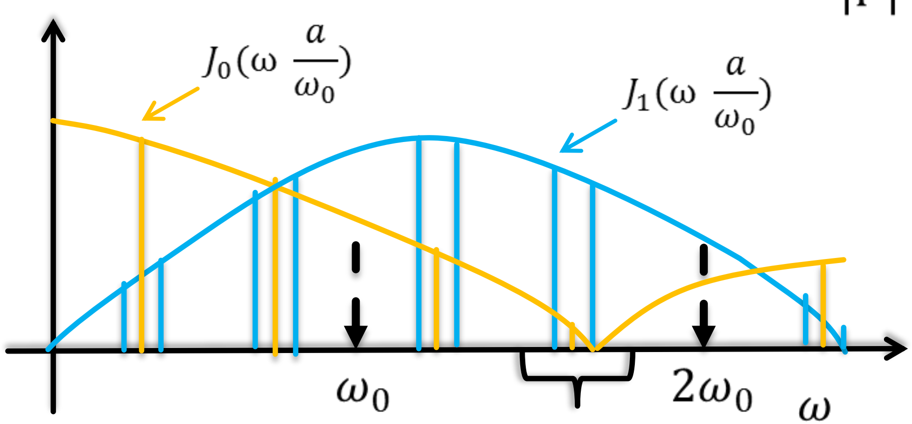
What do we measure? Coherent signals



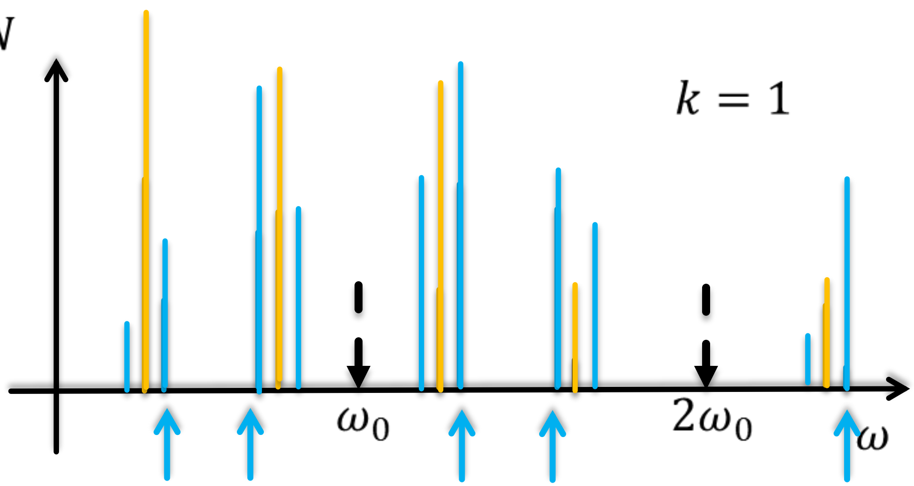


Excitation

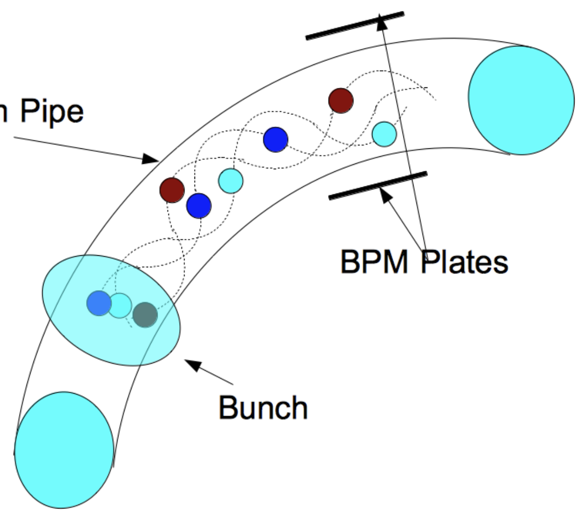
$|r| \propto \sqrt{N}$

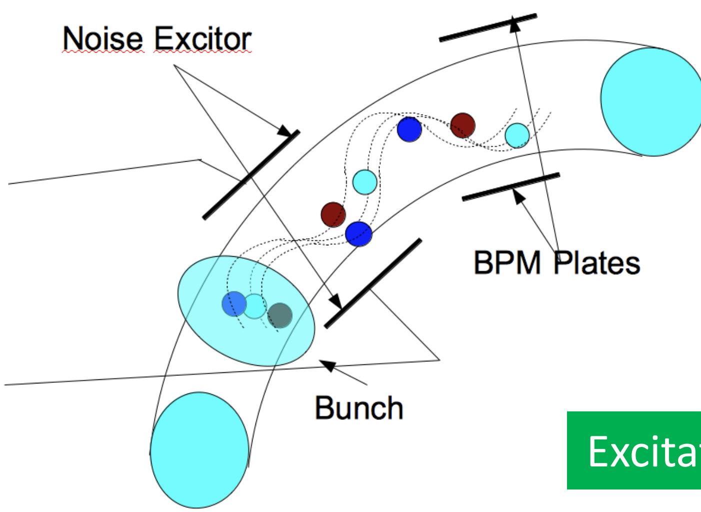


$|r| \propto N$

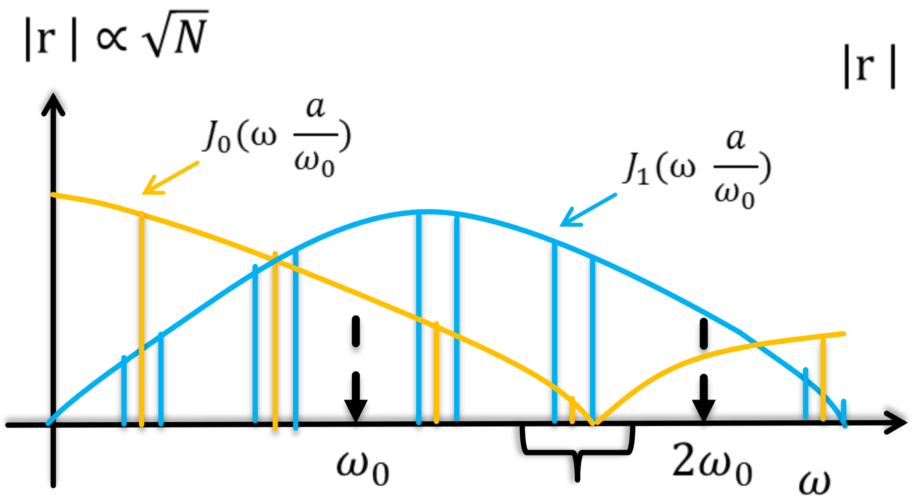


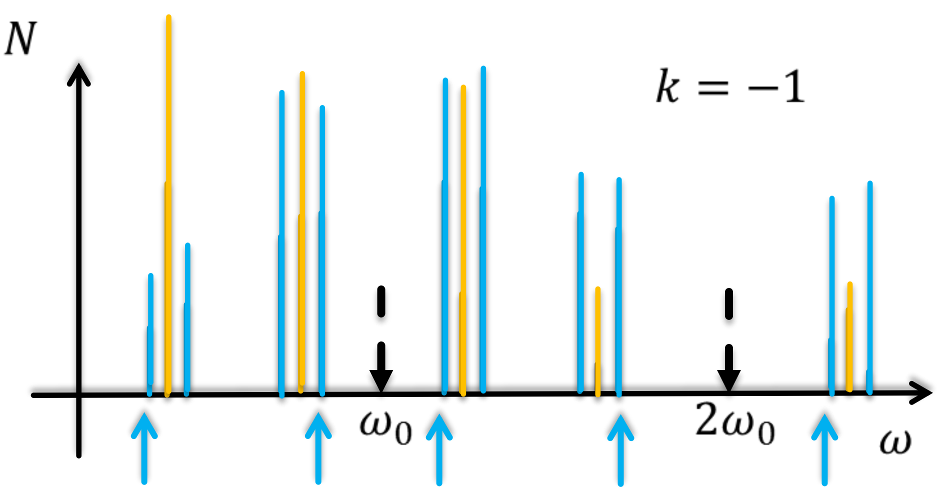
What do we measure? Coherent signals



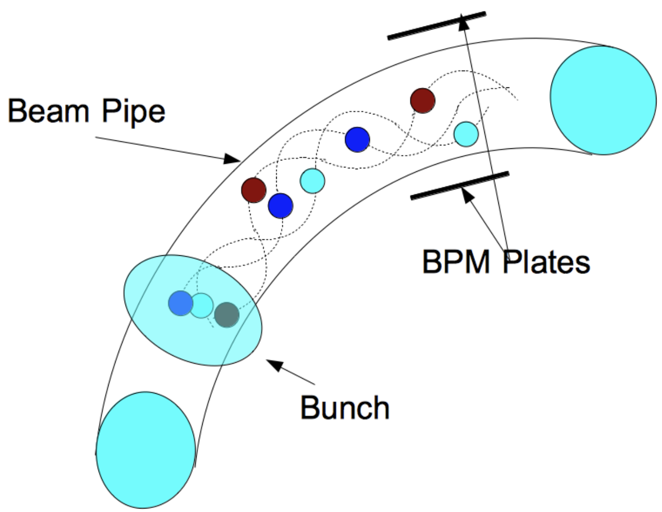


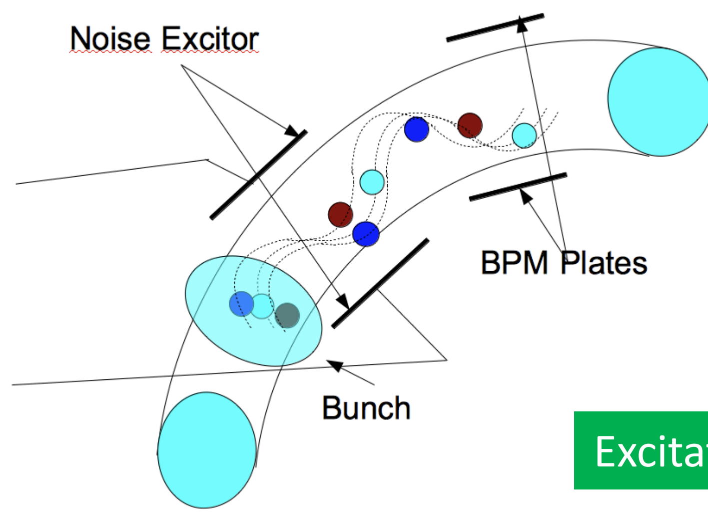
Excitation



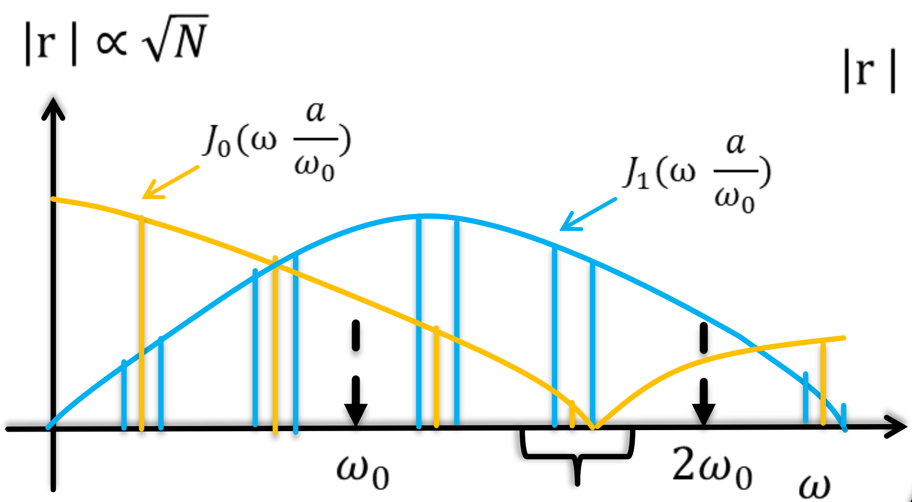


