Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

September 16-19, 2013

IBIC 2013
International Beam Instrumentation Conference

Resonant TE Wave Measurement of Electron Cloud Density Using Multiple Sidebands*

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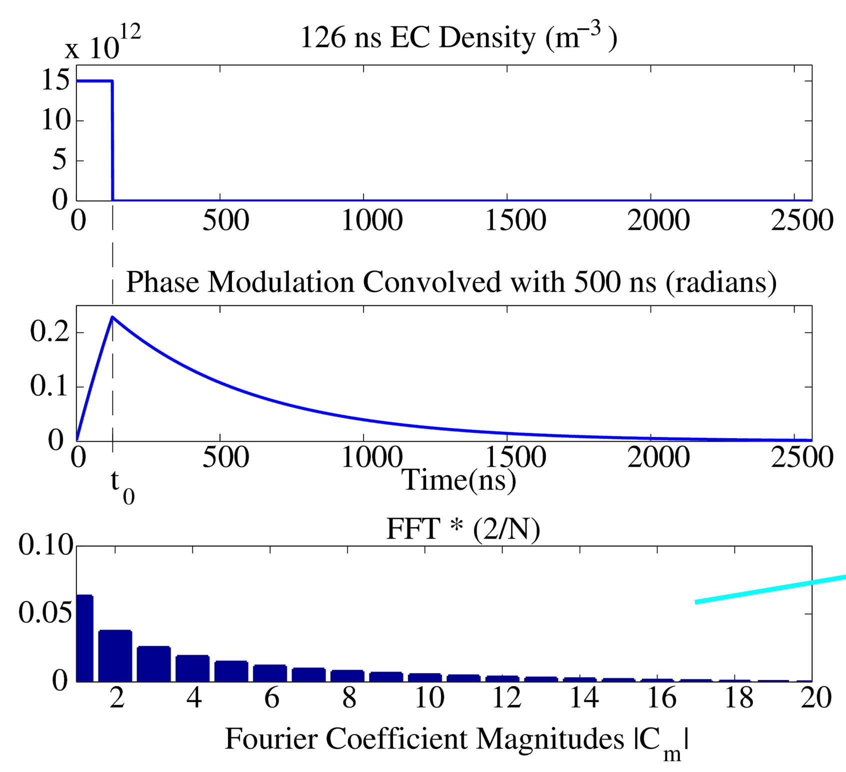
Abstract
A change in electron cloud (EC) density will change the resonant frequency of a section of beam-pipe. With a fixed drive frequency, the dynamic phase shift across the resonant section will include the convolution of the frequency shift with the impulse response of the resonance. The effect of the convolution on the calculated modulation sidebands is in agreement with measured data, including the absolute value of the EC density obtained from ECLOUD simulations. These measurements were made at the Cornell Electron Storage Ring (CESR) which has been reconfigured as a test accelerator (CESRTA) having positron or electron beam energies ranging from 2 GeV to 5 GeV

