A MULTICONDUCTOR TRANSMISSION LINE MODEL FOR THE BPMS AT THE 3-50 BEAM TRANSPORT LINE IN J-PARC

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Abstract

We have developed an accurate and efficient analysis method with a multi-conductor transmission line model. This method combines the two-dimensional electrostatic analysis including beams in the transverse plane and the transmission line analysis in the longitudinal direction. The loads are also included in the boundary condition of the transmission line analysis. Calculation of 2D electrostatic fields can be easily performed with the boundary element method. Taking low frequency limit of the formula, we have obtained an accurate expectation of the BPMs of 200 mm diameter at the 3-50 Beam Transport Line in J-PARC.

MODELING OF BPMS

A system we analyze is as follows. Pickups in a beam pipe, both of which are longitudinally uniform and made of a perfect conductor, are terminated by some load impedances at some longitudinal points. A beam runs longitudinally with ultra-relativistic velocity. We neglect higher order modes, which means there exist only TEM modes in the pipe.

Solution of the Maxwell Equation

Using the coordinate system, x, y, z where the z-direction coincides the beam and pipe axis, x and y are in the transverse plane. Under the above conditions the Maxwell's equations result in the Poisson equation for the transverse plane and a multi-conductor transmission line equation for the longitudinal plane. The electric and magnetic fields have the relation:

$$\mathbf{E}(x,y,z)=\mathbf{c}\times\mathbf{B}(x,y,z). \tag{1}$$

The electric fields can be expressed as

$$\mathbf{E}(x,y,z)=\tilde{\mathbf{E}}(\mathrm{x,y})\cdot \mathrm{e}^{\mp jkz}. \tag{2}$$

and obey the 2D Poisson equation:

$$\Delta V(\mathrm{x,y})=-\frac{\rho(x,y)}{\varepsilon_0}. \tag{3}$$

Using the potential coefficients and the line charge densities λ_n and λ_0 of n-th conductor and the beam,

$$\begin{bmatrix} \mathrm{V}_1 \\ \vdots \\ \mathrm{V}_n \end{bmatrix} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} + \begin{bmatrix} p_{10} \\ \vdots \\ p_{n0} \end{bmatrix} \lambda_0 \tag{4}$$

$$= \mathbf{p}\lambda + \mathbf{p}_0\lambda_0,$$

the traveling waves are expressed as

$$\mathbf{V}_+ \mathrm{e}^{-jkz} = \frac{1}{c}\mathbf{p}\ \mathbf{I}_+ \mathrm{e}^{-jkz} + \mathbf{p}_0\lambda_0\,\mathrm{e}^{-jkz}, \tag{5}$$

$$\mathbf{V}_- \mathrm{e}^{+jkz} = \frac{1}{c}\mathbf{p}\ \mathbf{I}_- \mathrm{e}^{+jkz}, \tag{6}$$

where λ and λ_0 are the charge densities of the conductors and beam, $\mathbf{V}_+$ and $\mathbf{V}_-$ ($\mathbf{I}_+$ and $\mathbf{I}_-$) are the voltages (currents) of forward and backward waves, k the wave number (=ω/c), c the light velocity, λ_0 the line charge density of the beam. The time variation is assumed to be $\mathrm{e}^{j\omega t}$. The beam propagating forward z-direction is expressed as e^{-jkz}. The beam affects only the forward wave as Eq. 5. There is subtlety that concerns the integral on the boundaries in the transverse plane [1]. The boundary conditions are

$$\mathbf{V}_+ + \mathbf{V}_- = -\mathbf{R}_0(\mathbf{I}_+ + \mathbf{I}_-), \tag{7}$$

$$\mathbf{V}_+ \mathrm{e}^{-jk\ell} + \mathbf{V}_- \mathrm{e}^{+jk\ell} = \mathbf{R}_\ell(\mathbf{I}_+ \mathrm{e}^{-jk\ell} + \mathbf{I}_- \mathrm{e}^{+jk\ell}), \tag{8}$$

if the terminations are done with the impedances, $\mathbf{R}_0$ and $\mathbf{R}_\ell$, at z = 0 and ℓ. As an example the geometry of one electrode and the beam is shown in Fig. 1. Other configurations as the center termination and so on can be also included in the boundary conditions in a similar manner.