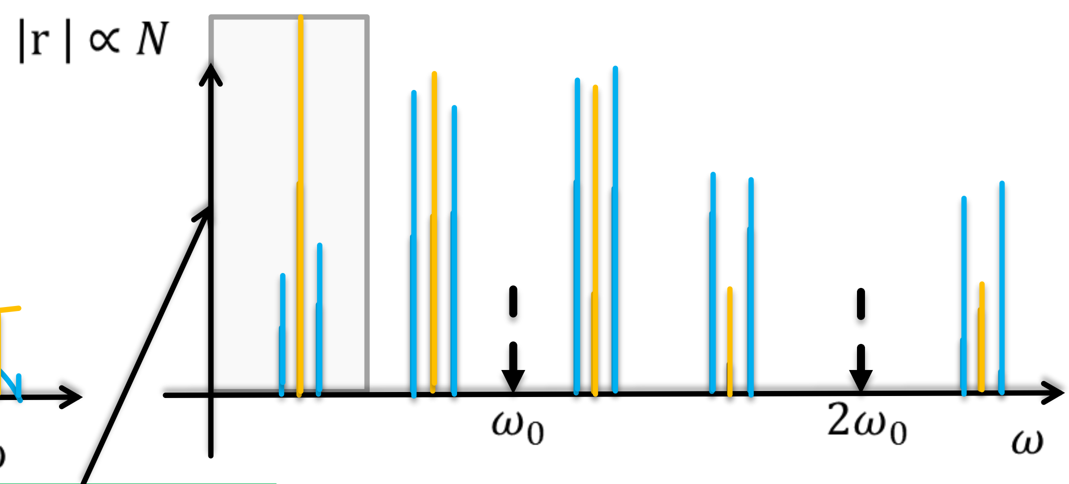
What do we measure? Coherent signals





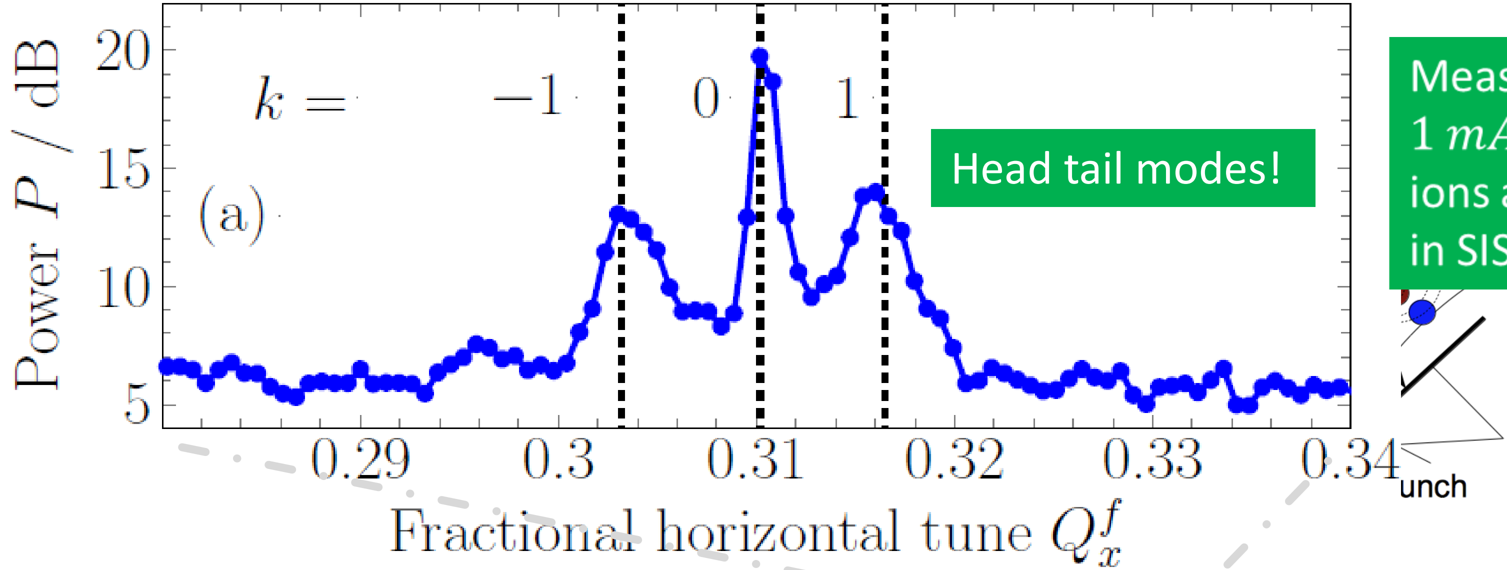
Excitation





Baseband spectra

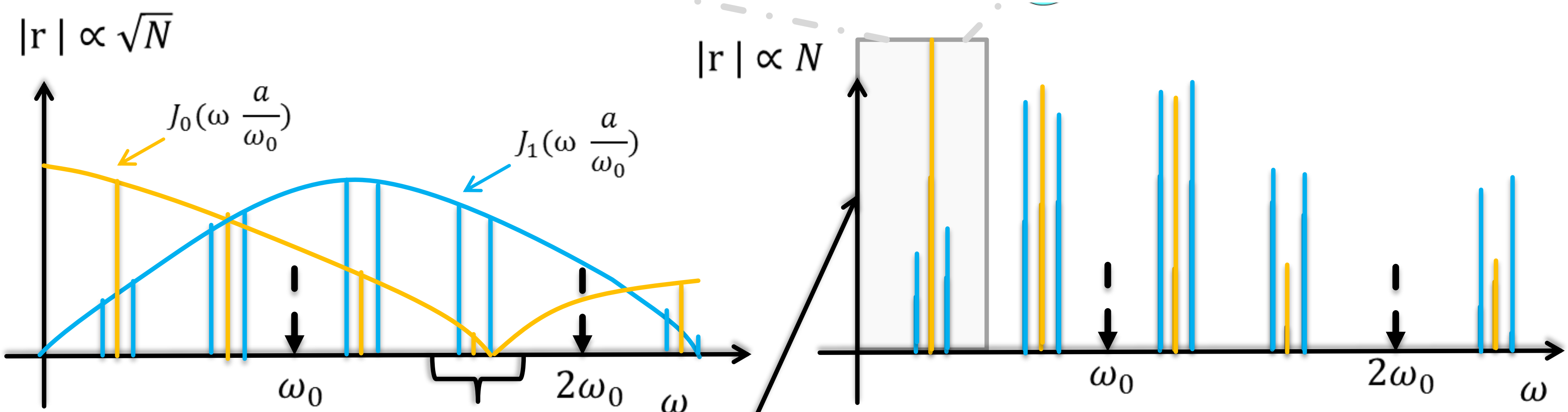
What do we measure? Coherent signals



Head tail modes!

Measurement with $1\ mA$ current of U^{73+} ions at injection energy in SIS-18

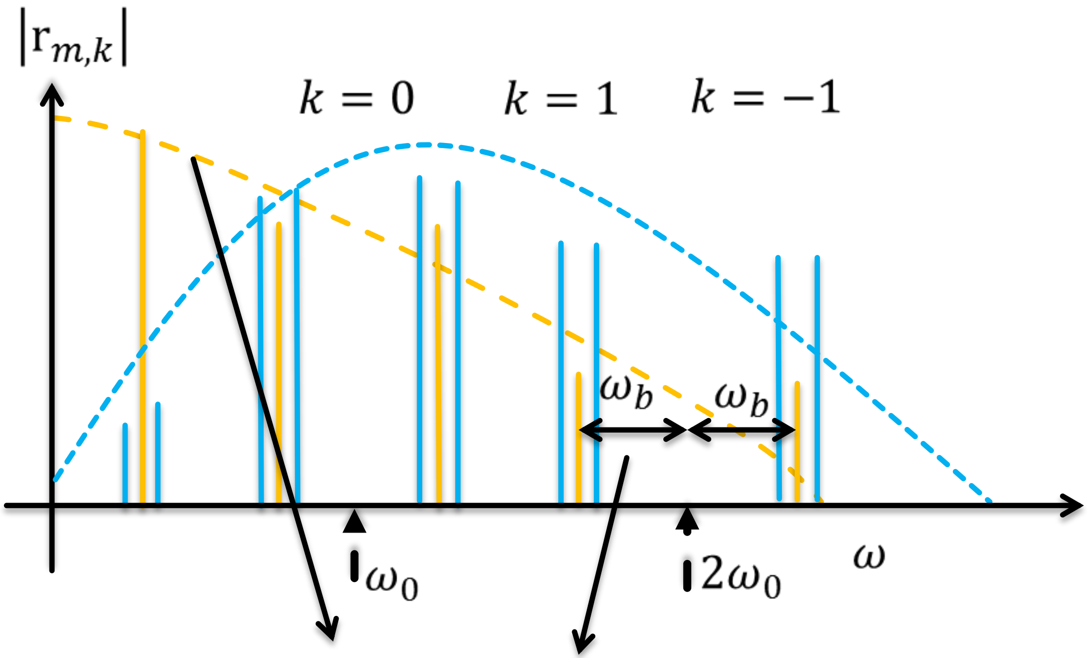
Excitation



Baseband spectra

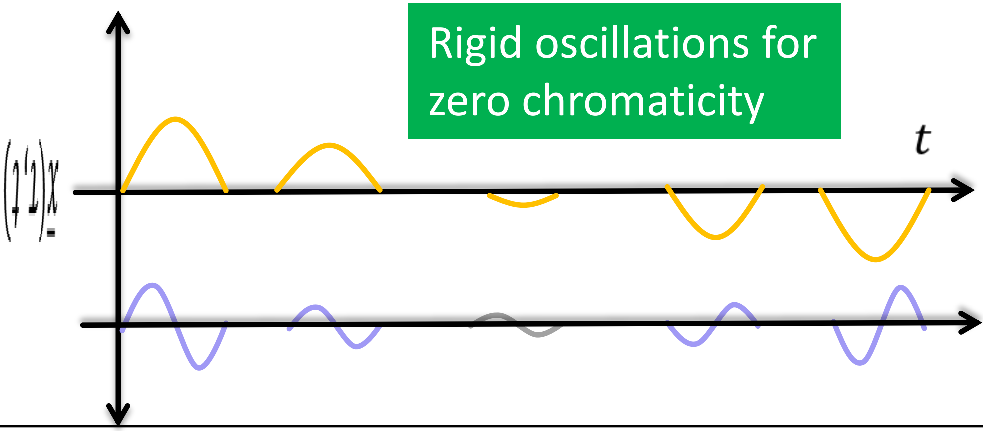
Coherent time domain signal

Non -zero chromaticity (non-rigid)

