COVARIANCE AND TEMPORAL CAUSALITY IN THE TRANSITION RADIATION EMISSION BY AN ELECTRON BUNCH

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Abstract

A model of the transition radiation emission by a N electron bunch must conform to covariance and causality. The covariance of the charge density must imprint the transition radiation energy spectrum via a proper formulation of the charge form factor. The emission phases of the radiation pulse must be in a causality relation with the temporal sequence of the N electron collisions onto the metallic screen. Covariance and temporal causality are the two faces of the same coin: failing in implementing one of the two constraints into the model necessarily implies betraying the other one. The main formal aspects of a covariance and temporal-causality consistent formulation of the transition radiation energy spectrum by an N electron beam are here described. In the case of a transition radiator with a round surface, explicit formal results are presented.

INTRODUCTION

Transition Radiation (TR) can be observed as a relativistic charge crosses the dielectric interface between two different media [1, 2]. For a relativistic charge at a normal angle of incidence onto the dielectric interface, the radiation is emitted backward and forward along the incidence axis according to a double conical spatial distribution with vertex at the collision point and angular aperture scaling down with the Lorentz factor γ of the relativistic charge ($\gamma = E/mc^2$). The bigger the discontinuity across the dielectric interface, the larger the intensity of the emitted radiation. The most suitable condition for the radiation emission is given by a metallic screen in vacuum. This case being the most common in a particle accelerator will be considered in the following. In beam diagnostics, TR is mainly observed in the visible [3] - to monitor the beam profile by imaging with a camera the so called Optical Transition Radiation (OTR) - or in the THz region to measure the bunch length from the analysis of the coherent enhancement of the TR spectral intensity [4]. In these relevant wavelength region for beam diagnostics, the TR radiator surface behaves practically as an ideal conductor. Ideal conductor properties of the TR radiator will be supposed in the following. Under such a hypothesis, the metallic surface of the TR radiator can be modeled as a double layer of charge. TR emission can be therefore interpreted as the result of the dipolar oscillation of the conduction electron that is induced by the incident relativistic beam on the ideal conductor surface, see Fig.(1). Such a dipolar model of the TR emission can indeed explain how a relativistic beam - colliding in a rectilinear and uniform motion onto a charge distribution (the charge double layer) at rest in the laboratory reference frame - can originate not only forward but also backward emitted radiation. This model also permits to deeper understand the kinematics of the radiative mechanism and to recognize the common relativistic nature that TR shares with other electromagnetic radiative mechanisms by relativistic beams. From the point of view of the kinematics, the collision at a normal angle of incidence of a relativistic charge onto a charge distribution at rest in the laboratory reference frame - so the TR kinematics can be indeed schematized - is equivalent to the head-on collision of two distributions of charge in a rectilinear and uniform motion when the collision is observed in the reference frame of rest of one of the two colliding charged distributions. The backward and forward double conical TR emission can be thus equivalently interpreted as the photon bremsstrahlung emission that two head-on colliding electron beams can originate. Taking into consideration the common kinematics and relativistic nature that TR shares with other electromagnetic radiative mechanisms - such as the synchrotron or the bremsstrahlung radiation - it is thus reasonable to expect that, even at a very short wavelength, some spectral modifications of the radiation intensity due to the beam transverse density should also affect the TR emission in a similar way as, in other electromagnetic radiative mechanisms of charged beams, the beam transverse size contributes to determine the so called Brilliance or Luminosity properties of the radiation source. The issue of the formal dependence of the TR emission on the transverse distribution of the N electron coordinates - even at a very short wavelength - is strictly joined to the issue of the covariance and causality in the TR model. The formal expression of the TR energy spectrum must indeed meet the two following constraints [5, 6, 7]: causality and covariance. The train of the emission phases - from the metallic surface - of the N single electron radiation field amplitudes must be in a causality relation with the temporal sequence of the N electron collisions onto the metallic screen. The distribution of the N electron transverse coordinates is a Lorentz invariant under a Lorentz transformation from the laboratory to the rest reference frame of the colliding bunch. The dependence of the charge density and of the electric field traveling with the N electrons - the so called *virtual quanta* field - on the distribution of the N electron transverse coordinates is invariant whether this is observed in the laboratory reference frame or in the reference frame of rest of the charged beam. The TR field resulting from the wave propagation of the *virtual quanta* field scattered