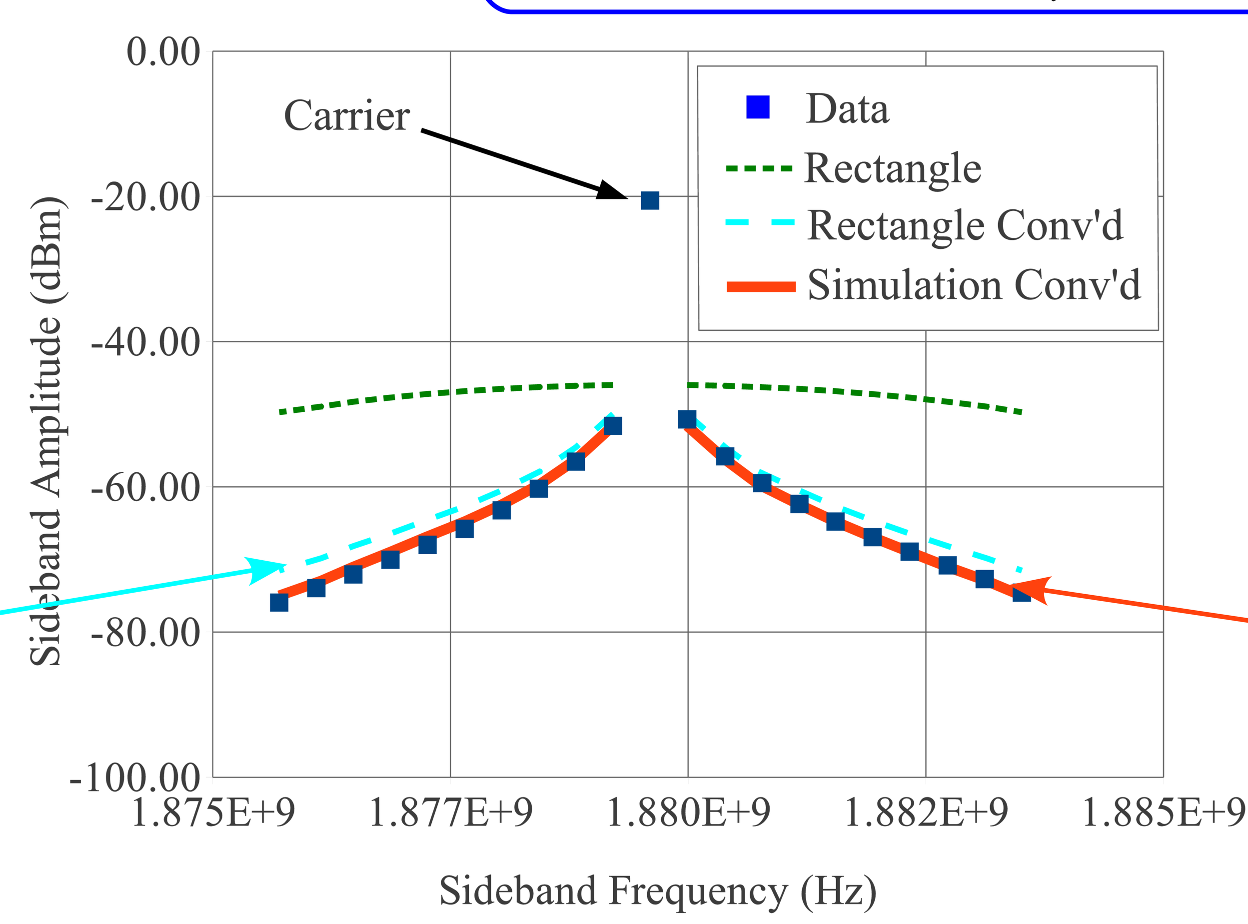
ECLOUD Simulations

The simulation code ECLOUD has been used extensively at CESRTA in understanding signals from shielded pickups (SPU) that sample the flux of cloud electrons onto the inner surface of the beam-pipe. There is an SPU detector within the resonant section of the TE wave measurement that can be seen in the "Beam-pipe Resonances" figure below. Simulation parameters as well as a model of the SPU are adjusted so that the simulation correctly predicts the signal recorded at the SPU.