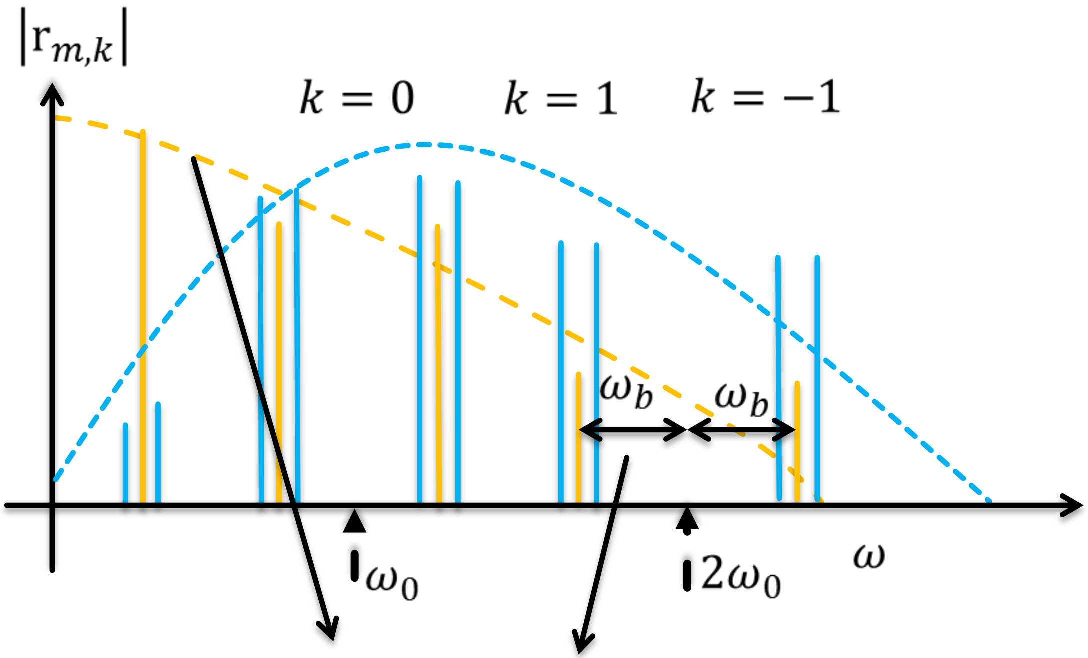


$$\bar{x}_k(\tau, t) = \bar{x}_k(\tau)\exp(j(\omega_b + k\omega_s)t)$$

Rigid oscillations for zero chromaticity

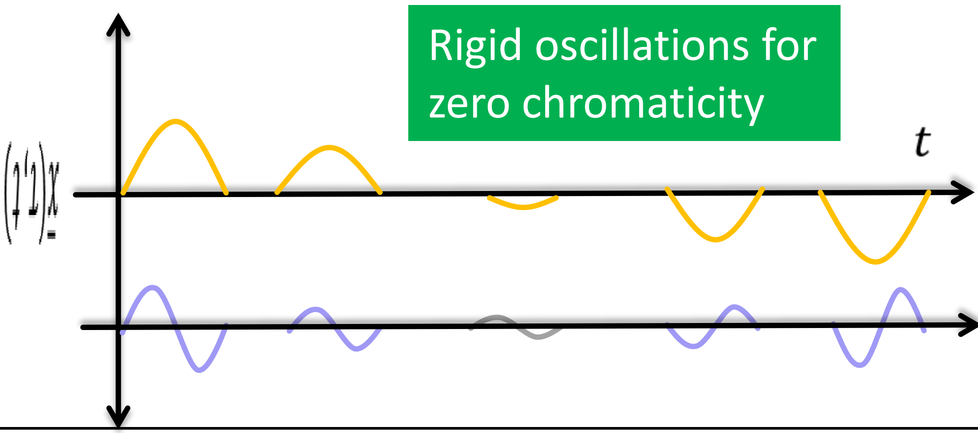


Coherent time domain signal

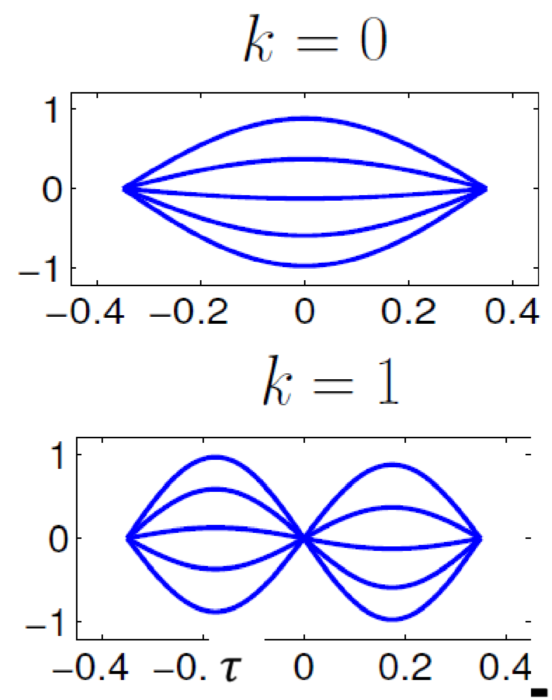


Non -zero chromaticity (non-rigid)

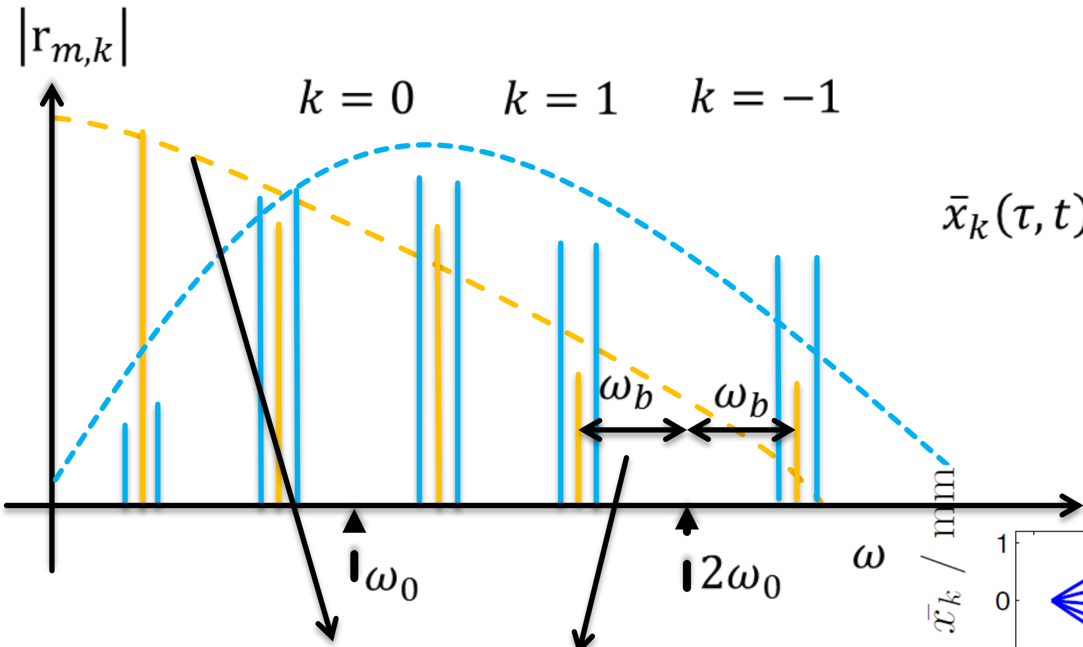
$$\bar{x}_k(\tau, t) = \bar{x}_k(\tau)\exp(j(\omega_b + k\omega_s)t)$$



Rigid oscillations for zero chromaticity

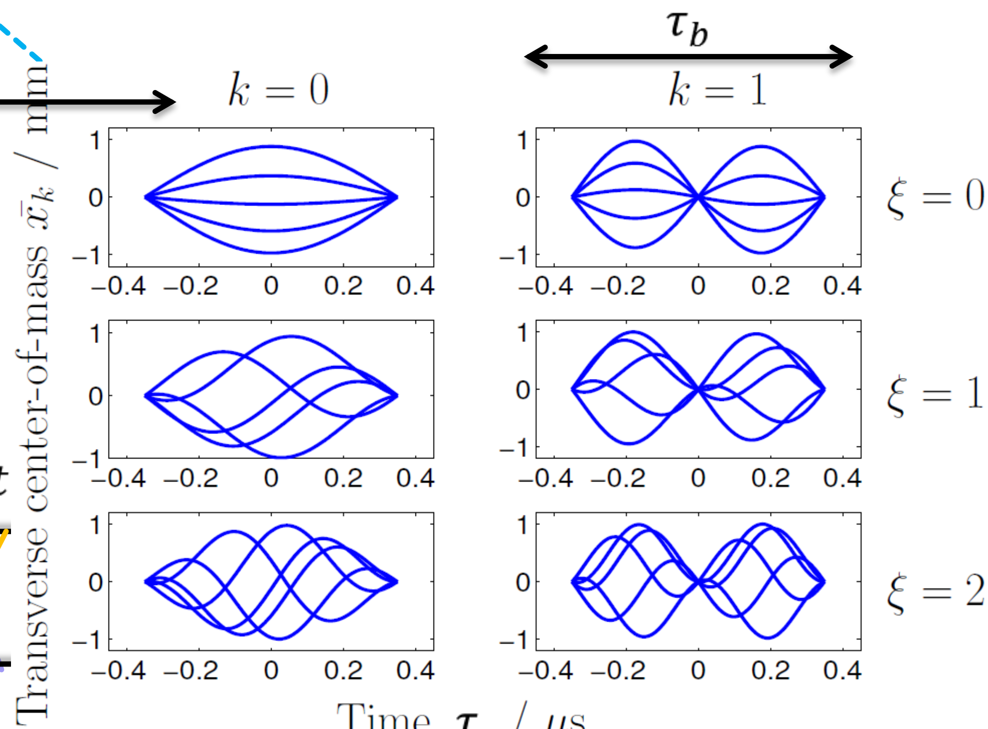


Coherent time domain signal



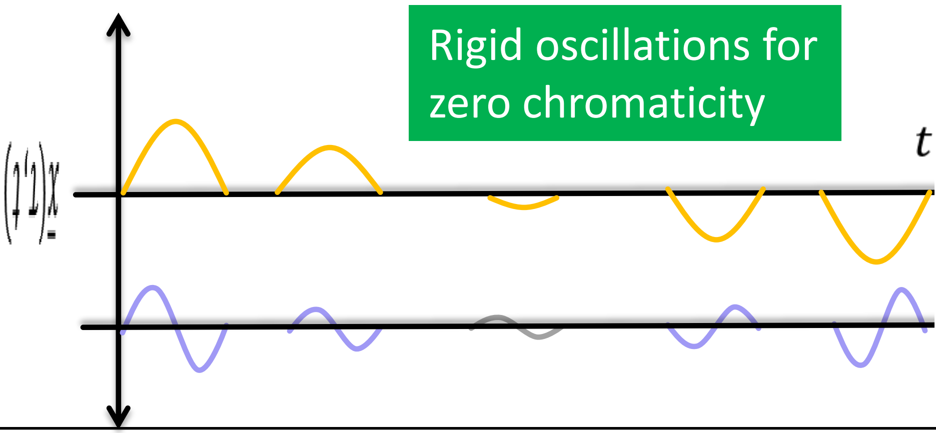
Non -zero chromaticity (non-rigid)

$$\bar{x}_k(\tau,t) = \bar{x}_k(\tau)\exp(j\omega_\xi\tau + j(\omega_b + k\omega_s)t$$



$$\bar{x}_k(\tau,t) = \bar{x}_k(\tau)\exp(j(\omega_b + k\omega_s)t)$$

Rigid oscillations for zero chromaticity



Recap : Introduction

- Transverse single particle signals have a rich spectra. Chromaticity shifts the amplitude envelope of the spectra.
- Transverse dipolar excitation aligns the phase of particles facilitating dipolar coherent motion.
- Typical tune measurement system measure head-tail modes in the baseband.

Head-tail modes at high intensity

- Head-tail mode frequencies for interacting particles are calculated by self-consistent evolution of the phase space taking all external forces into account. (Vlasov formalism)

- $$\underbrace{(Q_{k,coh} - Q_0 - kQ_s)\omega_0}_{\Delta Q_k \omega_0} = f(\Psi(x, x^{\cdot}, y, y^{\cdot}, z, z^{\cdot}, \mathrm{t}), Z_\perp)$$

6D phase space distribution and space charge

Transverse Impedance

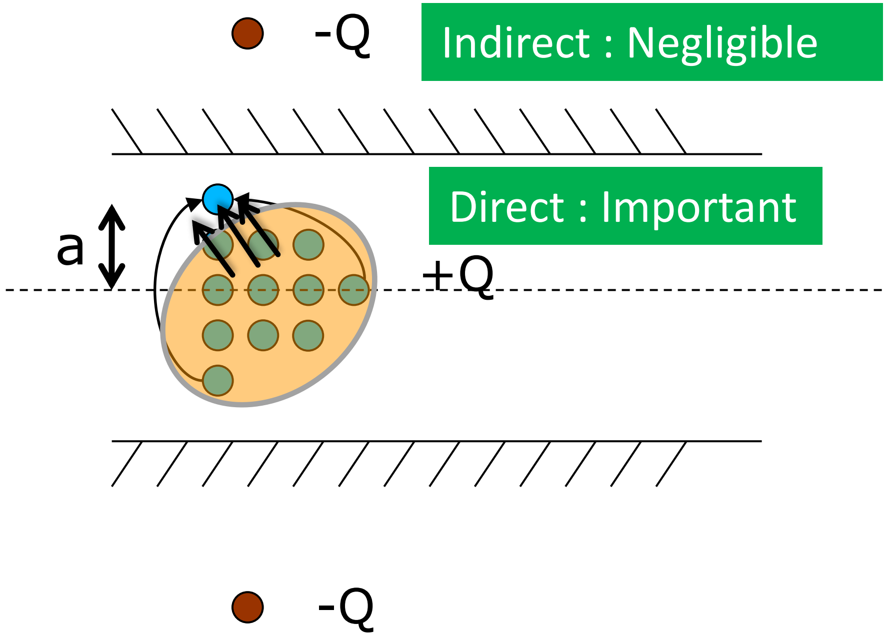
Usually solved numerically as an eigenvalue problem

What are

1) Space charge forces ?
2) Transverse coupling impedances ?