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by the metallic surface is expected to preserve the signature of such a Lorentz invariance and to show a covariant dependence on the distribution of the N electron transverse coordinates. In the following, it will be demonstrated how failing in implementing the causality constraints in the formal expression of the TR energy spectrum of a N electron beam necessarily implies a covariance defect in the formula and viceversa. From the derivation of the formal results, it will be also evident that the covariance in the formal expression of the TR energy spectrum is expected to manifest itself - as a function of the beam transverse size - as a spectral and angular modification of both the temporal incoherent and coherent components of the TR energy spectrum. It will be also demonstrated how the well-known results of the Frank-Ginzburg formula and of the TR emission of a single electron hitting a round metallic radiator can be obtained as a limit case of the covariance and causality consistent model of the TR emission from a round metallic surface.

TR ENERGY SPECTRUM

Causality and Covariance

In the following, a bunch of N electrons in a rectilinear and uniform motion along the z-axis of the laboratory reference frame with a common velocity $\vec{w} = (0, 0, w)$ is supposed to normally hit a round metallic surface with an arbitrary radius R which is placed on the plane $z = 0$. The harmonic component of the TR field resulting from the collision of the N electron reads

$$E^{tr}_{x,y}(\vec{\kappa}, \omega) = \sum_{j=1}^{N} H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})\, e^{-i(\omega/w)z_{0j}} \quad (1)$$

where $[\vec{\rho}_{0j} = (x_{0j}, y_{0j}), z_{0j}]$ $(j = 1, .., N)$ are the N electron spatial coordinates at the collision reference time. Under far-field approximation [5, 6, 7],

$$H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j}) = \frac{iek}{2\pi^2 Dw} \times \quad (2)$$
$$\times \int_S d\vec{\rho} \int d\vec{\tau} \frac{\tau_{x,y}\, e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2 + \alpha^2} e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}},$$

where D is the distance of the screen from the observation point, $k = \omega/c = 2\pi/\lambda$ is the wave number, $\vec{\kappa} = (k_x, k_y) = k \sin\theta(\cos\phi, \sin\phi)$ is the transverse component of the wave-vector, $\alpha = \frac{\omega}{w\gamma}$ (γ being the relativistic Lorentz factor), the vector $\vec{\rho} = (x, y)$ is the integration variable on the screen surface S whose size and shape are arbitrary, in principle. With reference to Eqs.(1,2), the TR energy spectrum can be finally obtained

$$\frac{d^2 I}{d\Omega d\omega} = \frac{cD^2}{4\pi^2} \sum_{\mu=x,y} \Bigg(\sum_{j=1}^{N} |H_{\mu,j}|^2 \quad (3)$$
$$+ \sum_{j,l(j\neq l)=1}^{N} e^{-i(\omega/w)(z_{0j}-z_{0l})} H_{\mu,j} H^*_{\mu,l} \Bigg).$$