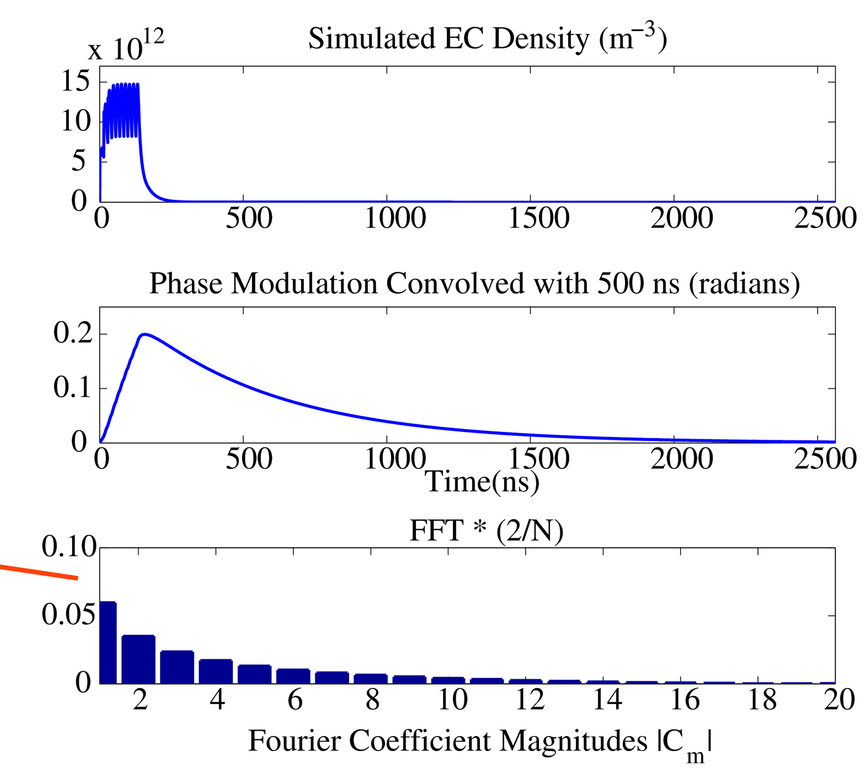
The 1-bunch and 10-bunch simulations of EC density used here have been tuned so that they agree with data taken with the SPU detectors. So the same EC density simulation is used for two independent measurement techniques.



A 126 ns rectangular EC density (top) is used to calculate the dynamic phase shift $\Delta\Phi(t)$ (middle) that is then Fourier transformed to obtain the sideband coefficients C_m (bottom) using a MATLAB script. 126 ns is the length of a 10 bunch train with 14 ns spacing.



10-bunch data (squares) at 60 mA total current (10^{11} particles/bunch), are compared with calculated envelopes of sideband amplitudes. The calculation based on convolved rectangular density (cyan dashed line) is in fair agreement with the measurement. The envelope obtained using the ECLOUD simulation is the solid orange line that (nearly) coincides with the data points. The FFT of the rectangular density *without* convolution is the green dashed line.



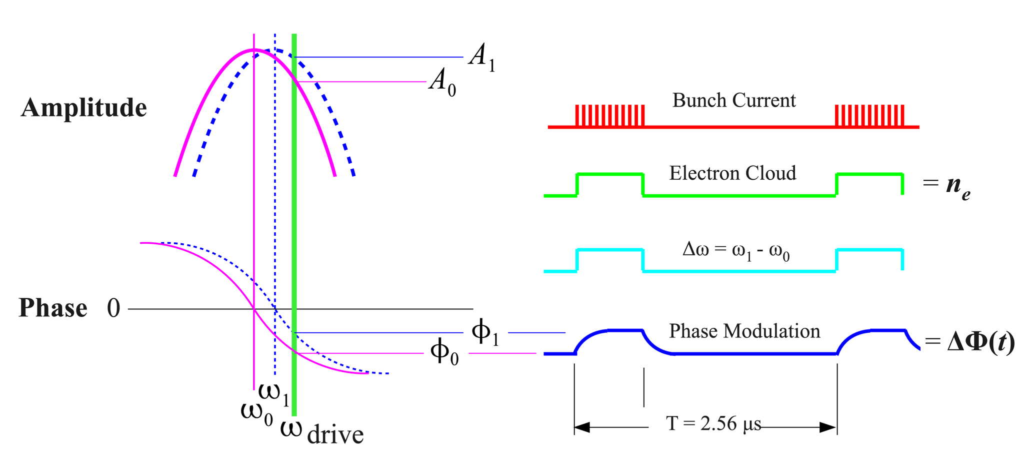
The ECLOUD simulation for a 10-bunch positron train at 60 mA total current (10^{11} particles/bunch) (top), is convolved with the impulse response of the beam-pipe resonance to calculate the expected sideband amplitudes dBc = 20 log$|C_m/2|$.