E. Metral, G. Rumolo : USPAS course on "Collective Effects in Beam Dynamics" in Albuquerque, New Mexico, USA

Space charge



- An incoherent effect.
- Inversely proportional to $\gamma_0{}^3$, prominent for low energy beams.

Space charge parameter!

$$q_{sc} = \frac{|\Delta Q_{sc}|}{Q_s}$$

For a uniformly distributed beam in the transverse phase space

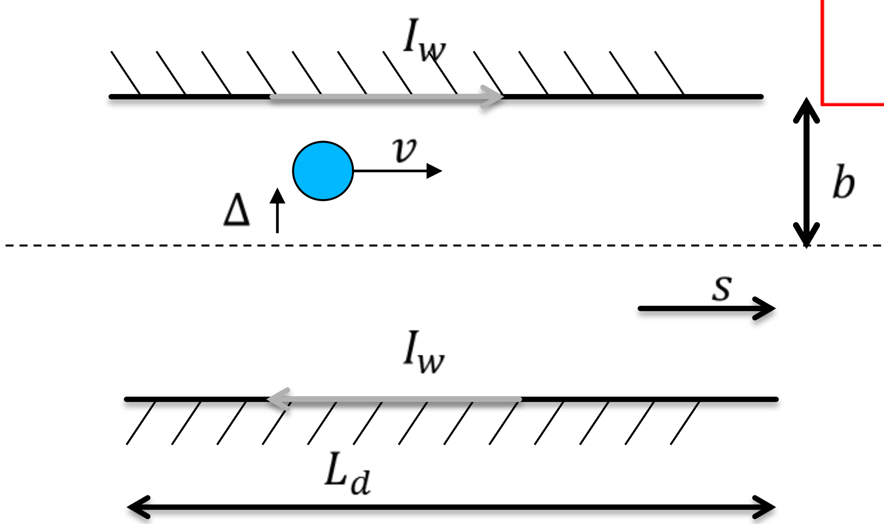
$$\boxed{\Delta Q_{sc} = \frac{q I_p R}{4\pi\epsilon_0 c W_0 \beta_0{}^3 \gamma_0{}^2 \varepsilon}}$$

Incoherent tune shift /tune spread

where,

I_p is the peak current
q is the particle charge
W_0 is the particle energy
R is the radius of the ring
β_0 is the relativistic beta
γ_0 is the relativistic gamma
$\varepsilon = \frac{a^2}{\beta}$ is the beam emittance

Transverse coupling impedances



$$Z_\perp(\omega) = -j\frac{\int_0^{L_d}(E(s,\omega) + v \times B(s,\omega))_\perp ds}{\beta I \Delta}$$

- Effect of beam on itself through its environment.
- In general, the impedance is complex and a function of frequency.
- $Im(Z_\perp)$ defines the frequency shift of the coherent modes. (Reactive)
- $\mathrm{Re}(Z_\perp)$ defines the growth rate of these modes. (Resistive)

Directly visible in tune spectra

For a perfectly conducting beam pipe, it reduces to

$$Z_\perp = -j\frac{Z_0}{2\pi{\beta_0}^2{\gamma_0}^2 b^2}$$

Characteristic impedance

$$\Delta Q_c = -j\frac{qI_pR^2Z_\perp}{2Q_0\beta_0W_0} \propto \frac{\Delta Q_{sc}a^2}{b^2}$$

Coherent tune shift

Head-tail modes at high intensity

$$\underbrace{(Q_{k,coh} - Q_{y0} - kQ_s)\omega_0}_{\boxed{\Delta Q_k \omega_0}} = f(\Psi(x, x^{\cdot}, y, y^{\cdot}, z, z^{\cdot}, \mathrm{t}), Z_{\perp})$$

6D phase space distribution and space charge

Transverse Impedance

Analytical solution of the Vlasov equation for ``square well airbag model", taking space charge forces into account gives the following eigenmodes,

$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc} - \Delta Q_c)^2}{4} + (kQ_s)^2}$$

For $\mathrm{k} = 0,\ \Delta Q_k = \Delta Q_c$

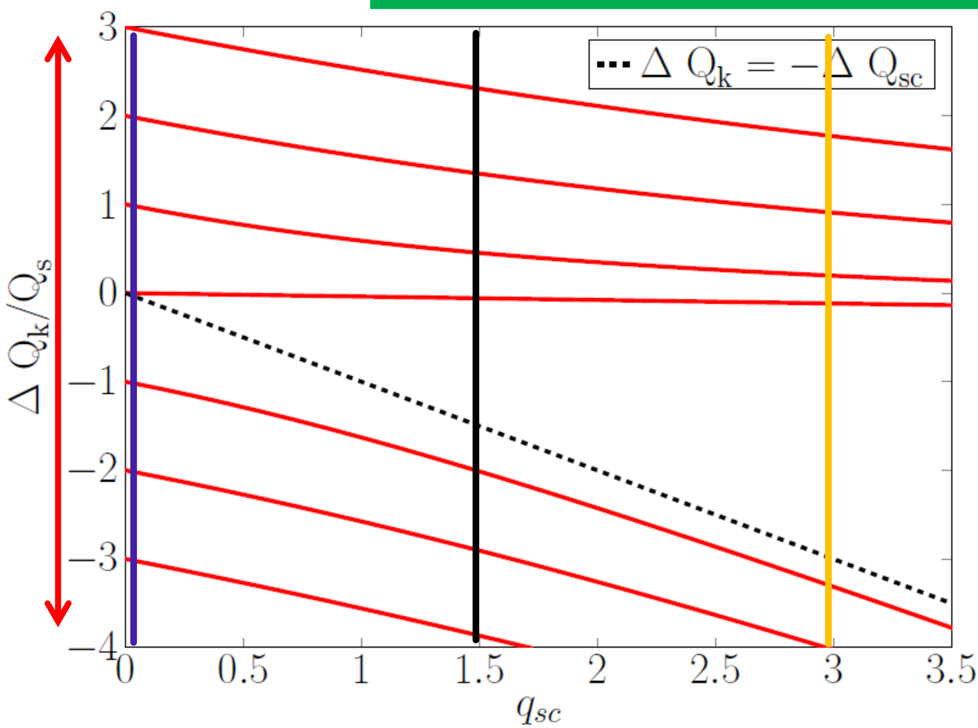
For $\mathrm{k} \neq 0,\ \Delta Q_k = f(\Delta Q_{sc})$

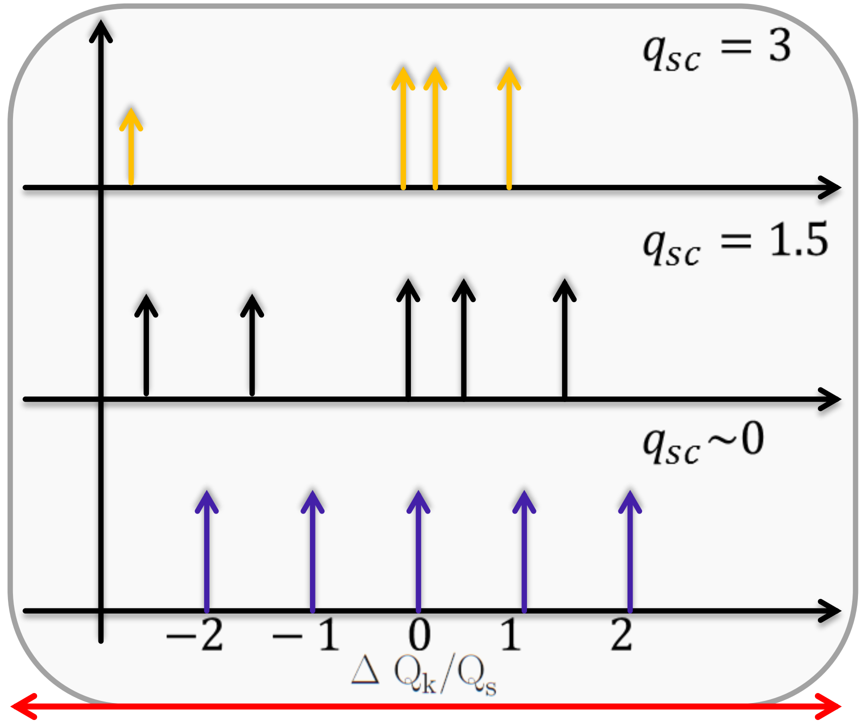
M. Blaskiewicz, "Fast head tail instability with space charge", Phys. Rev ST Accel. Beams 1, 044201, (1998)

Head-tail modes at high intensity

$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc}+\Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc}-\Delta Q_c)^2}{4} + (kQ_s)^2} \qquad q_{sc} = \frac{|\Delta Q_{sc}|}{Q_s}$$

As ΔQ_{sc} increases, the head-tail modes come closer for $k > 0$





A. Burov, "*Head-tail modes for strong space charge*", Phys. Rev ST Accel. Beams 12, 044202, (2009)
O. Boine-Frankenheim and V. Kornilov, "Transverse Schottky noise spectrum for bunches with space charge", Phys. Rev ST Accel. Beams 12, 114201, (2009)

Recap: tune spectra at high intensity

- The tune spectra of space charge dominated beams is significantly modified!

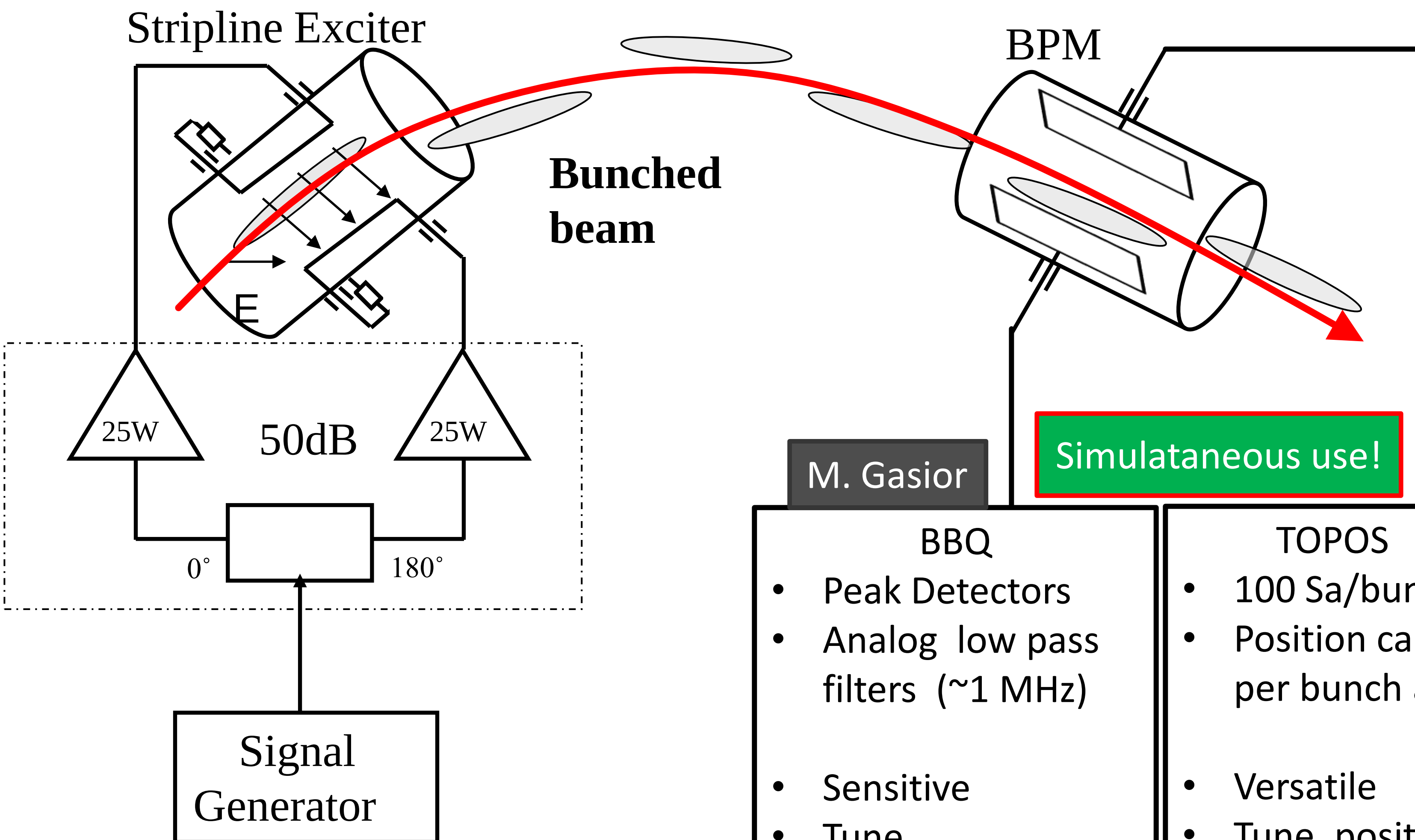
- The results are valid for realistic beam distributions shown by recent theoretical studies which also suggest (Landau) damping of $(-k)$ modes.

