Principal Component Analysis

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Principal Component Analysis

- To transform the raw data (either homodyne or heterodyne signals) into a basis which explains the variation within the data
- The data is a vector d with N variables in columns and M rows of repeated measurement, it can be transformed using an orthogonal matrix $\mathbf{W}^{\mathbf{T}}$ to $\mathbf{y}$ given by

$$\mathrm{y} = \mathrm{W}^{\mathrm{T}}\mathrm{d}$$

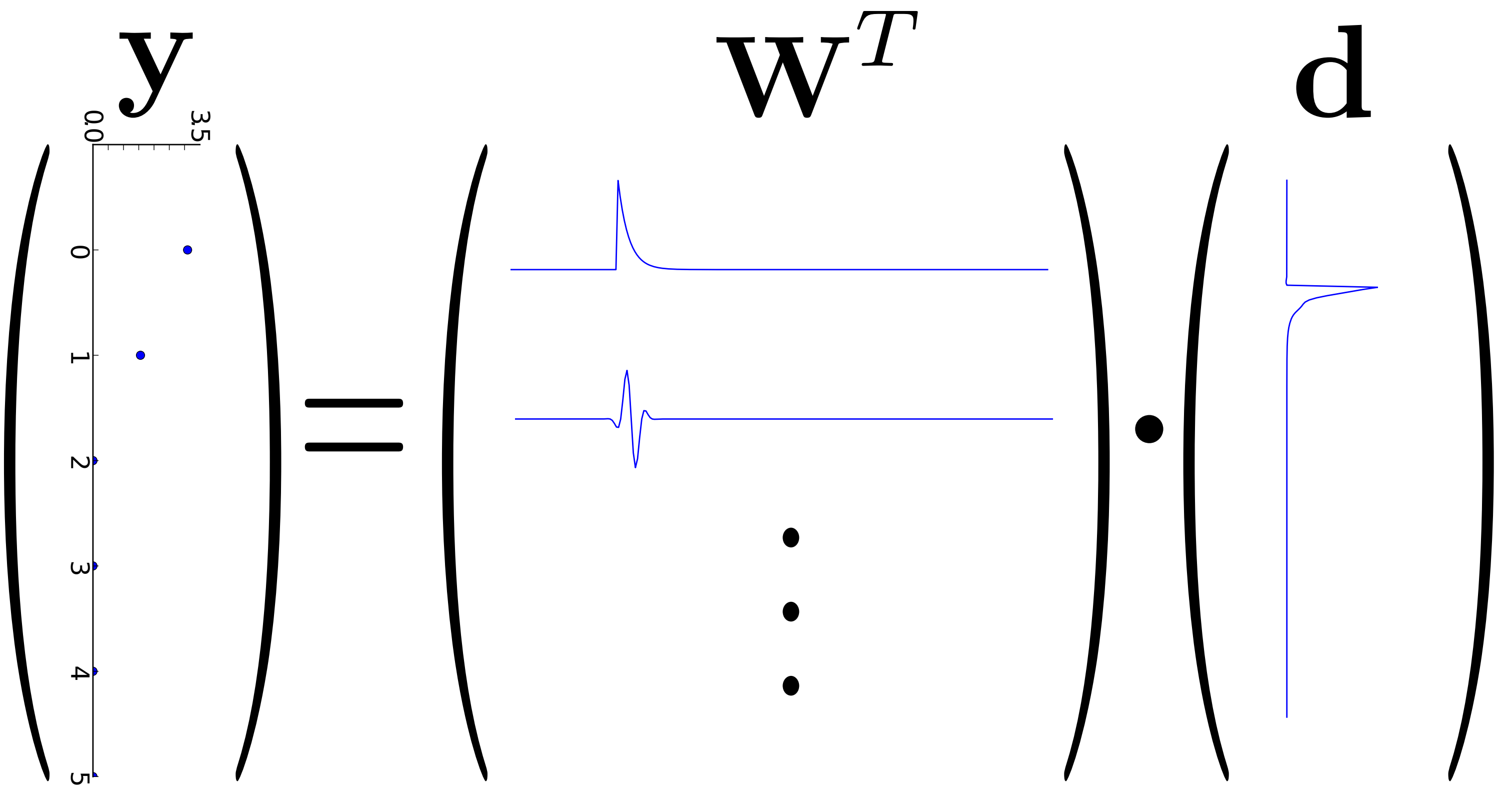
  - $\mathbf{W}^{\mathbf{T}}$: can be considered a rotation matrix that transforms the data into another linear vector space
- PCA is a method of determining the transformation matrix $\mathbf{W}^{\mathbf{T}}$ whilst keeping the variability of the original data
- PCA determines $\mathbf{W}^{\mathbf{T}}$ in such a way to make the covariance of $\mathbf{y}$ a diagonal matrix