Calculating Sidebands

Resonant frequency shift due to EC density: The resonant frequency shift produced by a low density plasma in the absence of magnetic field is given by the equation below, where the integral is over the resonant volume of the beam-pipe. For a uniform EC density n_e, constant over the volume, this value can be brought outside of the integral.

Electron Cloud Density

$$\frac{\Delta\omega}{\omega} \approx \frac{e^2}{2\varepsilon_0 m_e \omega^2} \frac{\int_V n_e E^2 dV}{\int_V E^2 dV} \rightarrow \frac{e^2}{2\varepsilon_0 m_e \omega^2} n_e$$



Equilibrium phase shift with fixed drive frequency: With a fixed drive frequency ω_{drive} near the resonant frequency ω_0, the equilibrium phase shift ϕ_0 across a resonator is shown in the sketch above.. For small shifts in the resonant frequency $\Delta\omega$ near resonance, the change in equilibrium phase $\Delta\phi$ is given by the equation below.

$$\Delta\phi \approx 2\,Q\,\frac{\Delta\omega}{\omega} \approx 2\,Q\,\frac{e^2}{2\varepsilon_0 m_e \omega^2} n_e = Q\,\frac{1.59\times10^3}{\omega^2} n_e$$

Dynamic phase shift obtained by convolution: If the rate of change in EC density n_e is rapid compared to the damping time of the resonance, the dynamic phase shift $\Delta\Phi$ is obtained by convolving the equilibrium phase shift with the impulse response of the resonance.

$$\Delta\Phi(t) = 2Q\frac{1.59\times10^3}{\omega^2}\int_{-\infty}^{t} n_e(\xi)\,e^{(t-\xi)/\tau}\,d\xi$$

Expressing the dynamic phase shift as a Fourier series: With a train of bunches in the storage ring, the growth and decay of the electron cloud will be periodic, the period being the revolution time T with the corresponding revolution frequency ω_T. The function $\Delta\Phi(t)$ can be expressed as a Fourier series of multiples of the revolution frequency ω_T.

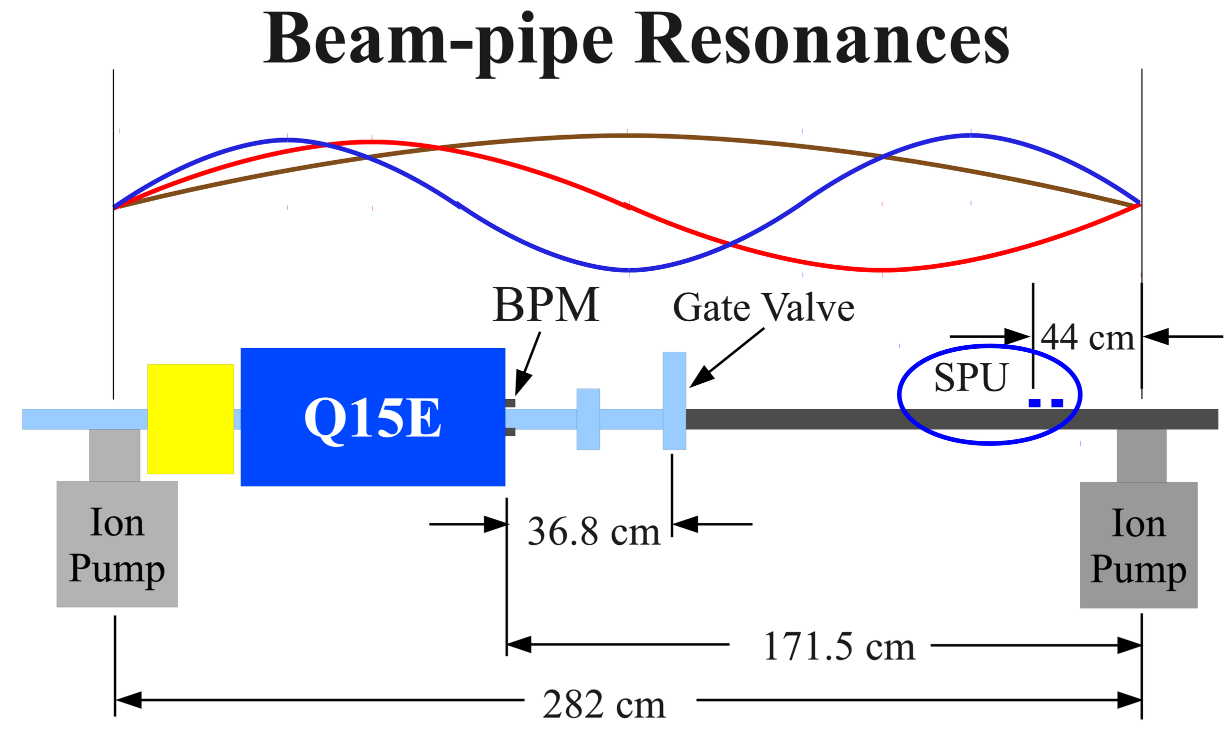
$$\Delta\Phi(t) = \sum_{m=0}^{+\infty} C_m \cos(m\omega_T t)$$