- Mode-structure similar to no space charge case i.e. mainly dependent on chromaticity.

$$\bar{x}_k(\tau, t) = \cos(\frac{k\pi\tau}{\tau_b}) \exp(j\omega_\xi \tau) \exp(j(\omega_b + k\omega_s)t \longrightarrow \text{From slide 11}$$

A. Burov, ”*Head-tail modes for strong space charge”*, Phys. Rev ST Accel. Beams 12, 044202, (2009)
O. Boine-Frankenheim and V. Kornilov, “Transverse Schottky noise spectrum for bunches with space charge”, Phys. Rev ST Accel. Beams 12, 114201, (2009)

(Typical) Tune measurement systems



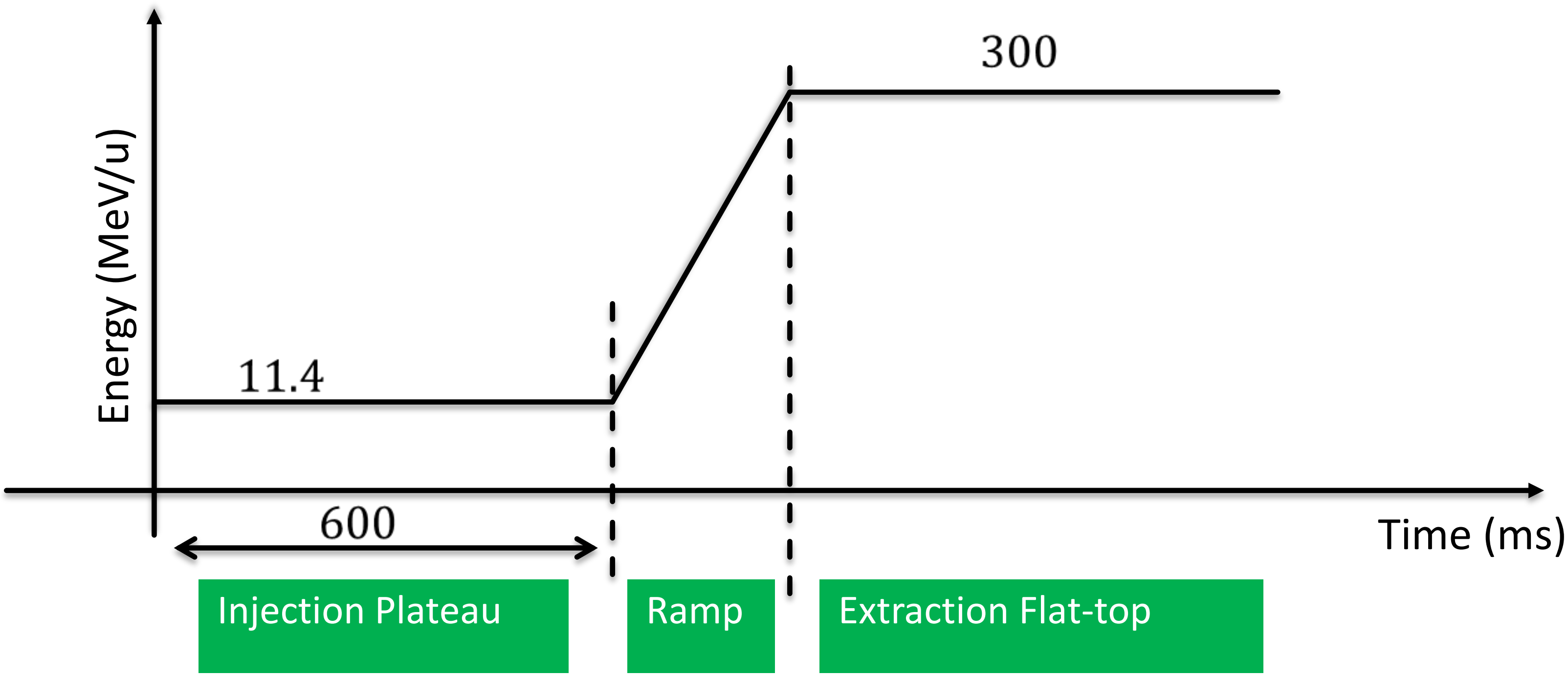
BBQ

- Peak Detectors
- Analog low pass filters (~1 MHz)
- Sensitive
- Tune

TOPOS

- 100 Sa/bunch
- Position calculation per bunch and FFT
- Versatile
- Tune, position, bunch structure

Beam experiments at GSI SIS-18



- Measurements primarily on injection plateau, due to high space charge effects
- Only measurements from selected beam times with U^{73+} and N^{7+} ions are shown

Measurement: beam parameters

Parameter	Symbols	Value	value
		U^{73+} beam	N^{7+} beam
Energy	W_{kin}	11.4 MeV/u	11.56 MeV/u
No. of particles	N_p	$1 \cdot 10^8 - 12 \cdot 10^8$	$1 \cdot 10^9 - 15 \cdot 10^9$
Emittance	$\varepsilon_x, \varepsilon_y (2\sigma)$	45,22 mm-mrad	33,12 mm-mrad
Bunching factor	B_f	0.4	0.37
Synchrotron tune	Q_{s0}, Q_{s1}	0.007, 0.0065	0.006, 0.0057
Tune	Q_{x0}, Q_{y0}	4.31, 3.27	4.16, 3.27
Chromaticity	ξ_x, ξ_y	$-0.94, -1.85$	$-0.94, -1.85$

Longitudinal Schottky

DC-CT

IPM

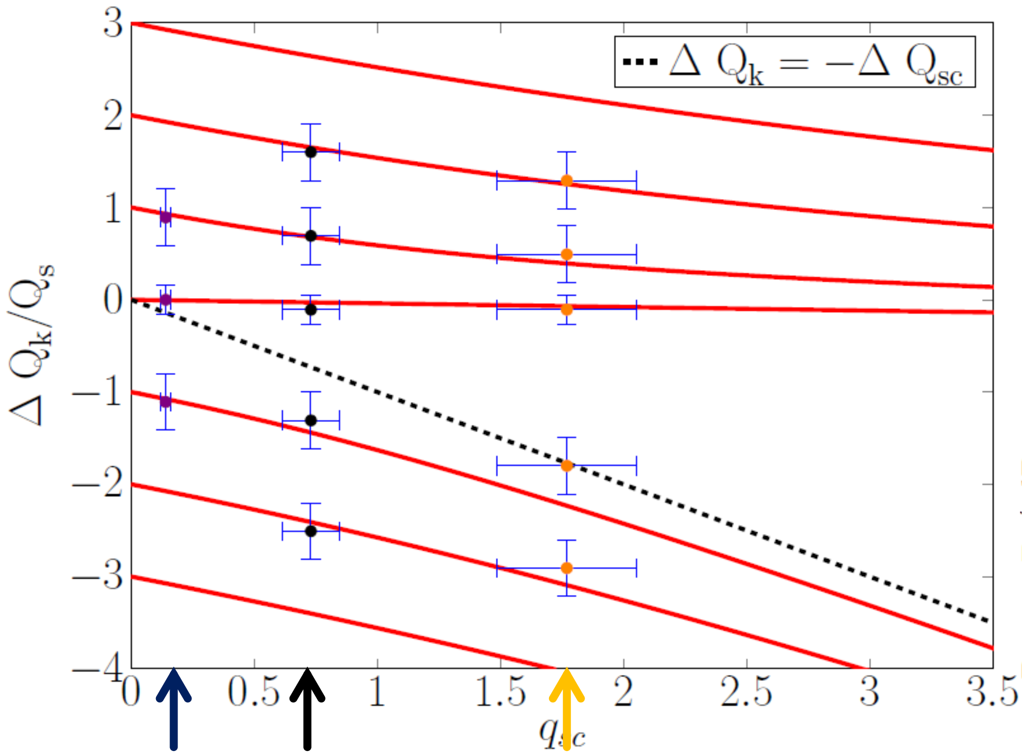
TOPOS

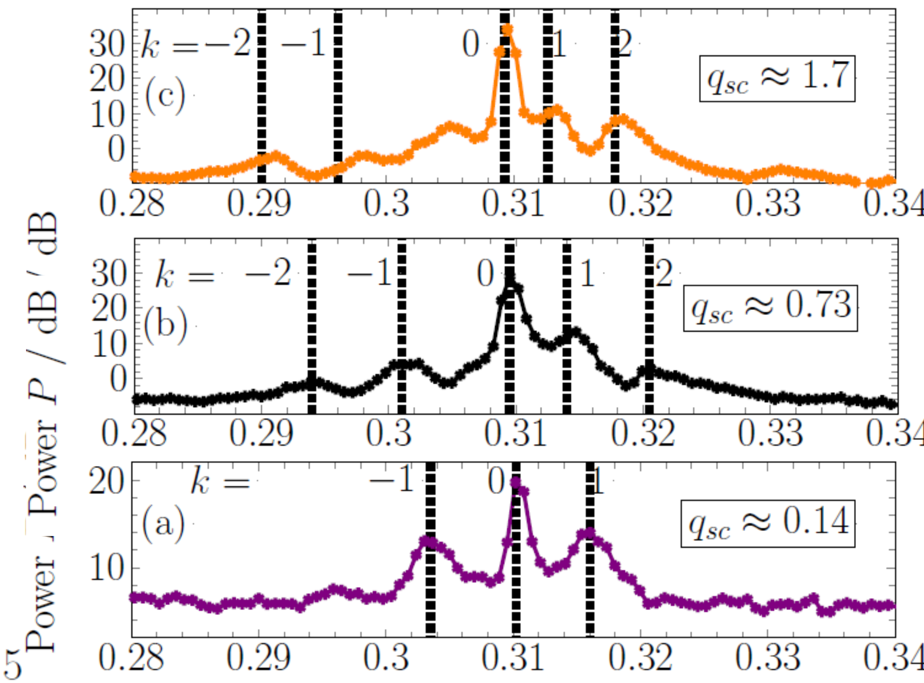
Measurement of ALL relevant beam parameters to calculate q_{sc}

$$q_{sc} = \frac{\Delta Q_{sc}}{Q_s} = \frac{q I_p R}{4\pi\epsilon_0 c W_0 {\beta_0}^3 {\gamma_0}^2 \varepsilon Q_s}$$