PCA example (cont.)

$$\mathbf{d} = \mathbf{d}_{\mathrm{d}} + \mathbf{d}_{\mathrm{u}}$$

d_d : Vary depending on the position of the beam and charge

d_u : Some variability but independent of the signal of interest



PCA determine the basis (cont.)

- Making a transformation W^T which makes the covariance of y a diagonal matrix
- The covariance of the transformed data y can be calculated by

$$\mathbf{y}\mathbf{y}^{\mathbf{T}} = \left(\mathbf{W}^T\mathbf{d}\right)\left(\mathbf{W}^T\mathbf{d}\right)^T = \mathbf{W}^T\mathbf{d}\mathbf{d}^{\mathbf{T}}\mathbf{W}$$

- The data matrix d can be decomposed using SVD

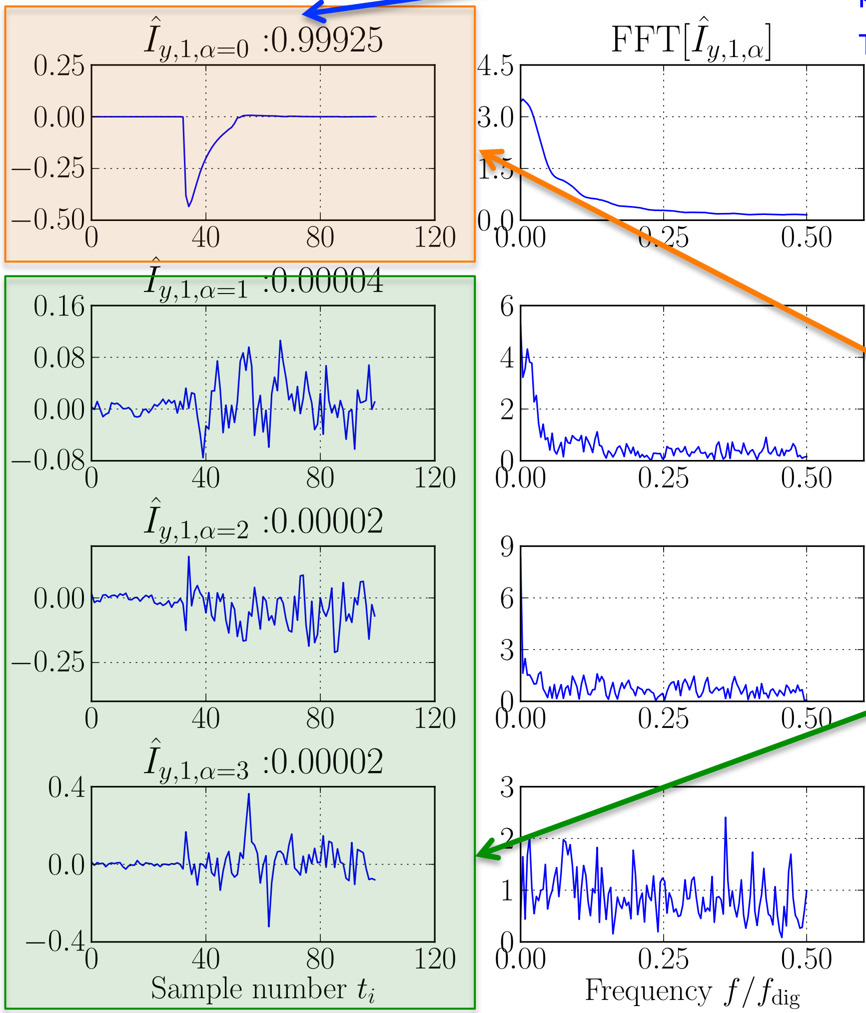
$$\mathbf{d} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