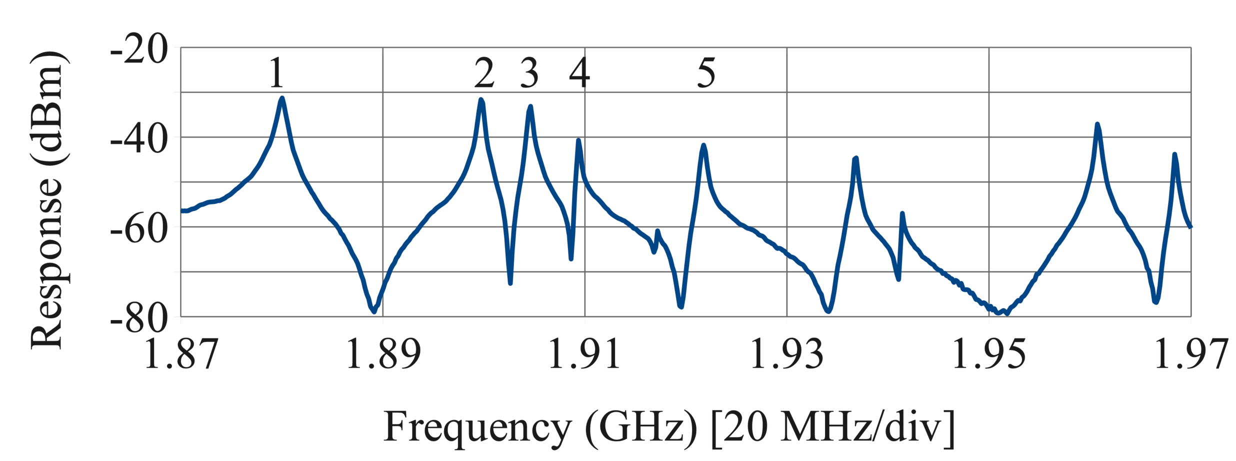
The amplitudes of modulation sidebands: A fixed frequency drive that is phase modulated by $\Delta\Phi(t)$ can be written (using trig identities) as the original sine wave plus multiples of the revolution frequency ω_T. -- upper and lower sidebands. For small modulation, the ratio of the amplitude of the sidebands to the drive amplitude is ½ C_m.

$$g(t) = \sin\left[\omega t + \sum_{m=0}^{+\infty} C_m \cos(m\omega_T t)\right]$$

$$\approx \sin(\omega t) + \sum_{m=0}^{+\infty} \frac{C_m}{2}\left\{\underbrace{\cos[(\omega+m\omega_T)t]}_{\text{Upper Sidebands}} + \underbrace{\cos[(\omega-m\omega_T)t]}_{\text{Lower Sidebands}}\right\}$$

Beam-pipe Resonances





Data presented here comes from a roughly 3 meter long resonant section of aluminum chamber near the quadrupole at 15E in CESRTA. Longitudinal slots at the ion pump ports generate reflections for TE waves and a number of resonances. The first resonance at about 1.88 GHz has a Q of about 3000 (τ = 500 ns). Phase modulation sidebands appear at multiples of the beam revolution frequency (390 kHz). Ten upper and lower sidebands of the first resonance were recorded.