Tune spectra are measured by BBQ and TOPOS

Modification of tune spectrum for U^{73+} beam





Predicted spectra : red lines

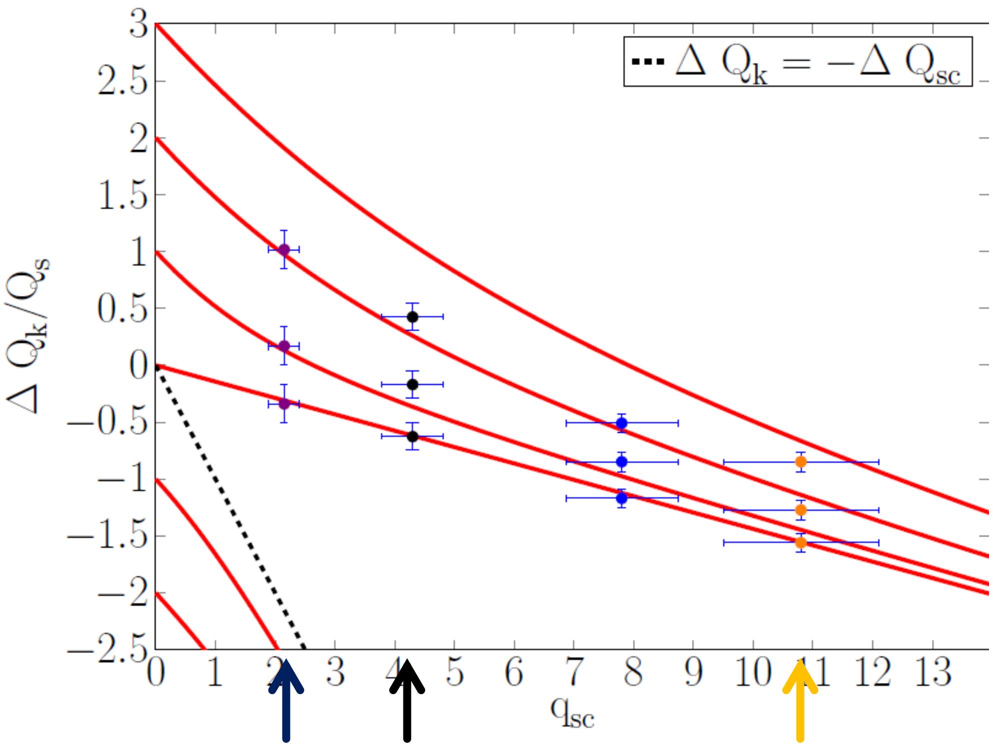
Measured spectra

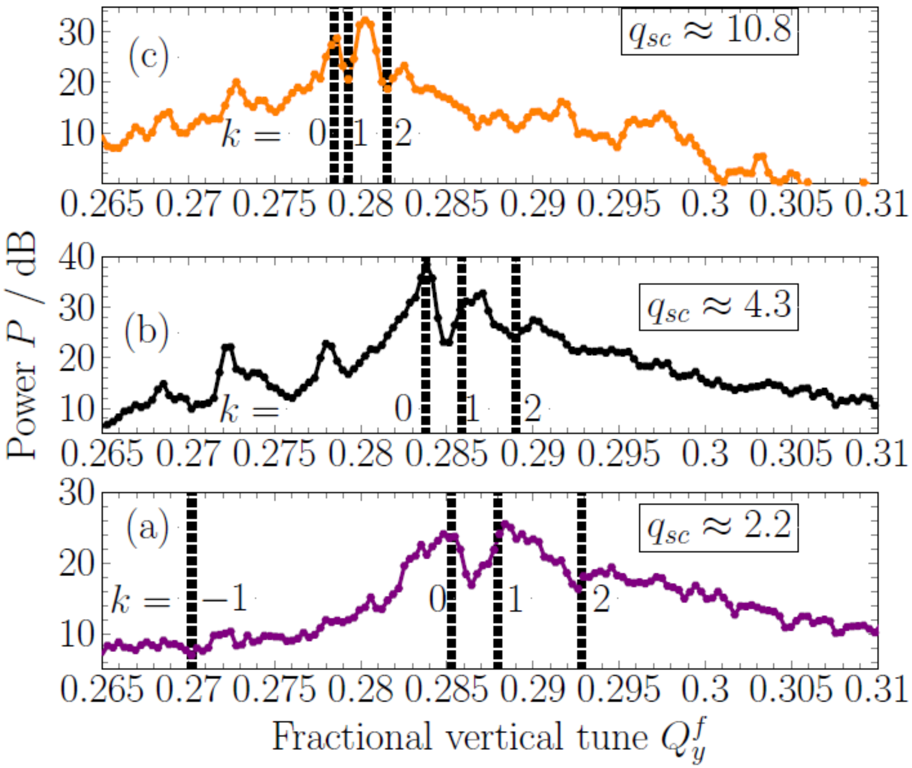
$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc} - \Delta Q_c)^2}{4} + (kQ_s)^2}$$

$$q_{sc} = \frac{|\Delta Q_{sc}|}{Q_s}$$

MEASUREMENTS : R. Singh et al., Interpretation of transverse tune spectrum in a heavy-ion synchrotron, Phys. Rev ST Accel. Beams 13, March, (2013)

Higher space charge with N^{7+} beam





Predicted spectra : red lines

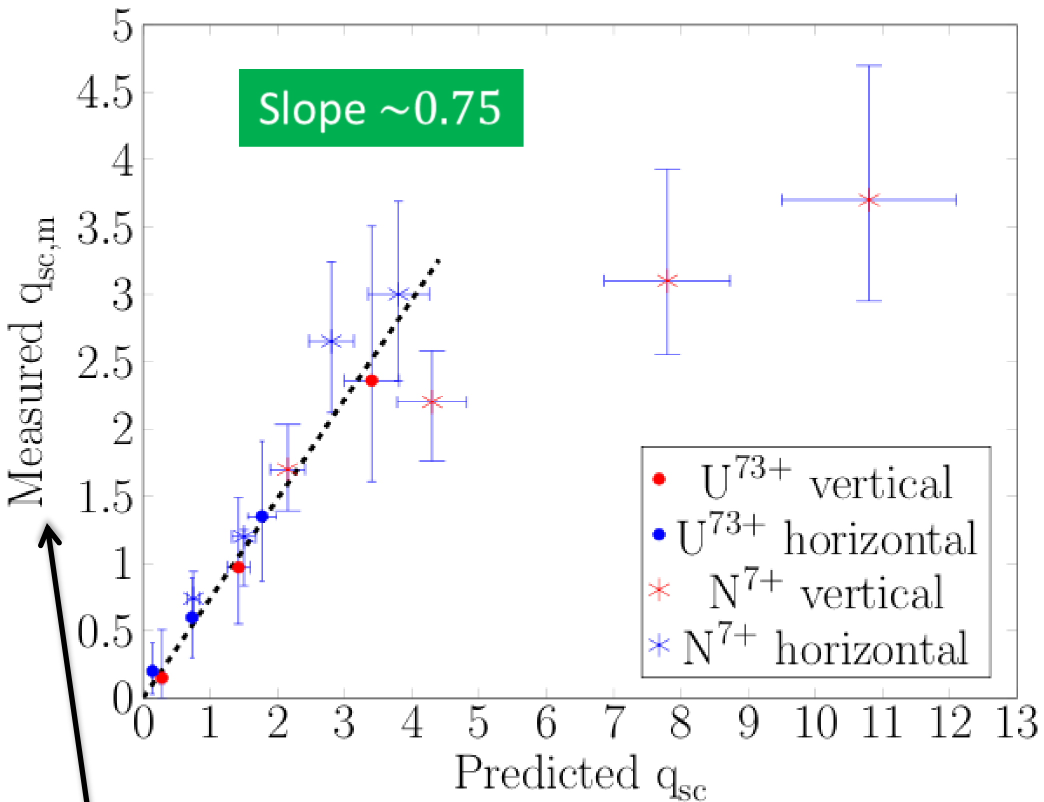
Measured spectra

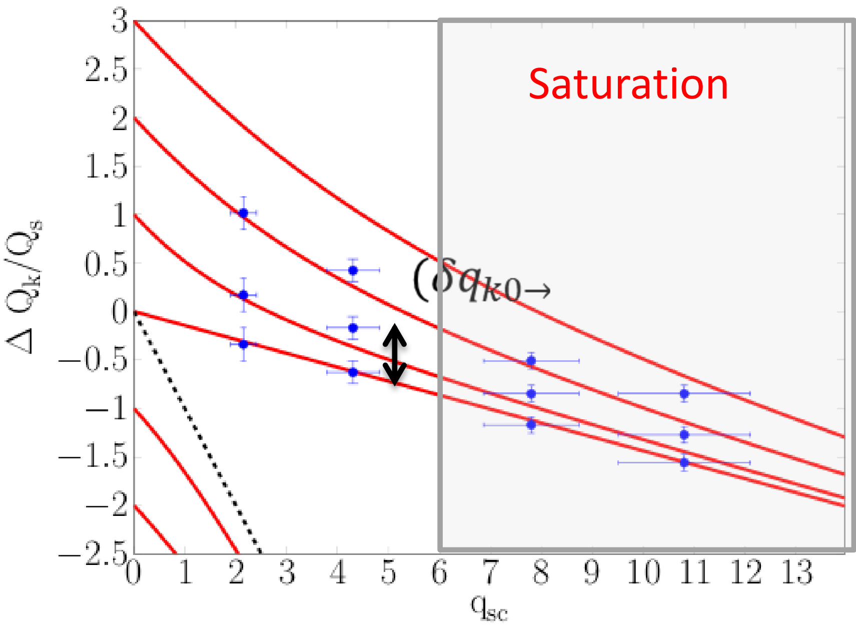
$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc} - \Delta Q_c)^2}{4} + (kQ_s)^2}$$

$$q_{sc} = \frac{|\Delta Q_{sc}|}{Q_s}$$

MEASUREMENTS : R. Singh et al., Interpretation of transverse tune spectrum in a heavy-ion synchrotron, Phys. Rev ST Accel. Beams 13, March, (2013)

Measurement of incoherent tune shifts



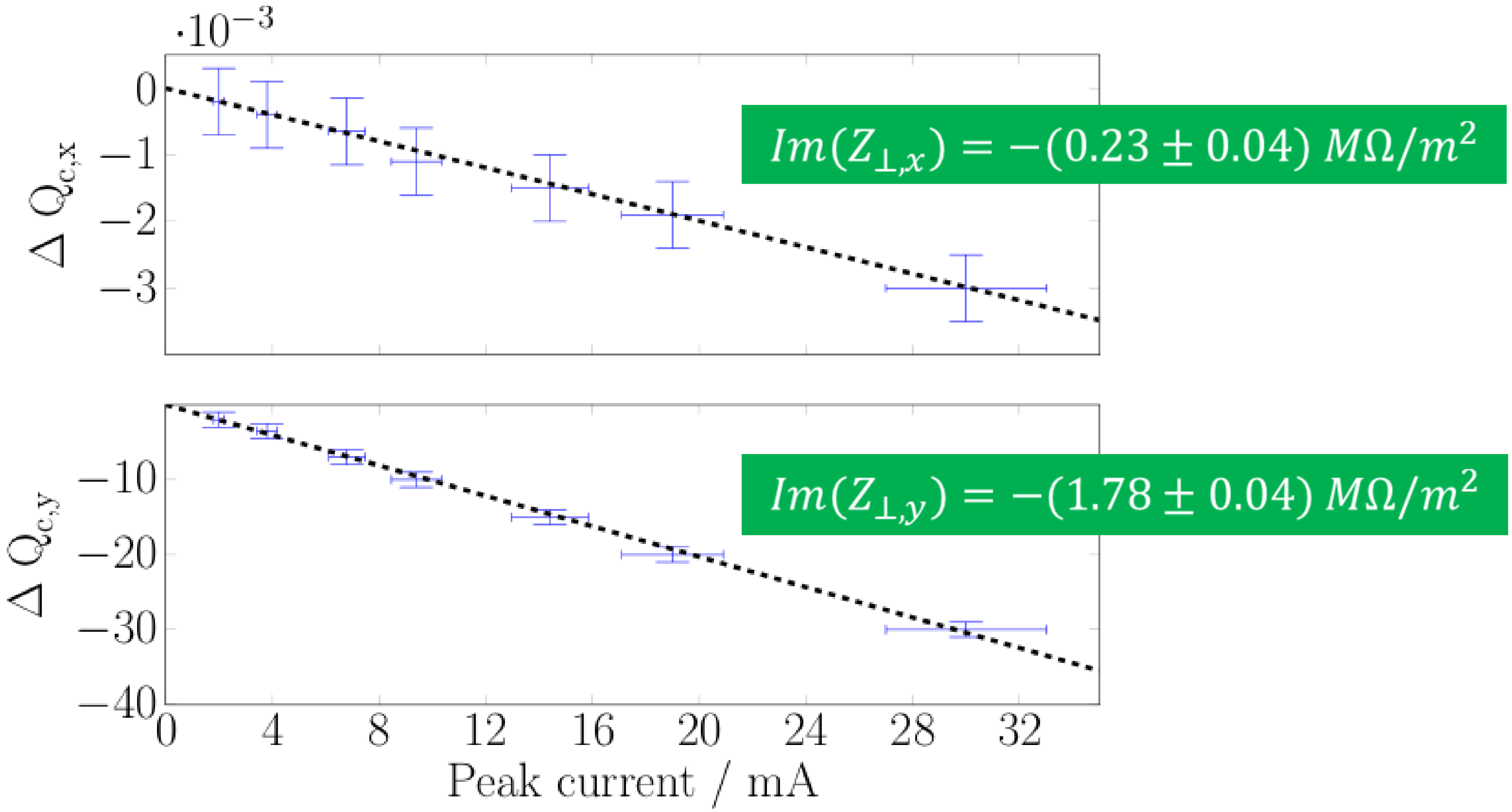


Predicted $q_{sc} = \dfrac{qI_pR}{4\pi\epsilon_0 cW_0\beta_0{}^3\gamma_0{}^2\varepsilon Q_s}$ based on measurements of various beam parameters