When the upstream termination is an open circuit as the 3-50 BT BPM (Fig. 2), and $\mathbf{R}_\ell = \mathrm{R}_\ell\,\mathbf{E}$, the voltage at z = ℓ is

$$\mathrm{V}(\ell) = \left[\cos k\ell + j\sin k\ell \cdot cR_\ell \mathbf{q}\right]^{-1} j\,\mathrm{e}^{-jk\ell}\sin k\ell \cdot R_\ell \mathbf{q}\cdot \mathbf{p}_0 I_0 \tag{9}$$

where **E** is the unit matrix and I_0 the beam current.

As described above the boundary conditions are included in the formulation without artificial

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manipulations or non-physical assumption as grounded electrodes.

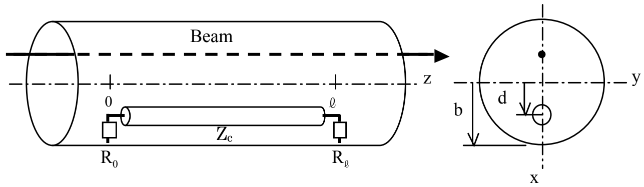

Figure 1: Example: one conductor and a beam.

Low Frequency Limit

At the low frequency ($k\ell = \omega\ell / c << 1$) Eq. 9 can be approximated up to the first order of $k\ell$. For the pickup with one conductor the voltage reads

$$V(\ell) = \frac{j\omega\ell C}{j\omega\ell C + \frac{1}{R_\ell}} \frac{P_0}{c} I_0 . \tag{10}$$

For the pickup with multi-conductor the voltage reads

$$\mathbf{V}(\ell) = \left[j\omega\ell\mathbf{q} + \mathbf{R}_\ell^{-1} \right]^{-1} j\omega\ell\mathbf{q} \frac{\mathbf{p}_0}{c} \mathrm{I}_0 . \tag{11}$$

The capacitance C in Eq. 10 corresponds to the coefficients of capacitances and inductions **q** in Eq. 11, the termination load R_ℓ to the matrix $\mathbf{R}_\ell$, the coefficient of potential P_0 to $\mathbf{P}_0$ and division to matrix inversion.

ANALYSIS OF 3-50BT BPM

There are 14 BPMs in the “3-50BT”, the beam transport line from the 3 GeV RCS to the 50 GeV main ring [2] [3]. The beam from the RCS is collimated by the collimator situated upstream of the 3-50 BT. Then the beam is bended down to fit the MR tunnel level.

BPM Structure

The schematic of the BPMs with inner diameter of 200 mm is shown in Fig. 2. The opening angle is 60 degree and the gap between the electrode and pipe wall is 1 mm. The loads, impedance matching transformers, are set at the downstream end of the electrodes.

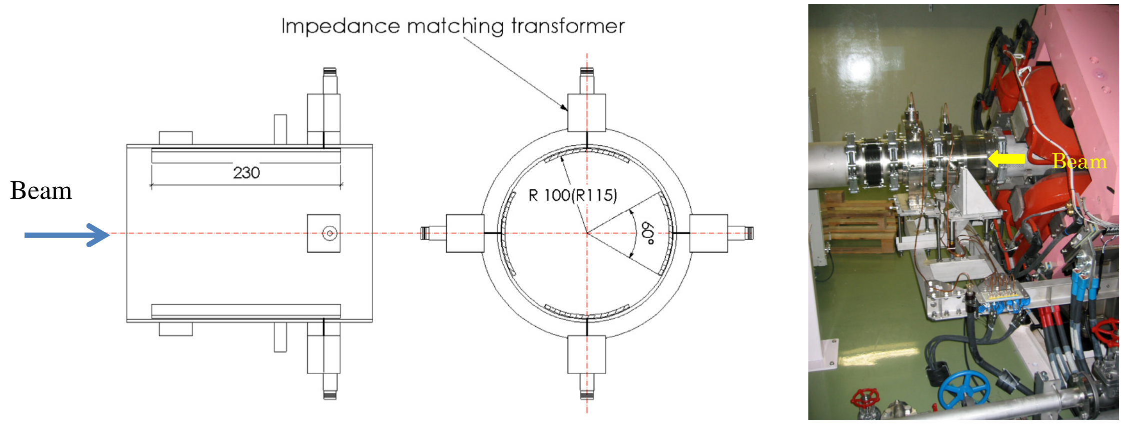

Figure 2: Geometry of the BPM at the 3-50 BT in J-PARC. Left to right, the side view, the cross section and the photograph of the BPM. The load is the impedance matching transformer (15:2) with 50 Ω load.