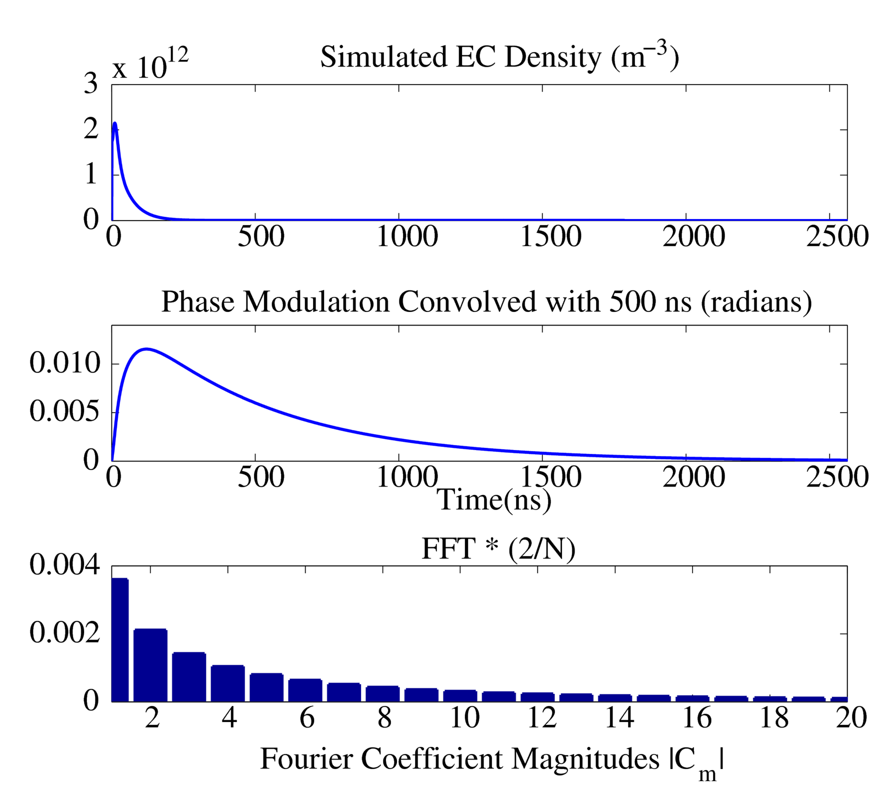
Summary and Discussion

The resonant response of beam-pipe to microwaves can be used to calculate electron cloud (EC) density produced by a train of bunches in a storage ring. When the calculation includes the convolution of the EC density with impulse response of the beam-pipe, the envelope of the phase modulation sidebands is in fair agreement with measured data.