- Where S is diagonal matrix, U and V^T are orthogonal matrices
- The covariance matrix of d is

$$\mathbf{d}\mathbf{d}^T = \mathbf{U}\mathbf{S}^{\mathbf{2}}\mathbf{U}^T = \mathbf{W}\mathbf{y}\mathbf{y}^{\mathbf{T}}\mathbf{W}^T$$

- So the PCA transformation matrix W can be identified with matrix U from SVD of the data matrix d. Also the covariance matrix of the transform data is diagonal and can be identified with the singular value matrix squared

PCA (cont.) Homodyne signal



Explained variability ratio (EVR):

percentage of variance explained by each of the Selected components

The sum of EVR is equal to 1.0

$$I(t_i) = \sum_j y_j \hat{I}^T_{ji}$$

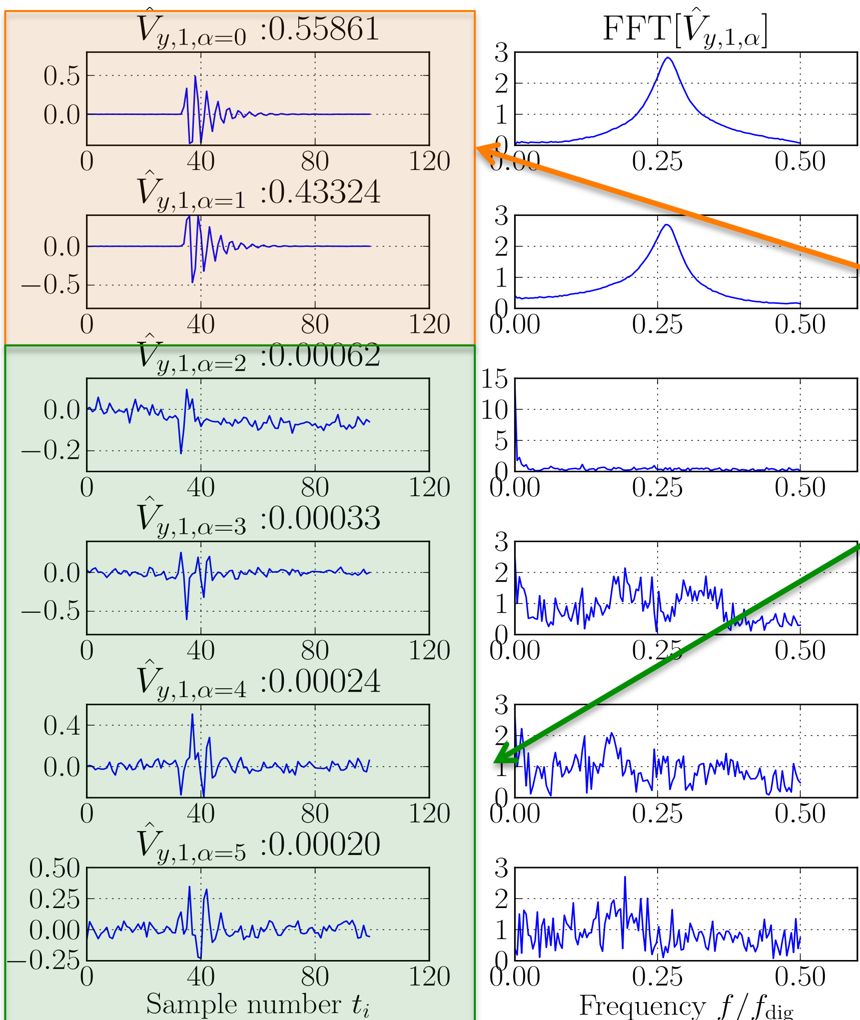
$$Q(t_i) = \sum_j y_j \hat{Q}^T_{ji}$$

Basis vectors

Coefficients, relative contribution of the basis vectors

- The principal component of the signal accounts for 99.9 % of the waveform variance
- Second and following components for almost none of the variation in the I signal