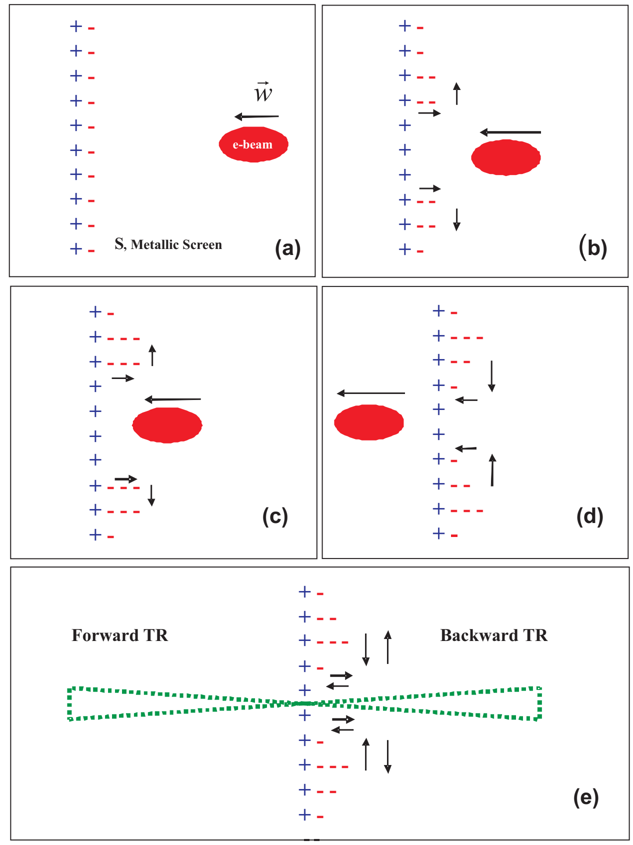

Figure 1: In the picture sequence $[(a) \rightarrow (e)]$, a qualitative and simplified representation of the TR emission is described. The charge double layer - i.e., the metallic foil - experiences a dipolar oscillation induced by the incident charge. With the charge approaching the metallic foil, the conduction electrons, initially at rest on the metallic surface, undergo a tangential displacement due to the electric component of the Lorentz force (the transverse component of the electric field is γ^2 stronger than the longitudinal one). Subsequent to the initial transverse motion, the conduction electrons are also displaced along the longitudinal direction because of the magnetic component of the Lorentz force. Resultant event is a dipolar oscillation of the double layer of charge that generates the TR emission. Due to the simplicity of this pictorial representation, charge oscillation and radiation emission are artificially distinct in two different phases. In reality, such two phases are intrinsically and temporally indistinguishable.

where $H_{\mu,j} = H_{x,y}(\vec{\kappa}, \omega, \vec{\rho}_{0j})$ $(\mu = x, y)$, see Eq.(2). The signature of the causality constraint in the both the expression of the TR field and energy spectrum is evident. The emission phases from the radiator of the N single electron radiation field amplitudes $H_{\mu,j}$ are in a causality relation with the temporal sequence of the N electron collision on the metallic screen, which only depends on the distribution of the N electron longitudinal coordinates z_{0j} $(j = 1, .., N)$, see Eqs.(1,2). Causality consistent is also the reciprocal interference of N single electron ampli-

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tudes in the TR energy spectrum - see Eqs.(3) - being this only ruled by the reciprocal emission delays $(z_{0j} - z_{0l})$, $j, l = 1, .., N$.

The covariant dependence of the TR energy spectrum formula - see Eqs.(1,2,3) - on the distribution of the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1, .., N)$ can be rendered explicit performing the integral calculus in the Eq.(2) in the case of a round radiator surface with a finite radius R, see [5] for more details. For a screen radius larger than the mean beam radius ($R >< \rho_{0j} >=< \sqrt{x_{0j}^2 + y_{0j}^2} >$), the TR field - Eqs.(1,2) - reads [5]

$$E^{tr}_{x,y}(\vec{\kappa}, \omega) = \sum_{j=1}^{N} \frac{2iek}{Dw} \frac{\kappa}{\kappa^2 + \alpha^2} e^{-i[(\omega/w)z_{0j} + \vec{\kappa}\cdot\vec{\rho}_{0j}]} \times \times \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \Big[\rho_{0j}\Phi(\kappa, \alpha, \rho_{0j}) - (R + \rho_{0j})\Phi(\kappa, \alpha, R + \rho_{0j})\Big], \quad (4)$$

where, with (J,K) Bessel function of $1st$ and $2nd$ kind,

$$\Phi(\kappa, \alpha, \rho_{0j}) = \alpha J_0(\kappa\rho_{0j})K_1(\alpha\rho_{0j}) + \frac{\alpha^2}{\kappa} J_1(\kappa\rho_{0j})K_0(\alpha\rho_{0j}). \quad (5)$$

With reference to Eqs.(3,4,5), the explicit expression of the TR energy spectrum of a N electron bunch normally hitting a round radiator with a finite radius R $(0 \le R < \infty)$ reads

$$\frac{d^2 I}{d\Omega d\omega} = \frac{d^2 I_e}{d\Omega d\omega} \Bigg(\sum_{j=1}^{N} |A_j|^2 + \sum_{j,l(j\neq l)=1}^{N} A_j A_l^* \, e^{-i[(\omega/w)(z_{0j} - z_{0l}) + \vec{\kappa}\cdot(\vec{\rho}_{0j} - \vec{\rho}_{0l})]} \Bigg) \quad (6)$$

where

$$\frac{d^2 I_e}{d\Omega d\omega} = \frac{(e\beta)^2}{\pi^2 c} \frac{sin^2\theta}{(1 - \beta^2 cos^2\theta)^2} \quad (7)$$

is the well-known Frank-Ginzburg formula and

$$A_j = \rho_{0j}\Phi(\kappa, \alpha, \rho_{0j}) - (R + \rho_{0j})\Phi(\kappa, \alpha, R + \rho_{0j}). \quad (8)$$