Measured q_{sc} based on the distance between lines of modes $k =$ 0 and 1 $(\delta q_{k0\rightarrow}$

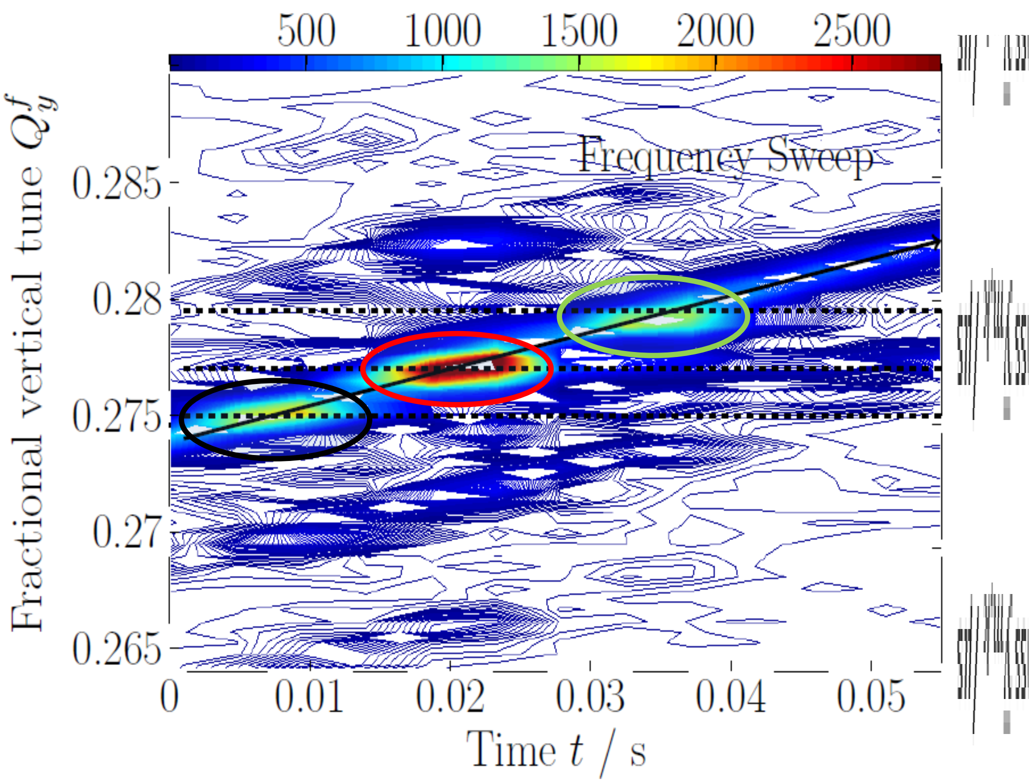
First measurements of this type, but works well only for $q_{sc} \leq 5$

Measurement of transverse impedances

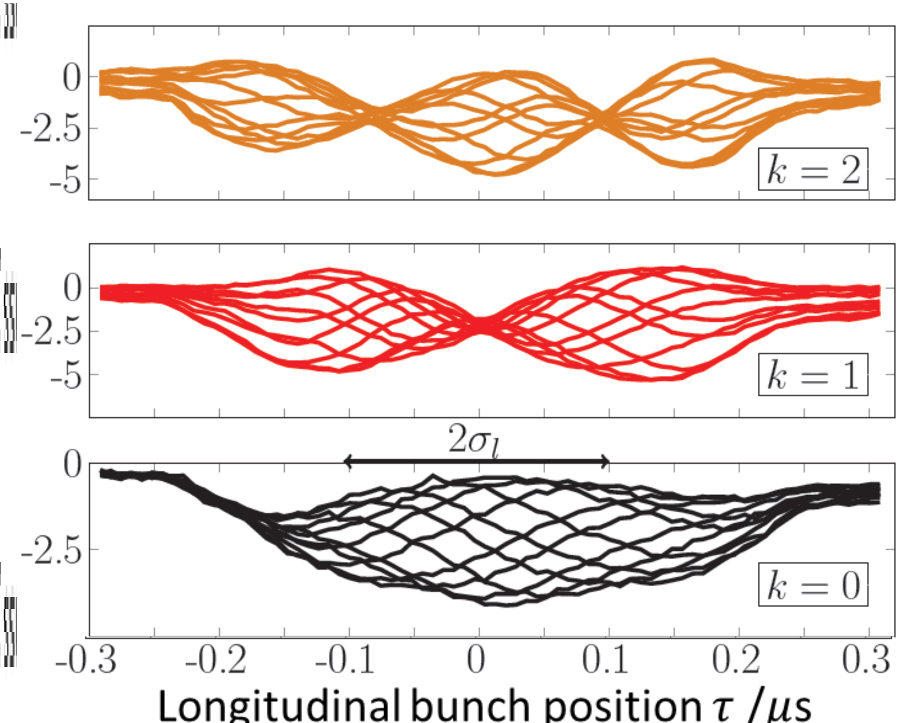


- Slope of coherent tune shifts ΔQ_c ($\Delta Q_{k=0}$) versus peak beam current I_p
- Average beam pipe radius can be calculated (~ 115 and 35 mm)

Time domain : head-tail modes



Time domain structure



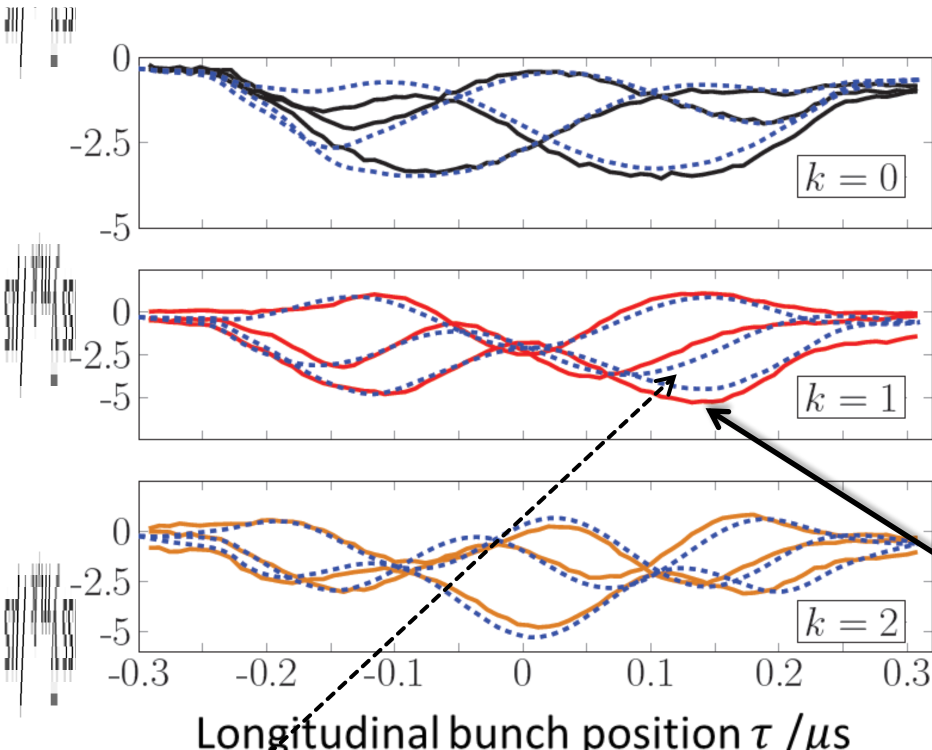
Sweep Excitation allows excitation of individual head-tail modes

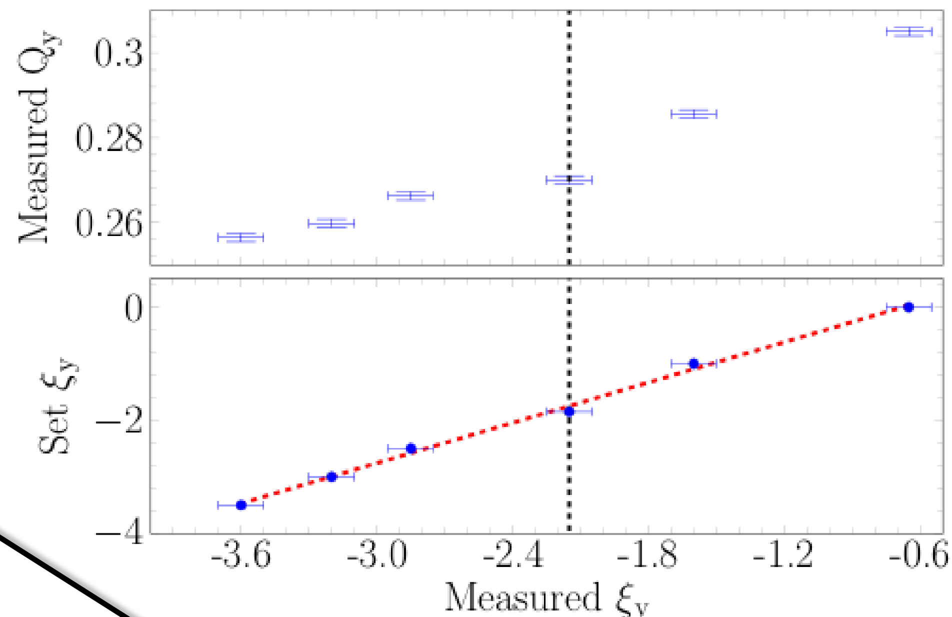
The mode-structure verifies the spectra interpretation

Animation!

