ANALYSIS OF MODULATION SIGNALS GENERATED IN THE TE WAVE DETECTION METHOD FOR ELECTRON CLOUD MEASUREMENTS*

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Abstract

The Evaluation of the electron cloud density (ECD) by measuring its effects on the propagation of electromagnetic signals across portions of the beampipe is a widely used technique and the most suited for measurements over extended regions. Recent results show that in a majority of cases the RF signal transmission takes place by coupling to standing waves excited in the vacuum chamber, rather than through propagating modes. In such an event the effect of a varying cloud density results in a simultaneous amplitude, phase and frequency modulation of a fixed frequency drive signal. The characteristics of the modulation depend not only on the cloud density values as a function of time and its special distribution, but also on the damping time of the standing waves. In this paper we evaluate the relationship between measured modulation sidebands amplitude and the electron cloud density when cloud and electromagnetic resonance rise and fall times are of the same order of magnitude, as it is the case in the accelerators where we have conducted our experiments.

INTRODUCTION

The fundamental principles of the TE wave detection method have been described in [1-3]. Essentially, an electromagnetic wave is excited at one point and detected at another point after it has propagated along the beam pipe, usually using existing beam position monitors (BPM). The presence of the electron cloud changes the propagation characteristics introducing a phase delay proportional to the ECD.

More recently [4] it has been observed that in many instances the signal that is transmitted from one BPM to another is not due to a propagating wave, but to a standing wave corresponding to a vacuum chamber resonance excited in the region where the two BPM's are located.

The shift in resonant frequency due to the ECD has been verified experimentally on Cesr-TA and DAΦNE [5]

In a previous paper [6] we have presented an analysis of the signals detected under such circumstances. Instead of a variable delay, resulting in the phase modulation of the transmitted signal, simultaneous amplitude, phase and frequency modulations are induced when the varying ECD changes the resonant frequency of the standing wave.

In our analysis we have limited ourselves to the case of resonant frequencies instantaneously changing between steady-state values. That is, we assumed that the changes in ECD, which in turn cause the resonant frequency of the standing wave to change, take place in a time scale much shorter than the resonance time constant $\tau = 2Q_0 / \omega_0$, where Q_0 is the resonance quality factor and ω_0 its angular frequency. We also assumed that the changes in ECD are sufficiently spaced so that the transient response generated by each change is completely vanished before the next change takes place.

Actual situations encountered in practice are generally different from such an idealized model: first of all, changes in the ECD take a finite time for either filling the pipe with electrons, or dissipating the accumulated density. Also, the bunch pattern structure does not necessarily present gaps or trains much longer than τ, so that the description using a succession of steady-state responses may not be an appropriate one.

In fact, our measurements on Cesr-TA and DAΦNE show that, although the two machines have rather different vacuum chamber geometries, their resonances near the beampipe cutoff, which are the ones used for the ECD measurements, have similar time constants, of the order of a few hundreds of nanoseconds (Table1).

Table 1: Standing Waves Typical Parameters

	Cesr-TA	DAΦNE
Frequency (MHz)	1900	300
Q_0	1000's	100's
τ (ns)	100's	100's

As the rise and fall times of the ECD are concerned, we have observed that the formers are related primarily to the bunch train structure, i.e. it takes up to a few 10's of bunches to reach a saturation level and depending on the particular bunch spacing that means the ECD reaches a constant value in the space of several tens up to a few hundreds of nanoseconds. The decay time of the cloud has been observed with localized measurements using shielded pickups [7] and estimated of the order of a few hundreds of nanoseconds. Since the beampipe size and material are the main factors in determining the cloud decay time, it wouldn't be farfetched to assume that these results are representative of the ECD decay in a substantial number of accelerators.

These measurements show that ECD changes take place on the same timescale of the transients they induce and

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therefore the analytical treatment presented in [6] is best suited for application to long bunch trains and long gaps, in large rings.