In Eqs.(6,7,8), the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1, .., N)$, on the one hand, contribute to determine the well-known three-dimensional form factor, on the other hand, leave a covariant mark on both the temporal incoherent and coherent components of the TR energy spectrum. The case of the TR emission from an infinite metallic surface $(S = \infty)$ can be obtained by applying the limit $R \to \infty$ to the above results. Under the limit $R \to \infty$, the quantity in Eq.(8) - see also Eq.(10) - reads

$$A_j \to \rho_{0j}\Phi(\kappa, \alpha, \rho_{0j}). \quad (9)$$

Finally, with reference to Eqs.(6,7,9), the formula of the TR energy spectrum of N electrons hitting an infinite radiator $(S = \infty)$ can be obtained. The formal procedure leading to the infinite radiator $(S = \infty)$ results deserves to be emphasized: first, in the implicit expression of the TR field - see Eq.(1,2) - the integral calculus with respect to the radiator surface S is performed for a finite screen radius R; finally, the limit $R \to \infty$ is applied to the so obtained explicit expression of the TR field. In the following subsection, the consequences of applying directly the limit $S \to \infty$ to the implicit expression of the radiation field - Eq.(1,2) - will be described. Numerical simulations of the angular distribution of the TR intensity - refer to the temporal incoherent part of the TR energy spectrum, see Eqs.(6,7,9) - are shown in Fig.(2) for a given value of the beam transverse size and different values of the observed wavelength and beam energy. A beam transverse size dependent diffractive cut-off clearly affects both the angular and spectral distributions of the TR intensity, see Fig.(2) and [5, 6, 7] for more details. Some relevant results already well-known in literature can be derived from the above reported results, see Eqs.(6,7,8). In the case of a single electron travelling on the z-axis, it can be indeed demonstrated [5] that under the limit $R \to \infty$ and $\rho_{01} \to 0$

$$\begin{cases} (R + \rho_{0j})\Phi(\kappa, \alpha, R + \rho_{0j}) \to 0 \\ \rho_{0j}\Phi(\kappa, \alpha, \rho_{0j}) \to 1 \end{cases} \quad (10)$$

the TR field - Eq.(4) - tends to

$$E^{tr,e}_{x,y}(\vec{\kappa}, \omega) = \frac{2iek}{Dw} \frac{\kappa}{\kappa^2 + \alpha^2} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \quad (11)$$

from which the Frank-Ginzburg formula follows, see Eq.(7). Furthermore, in the case of a single electron with $\rho_{01} \to 0$ and a radiator with a finite radius R the well-known result of the TR field of a single electron hitting a round radiator can be obtained from Eq.(4,5,10)

$$E^{tr}_{x,y}(\vec{\kappa}, \omega) = \frac{2iek}{Dw} \frac{\kappa}{\kappa^2 + \alpha^2} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \times \times \Big[1 - \alpha R J_0(\kappa R)K_1(\alpha R) - \frac{\alpha^2 R}{\kappa} J_1(\kappa R)K_0(\alpha R)\Big]. \quad (12)$$

See Eq.(12) and compare it with Eqs.(8, 9) in [8].

Causality and Covariance Defect

In previous section, the formula of the TR energy of an N electron beam normally hitting an infinite metallic surface $(S = \infty)$ - see Eqs.(6,7,9) - was derived according to the following procedure: first, the integral calculus in Eq.(1,2) with respect to a finite screen size $(S < \infty)$ is performed; finally, the limit $S \to \infty$ is applied to the so obtained result. If this procedure is inverted, i.e., if the limit $S \to \infty$ is directly applied to the implicit expression of the TR field - see Eq.(1,2) - before performing the integral calculus with respect the radiator surface S, what are the consequences?