Adobe Acrobat Document

Chromaticity measurements





$$F_k(\xi,\tau,A) = S(\tau)\left(\Delta y + A\bar{y}_k(\xi,\tau)\right)$$

Longitudinal profile

Beam offset

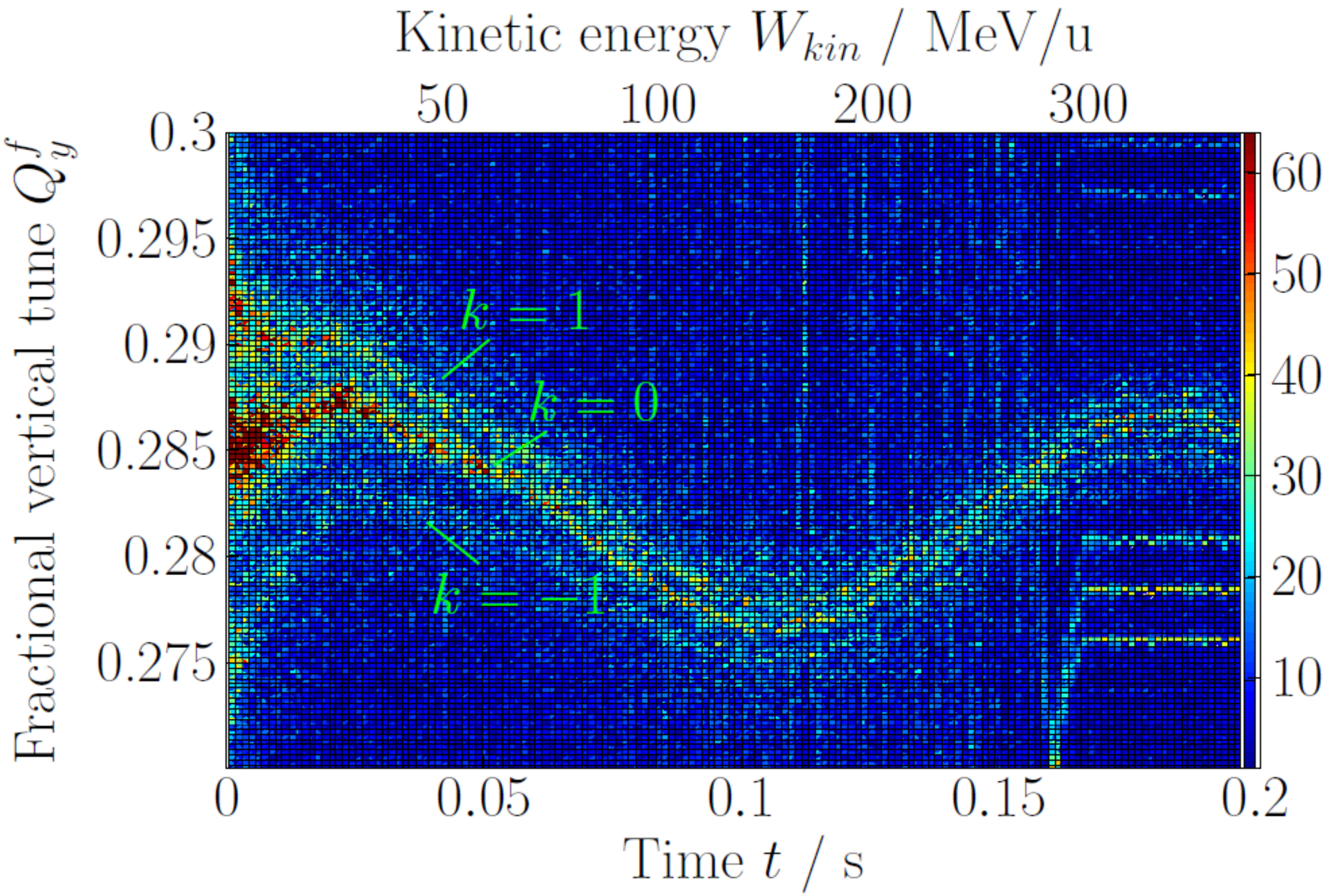
Analytical mode-structure

$$E(\xi,A) = (\bar{y}_{m,k} - F_k(\tau,\xi,A)\,)^2$$

Minimize $E(\xi,A)$ as a function of ξ and A

Similar principle: D. Cocq et al., “The Measurement of Chromaticity via a Head-Tail Phase Shift” , BIW’98

Head-tail modes during acceleration



The mode shifts also visible during acceleration !

Where and why?

Where could these effects be relevant?

At most hadron synchrotrons, especially at injection.

Parameter	SIS-18	SPS	RHIC	ANKA (B)
Peak current / mA	10	1400	500	12
Emittance / $mm - mrad$	22	0.2	10	0.153
Relativistic gamma (γ)	~ 1	~ 27	~ 4	~ 100
ΔQ_{sc}	-0.05	-0.1	-0.02	-10^{-4}
Q_s	0.007	0.015	0.0015	0.008
q_{sc}	~ 7	~ 7	~ 12	~ 0

For $q_{sc} \gtrsim 1$, the space charge effects will play a role!

Where and why?

Where could these effects be relevant?

At most hadron synchrotrons, especially at injection.

Why are these measurements relevant?

- If the coherent spectrum is modified by high intensity effects, they can be measured and studied.

- Feedback systems using tune spectra as input might have problems. Understanding of spectra needed.

  An example from RHIC!

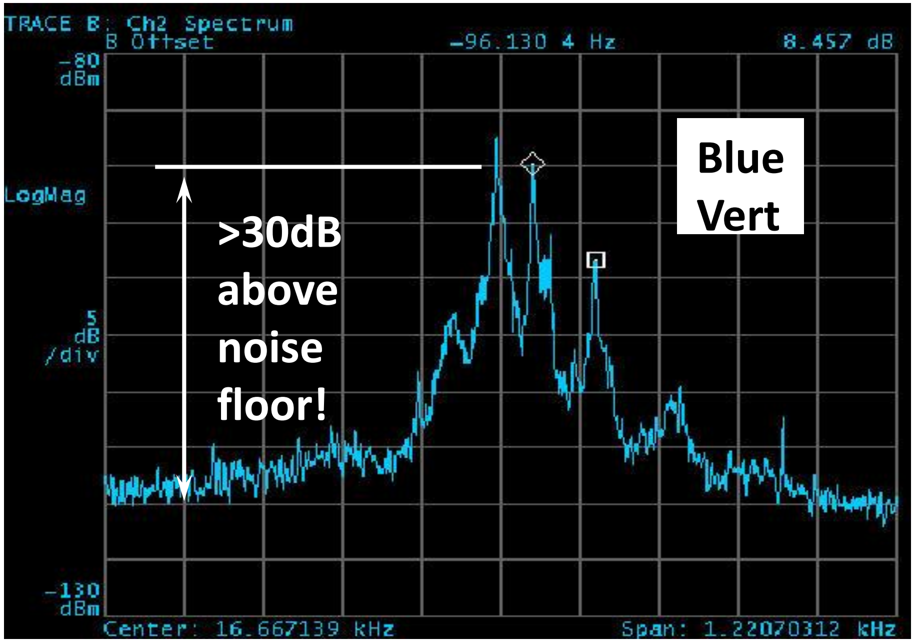
Spectrum from RHIC

Courtesy : P. Cameron

Observations reported in 2007!

Anomalous beam response at injection

- **tunes separated and well decoupled**
- **not power line, synchrotron freqs**
- **similar in all 4 planes**
- **serious obstacle to acquiring lock**
- **disappeared with start of ramp**
- **not understood – speculation on power supply regulation/phase shift at low current**

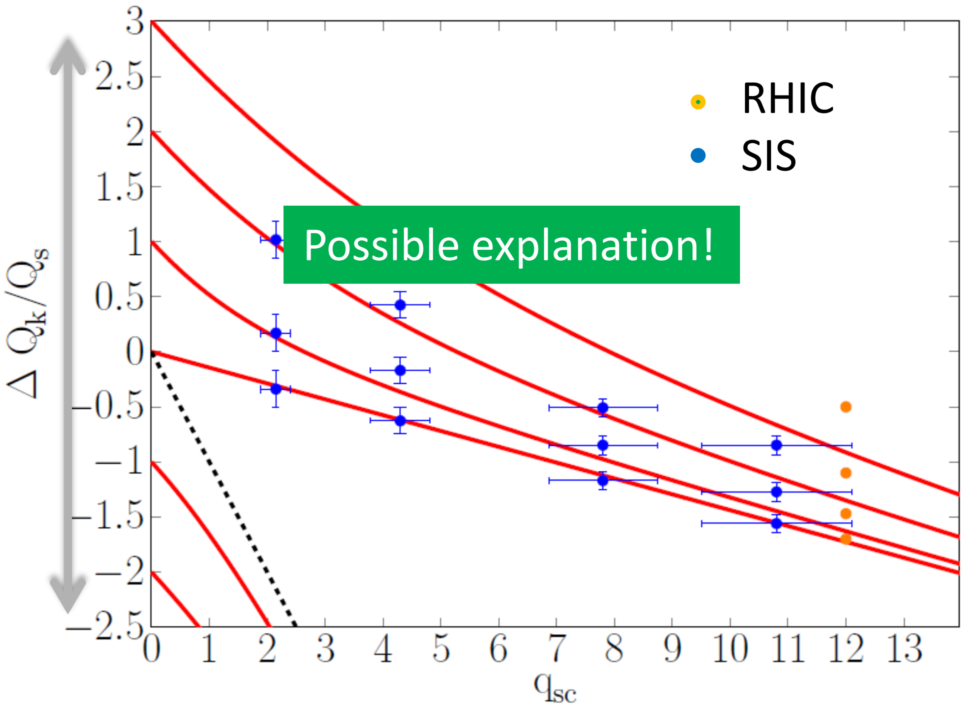


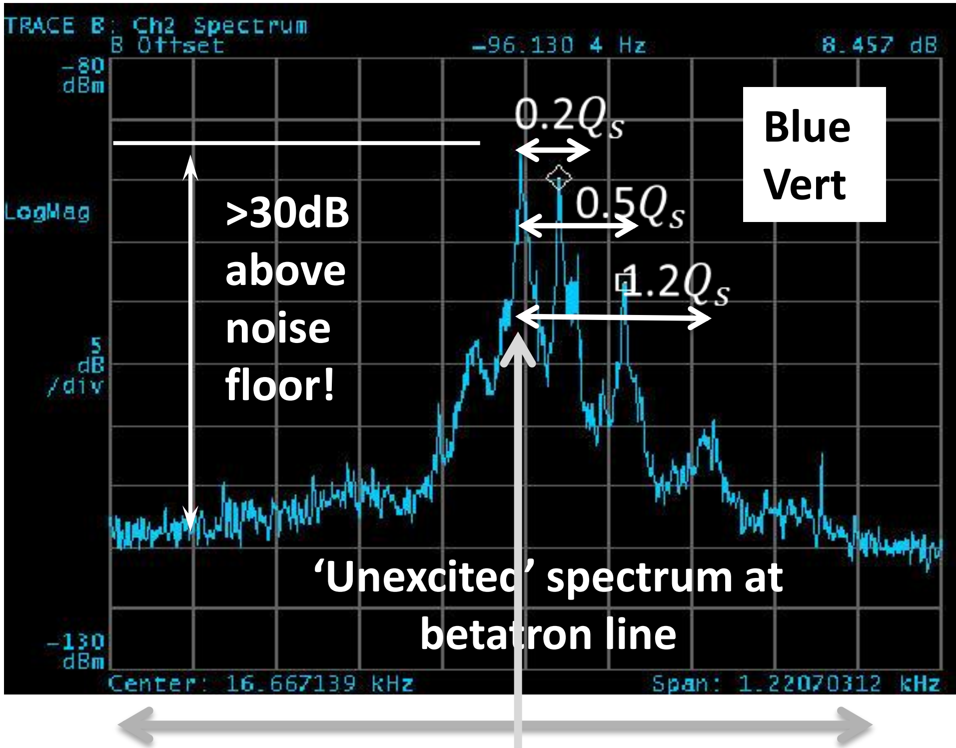
Problems reported

- **The phase locked loop (PLL) has difficulty locking to the tune frequency**
- **Feedbacks using PLL output were sometimes unstable**

P. Cameron et al., "TUNE, COUPLING, AND CHROMATICITY MEASUREMENT AND FEEDBACK DURING RHIC RUN 7*", DIPAC 2007, Venice
P. Cameron et al., "PROGRESS IN TUNE, COUPLING, AND CHROMATICITY MEASUREMENT AND FEEDBACK DURING RHIC RUN 7*", PAC'07

Spectrum from RHIC





Excellent logbook at BNL

$I_p\,/\,mA$	~ 500
γ	~ 4
ΔQ_{sc}	~ -0.02
Q_s	~ 0.0015
q_{sc}	~ 12

The left peak can be considered as tune.

Conclusion and outlook

- **Tune spectra has a characterstic modification at high intensities due to space charge forces.**

- **To extract machine tune from the spectra, the inter-dependence of various beam and machine parameters should be carefully considered.**

- **Correlating with simplified analytical models gives access to elusive quantities such as incoherent tune spreads, effective impedances, chromaticity.**

Quadrupolar modes (beam size oscillations) planned to be measured

Direct and fast measurement of incoherent tune spread

Acknowledgement

- *Contributors : P. Forck, P. Kowina, W. Kaufmann, K. Lang, O. Chorniy, R. Haseitl , O. Boine-Frankenheim*
- *Marie Curie ITN-DITANET for funding the work and Beam diagnostics group at GSI*
- *Marek Gasior from CERN-Beam Instrumentation group for great support with BBQ system*