PCA (cont.) Heterodyne signal



Basis vectors

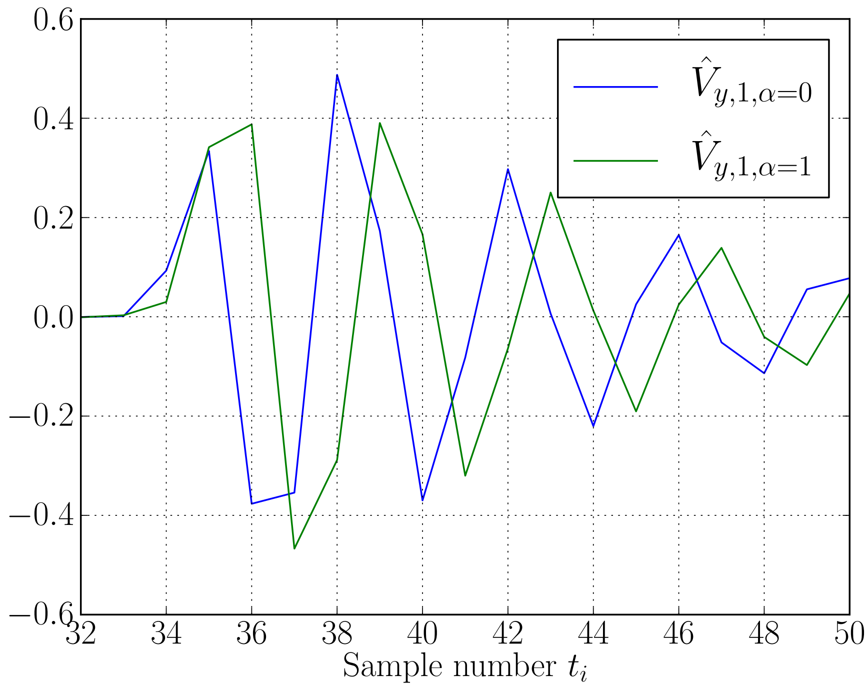
$$V(t_i) = \sum_j y_j \hat{V}^T_{ji}$$

Coefficients, relative contribution of the basis vectors

- First and second modes are clearly dipole like
- Below these are other contaminating modes at the few percent level

PCA (cont.)

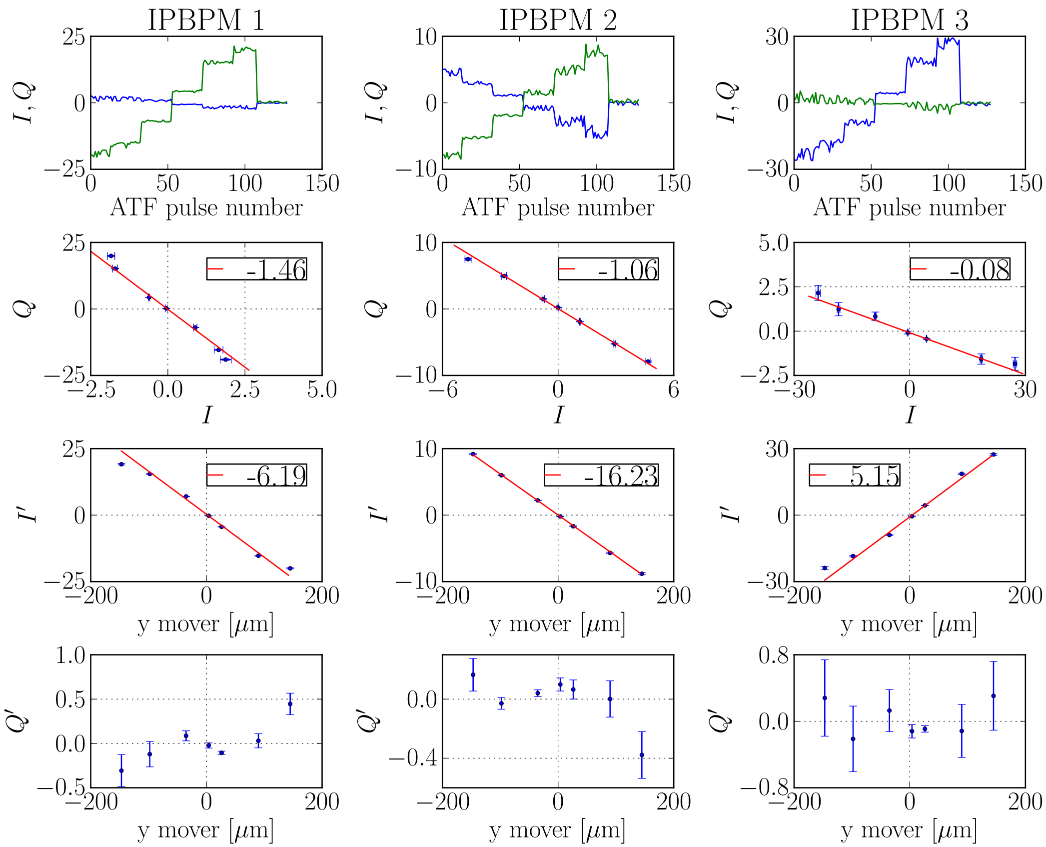
Heterodyne signal



To understand the physical interpretation of the two highest variance modes, plotted together