A first approach to including the effects of the finite time constant of the standing waves used in the ECD measurement is given in [8-9], where a transfer function for the beampipe response is introduced, corresponding to an exponentially decaying impulse response with a 400÷500 ns time constant. Such a method agrees quite well with the experimental data in the case of both frequency domain measurements, and direct phase measurements in the time domain. Furthermore, the beampipe response time constant value which best fits the experimental data coincides with the actual standing wave time constant, as measured experimentally.

In this paper we extend the analytical treatment of the signal modulation from changing resonant frequency presented in [6] to the case of a slowly varying ECD.

ANALYTICAL EXPRESSION OF THE RECEIVED SIGNAL

We assume to have a fixed sinusoidal excitation signal of amplitude A_x, and angular frequency ω_x. It excites a standing wave of frequency ω_0, when no ECD is present, and a fixed quality factor Q_0 (this is true for high enough values of Q_0).

In the presence of a time varying ECD $n(t)$, the resonant frequency also become a function of time given by:

$$\omega_r(t) \approx \omega_0 + \frac{(18\pi)^2}{2}\frac{n(t)}{\omega_0} \tag{1}$$

A steady-state response is reached after the ECD remains constant for a few decay times and can be written as:

$$s(t) = A_0 \sin(\omega_x t + \varphi_0) \tag{2}$$

where amplitude and phase depend on the difference between ω_x and ω_r:

$$A_0 = \frac{A_x C}{\sqrt{\left(\omega_x^2 - \omega_r^2\right)^2 + \dfrac{\omega_x^2\omega_r^2}{Q_0^2}}} \tag{3}$$

and

$$\varphi_0 = \tan^{-1}\left(Q_0 \frac{\omega_x^2 - \omega_r^2}{\omega_x \omega_r}\right) \tag{4}$$

For a change in the resonant frequency at time $t = t_1$, Eq.(2) becomes

$$\begin{aligned} s(t \ge t_1) = {} & A_1 \sin(\omega_x t + \varphi_1) + \\ & - A_1 e^{-(t-t_1)/\tau} \sin\left[\omega_1 t + (\omega_x - \omega_1)t_1 + \varphi_1\right] + \\ & + A_0 e^{-(t-t_1)/\tau} \sin\left[\omega_1 t + (\omega_x - \omega_1)t_1 + \varphi_0\right] \end{aligned} \tag{5}$$

where for the sake of brevity we have indicated $\omega_r(t_1)$ with ω_1 and A_1 and φ_1 are the quantities in Eqs.(3) and (4) calculated for $\omega_r=\omega_1$. The two exponentially decaying terms are the transient response generated by the ECD change.

When the resonant frequency changes again at $t = t_2 > t_1$ Eq.(5) becomes:

$$\begin{aligned} s(t \ge t_2) = {} & A_2 \sin(\omega_x t + \varphi_2) + \\ & - A_2 e^{-(t-t_2)/\tau} \sin\left[\omega_2 t + (\omega_x - \omega_2)t_2 + \varphi_2\right] + \\ & + A_1 e^{-(t-t_2)/\tau} \sin\left[\omega_2 t + (\omega_x - \omega_2)t_2 + \varphi_1\right] + \\ & - A_1 e^{-(t-t_1)/\tau} \sin\left[\omega_2 t + (\omega_1 - \omega_2)t_2 + (\omega_x - \omega_1)t_1 + \varphi_1\right] + \\ & + A_0 e^{-(t-t_1)/\tau} \sin\left[\omega_2 t + (\omega_1 - \omega_2)t_2 + (\omega_x - \omega_1)t_1 + \varphi_0\right] \end{aligned} \tag{6}$$

and a new transient is generated by the new frequency change.