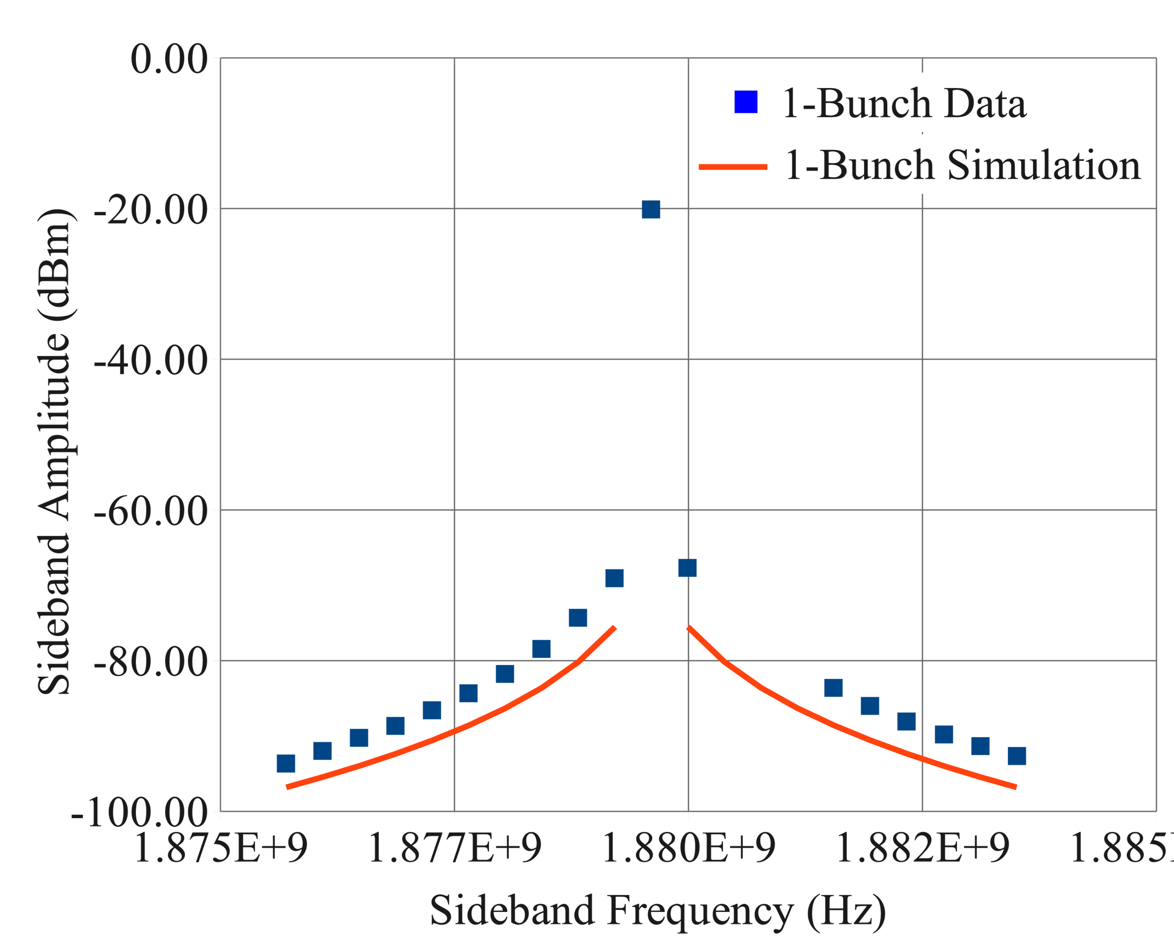
In this analysis, I have ignored contributions from amplitude modulation and frequency modulation (AM, FM). Near resonance, AM should be small. FM is under investigation.

For the 10-bunch comparison, the calculated values based on ECLOUD simulations nearly coincide with the data.

For the 1-bunch comparison, the calculated values are lower than the data by about 6 dB. This is probably explained by the following argument. For the 1-bunch data, the EC density will be more or less proportional to the amount of synchrotron light at each longitudinal position. The ECLOUD simulation was made for the location of the SPU, which is near the end of the resonant TE wave section. The synchrotron light in this section decreases by a factor of 3, being smallest at the end nearest to the SPU. In contrast, for the 10-bunch data at 6 mA/bunch the EC density is saturated at the SPU and the increased amount of synchrotron light in the other parts of the chamber do not result in a significantly higher EC density there. So for the 10-bunch data, the EC density should be more nearly uniform over the resonant section of beam-pipe.



An ECLOUD simulation (top) for a 1-bunch positron beam of 3 mA, 4.8 x 10^{10} particles, is scaled up to 8 mA and convolved with the 500 ns impulse response of the beam-pipe resonance to calculate the expected sideband amplitudes dBc = 20 log $|C_m/2|$.



A comparison of 1-bunch data at 8 mA (squares) with the simulated sideband amplitude envelope (solid lines) given by the $C_m/2$ of the simulation at left, but scaled up from 6 to 8 mA.

*This work is supported by the US National Science Foundation PHY-0734867, PHY-1002467 and the US Department of Energy DE-FC02-08ER41538, DE-SC0006505.

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