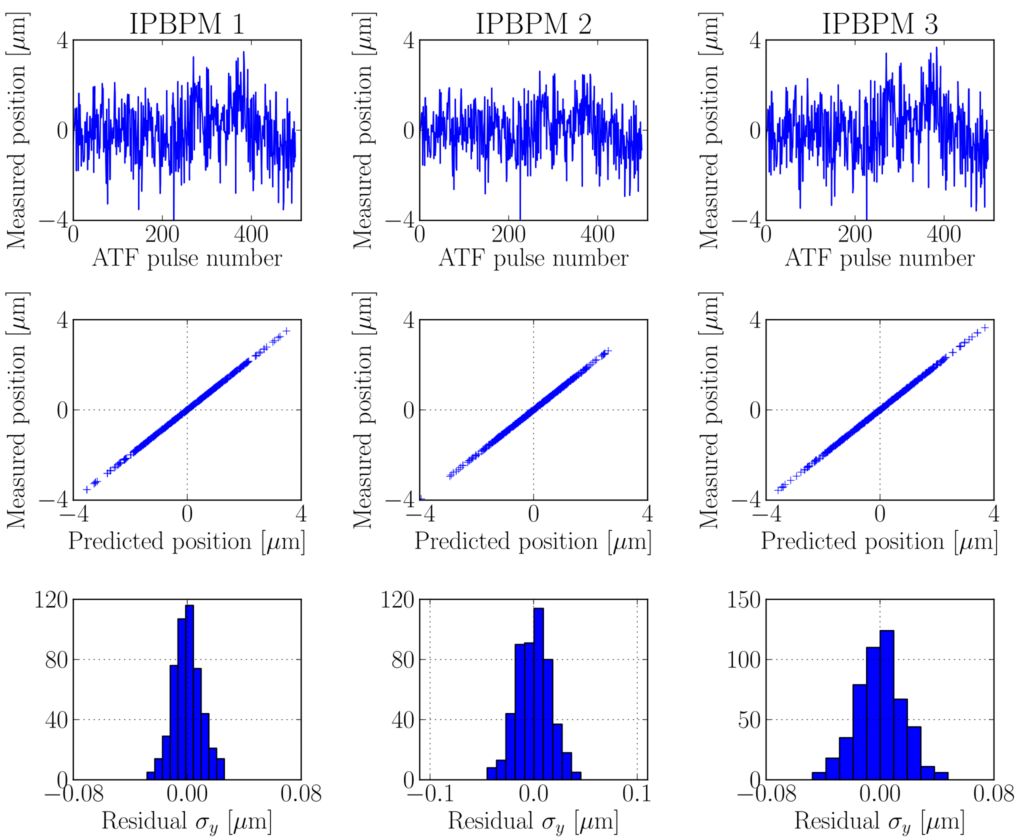
Clearly the principal and secondary components are orthogonal so much the same as I and Q like signals

PCA calibration (cont.)



- Application of the calibration is just the dot product of the basis vector with a pulse of recorded data
- The value of this dot product can be used for I or Q analysis
- The calibration plot looks very similar to the traditional signal processing

PCA triplet resolution (cont.)



- Gives similar results compare to other digital signal processing methods with the parameter optimization
- 4.3 nm position resolution with 0.79 x 10^{10} electron/ pulse

Summary

- A Principal Component Analysis (PCA) is a promising idea to determine the beam position in a model independent way
- The method clearly determines the principal component which is easily interpreted in a physical sense
- The IPBPM data are used to test this new technique, which give similar results which the RMS vertical position residual is 4.3 nm compared to more standard processing methods with parameter optimization (3.6 nm)
- Very simple to apply to the cavity BPM data when the digital signal processing method is not clearly identified
- Could be very useful in the early stages of BPM commissioning when the optimal parameters are poorly known
- Will be published soon