For a cyclical transition between two ECD, and therefore two resonant frequencies, we can rewrite Eq.(6) with $A_2=A_0$, $\omega_2=\omega_0$, and $\varphi_2=\varphi_0$. We can also assume that $\omega_x = \omega_0 \Rightarrow \varphi_0 = 0$ in order to make the expression simpler (on resonance excitation) and obtain:

$$\begin{aligned} s(t \ge t_2) = {} & A_0 \sin(\omega_0 t) + \\ & - A_0 e^{-(t-t_2)/\tau} \sin(\omega_0 t) + \\ & + A_1 e^{-(t-t_2)/\tau} \sin(\omega_0 t + \varphi_1) + \\ & - A_1 e^{-(t-t_1)/\tau} \sin\left[\omega_0 t + (\omega_1 - \omega_0)(t_2 - t_1) + \varphi_1\right] + \\ & + A_0 e^{-(t-t_1)/\tau} \sin\left[\omega_0 t + (\omega_1 - \omega_0)(t_2 - t_1)_1\right] \end{aligned} \tag{7}$$

This can be further simplified assuming that $A_0 \approx A_1$, which is certainly true for small frequency changes around the resonant frequency (i.e. small changes in $n(t)$). We also can write $\omega_1 - \omega_0 = \delta\omega$ and $t_1 = t_2/2 = \delta t$ and write

$$\begin{aligned} s(t \ge t_2) = A_0 \Big[& \sin(\omega_0 t) + \\ & + e^{-(t-2\delta t)/\tau} \cos(\omega_0 t + \varphi_1/2)(\varphi_1/2) \\ & - e^{-(t-\delta t)/\tau} \cos(\omega_0 t + \delta\omega\delta t + \varphi_1/2)(\varphi_1/2) \Big] \end{aligned} \tag{8}$$

which represents the response measured when the resonance is at $\omega=\omega_0$. When it is at $\omega=\omega_1$ when can write the following expression:

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$$s(t \geq t_1) = A_0 \Big[\sin(\omega_0 t + \varphi_1) + \\ + e^{-(t-2\delta t)/\tau} \cos(\omega_1 t + \varphi_1/2)(\varphi_1/2) \\ -e^{-(t-\delta t)/\tau} \cos(\omega_1 t + \delta\omega\delta t + \varphi_1/2)(\varphi_1/2) \Big] \quad (9)$$

and $\varphi_1 \approx 2Q_0\delta\omega/\omega_0$.

PHASE MODULATION

From the comparison of Eqs.(8) and (9) it is possible to calculate the phase modulation index. A further simplification comes from assuming that the term $\delta\omega\delta t << \varphi_1/2$, which is true if δt << τ. In such case Eqs.(8) and (9) can be rewritten respectively as

$$s(t \geq t_2) = A_0 \Big[\sin(\omega_0 t) + \\ + \tau\delta\omega e^{-(t-\delta t)/\tau} \cos(\omega_0 t + \varphi_1/2) \Big] \quad (10)$$

and

$$s(t \geq t_1) = A_0 \Big[\sin(\omega_0 t + \varphi_1) + \\ + \tau\delta\omega e^{-(t-\delta t)/\tau} \cos(\omega_1 t + \varphi_1/2) \Big] \quad (11)$$

In Eqs.(10) and (11) exponentially decaying terms are added to the steady-state responses. These terms vanish if the resonance damping time is short enough, or if the time derivative of the ECD change $\frac{\delta\omega}{\delta t} = \frac{dn_e}{dt}$ is small enough. When neither condition is verified, the transient part of the response contains a phase difference proportional to $\delta\omega$ since $\omega_1 = \omega_0 + \delta\omega$. This term can become dominant over the steady-state response if the factor $\tau\delta\omega >> 1$.

CONCLUSIONS

In this paper we have calculated the RF signal generated by the excitation with a fixed frequency sinusoid of a standing wave in an accelerator vacuum chamber, when its resonant frequency shifts under the effect of a varying ECD. We have considered time constants longer than the ECD characteristic times, so that transients are always present in the response. The phase modulation index calculated when the ECD is cycling between two values can be dominated by the transient part of the response if the time derivative of the ECD is large enough.

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