If the limit $S \to \infty$ is directly applied to Eq.(1,2), then the following formula for the TR field can be obtained [7]

$$E^{tr}_{x,y}(\vec{\kappa}, \omega) = \sum_{j=1}^{N} E^{tr,e}_{x,y}(\vec{\kappa}, \omega) \, e^{-i[(\omega/w)z_{0j} + \vec{\kappa}\cdot\vec{\rho}_{0j}]}, \quad (13)$$

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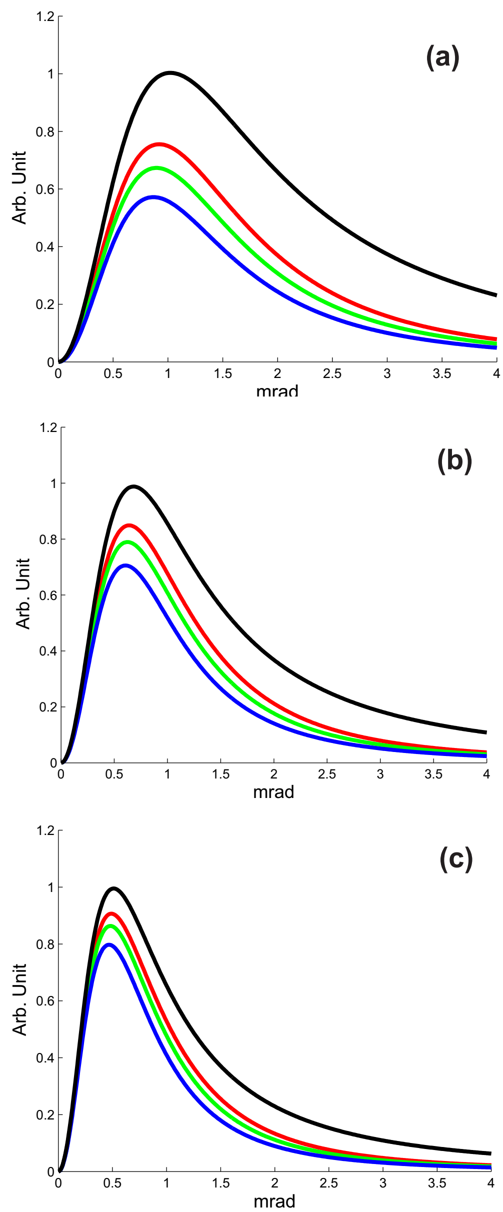

Figure 2: TR angular distribution for different beam energy: (a) 500, (b) 750 and (c) 1000 MeV; wavelength $\lambda = 680\ nm$ (Red curve), $\lambda = 530\ nm$ (Green curve), $\lambda = 400\ nm$ (Blue curve); Gaussian bunch of $N = 10^5$ electrons with $\sigma = 50\ \mu m$. Blue, Red and Green curves from Eqs.(6,7,9), (Black curve) from Eq.(7).

where $E^{tr,e}_{x,y}(\vec{\kappa}, \omega)$ is given in Eq.(11), while the related formula of the TR energy spectrum reads:

$$\frac{d^2 I}{d\Omega d\omega} = \frac{d^2 I_e}{d\Omega d\omega}\Bigg(N + \sum_{j,l(j\neq l)=1}^{N} e^{-i[(\omega/w)(z_{0j}-z_{0l})+\vec{\kappa}\cdot(\vec{\rho}_{0j}-\vec{\rho}_{0l})]}\Bigg) \quad (14)$$

According to the above reported results, the covariant role of the N electron transverse coordinates is completely lost as the absence of any dependence on the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1, .., N)$ in the N single electron radiation amplitudes - see Eq.(13) - and in the temporal incoherent part of the TR energy spectrum - see Eq.(14) - is indicating. Moreover, looking at the formula of the TR field - Eq.(13) - it turns out that the causality role played by the N electron longitudinal coordinates z_{0j} $(j = 1, .., N)$ in determining the emission phases of the N single electron radiation amplitudes is completely mixed up (indistinguishable) with the role of the N electron transverse coordinates (x_{0j}, y_{0j}) $(j = 1, .., N)$ which do not determine the emission phases but only contribute to determine the observation phases as a function of the N electron distances from the z-azis of the reference frame.

CONCLUSIONS

Causality and covariance are physical constraints which a model of the TR emission of a N electron bunch must meet. In case of normal incidence, the covariance signature in the TR model is represented by the intrinsic dependence of the N single electron radiation field amplitudes on the related electron transverse coordinates whose distribution is a Lorentz invariant. The direct connection between the temporal sequence of N electron collisions onto the metallic screen - in the present context, only dependent on the distribution of the N electron longitudinal coordinates - and the emission phases of the N single electron radiation field amplitudes from the radiator surface constitutes the causality signature in the TR model. The improper formal implementation of the limit of infinite surface into the integral calculus of the TR radiation field is the cause of a causality and covariance defect in the formula of the TR energy spectrum.

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