Numerical Calculation with BEM

The coefficients of capacitance and induction **q**, and the coefficient of potential $\mathbf{P}_0$ are obtained using the boundary element method (BEM) with two-dimensional BPM geometry and the beam point charge [4]. These quantities are independent on a conductor’s specific boundary condition.

Using Eq. 11 we map the transverse beam position (Fig. 3) onto the BPM Δ/Σ space (Fig. 4). In this mapping we use the same **q** and varying $\mathbf{q}_0$ depending on the beam position (x,y). The 10-mm-interval grid lines in Fig. 3 are mapped onto the blue lines in Fig. 4. The difference between the present calculation and the usual calculation with grounded electrodes was remarkable at the radius > 70 mm.

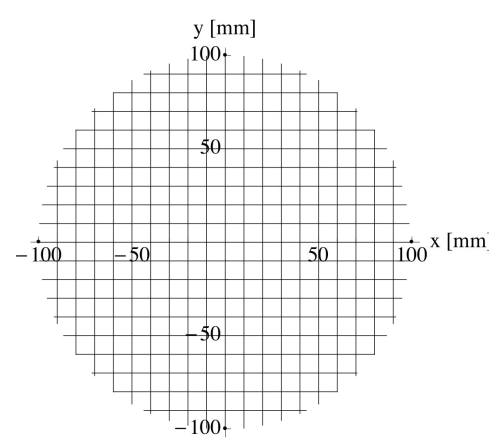

Figure 3: Mapping of the BPM of the 3-50BT. The grid lines represent the beam position, spanning to the BPM aperture with 10 mm interval in the transverse plane.

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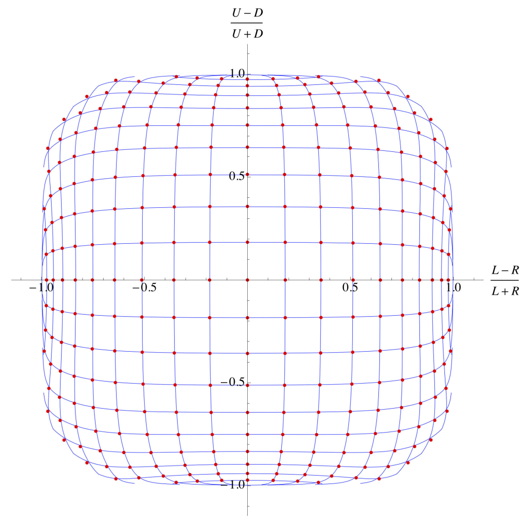

Figure 4: BPM response to the beam or wire. Blue lines are calculated response to the beam on the grid lines in Fig. 3. Dots are the results of the stretched-wire method measured with 10-mm step.

Stretched-Wire Measurement

The stretched-wire has been utilized for BPM calibration [5][6]. The setup is shown in Fig. 5. A copper-plated steel wire of 0.26 mm diameter is stretched in the BPM. Dummy pipes are connected at both ends of the BPM to attenuate HOMs which may otherwise disturb the TEM signal. The wire is tensioned with the weight and terminated with the matching resister. The BPM and dummy pipes are moved in x- and y-direction with stepping motors. The positioning error is estimated less than 30 μm. The measured frequency range is 0.85 M – 10.2 MHz. The result measured at 1.7 MHz is superposed on the calculated lines in Fig. 4. In Fig. 4 the x- and y-offset and tilt in the measured data was corrected by shifting and rotating all the data. The calculated Δ/Σ vs. the beam position and the measured Δ/Σ vs. the wire position on the x-axis are plotted in Fig. 6. The both plots agree very well. The straight line is a linear approximation at the origin. To reduce the error with the linear approximation, a 3rd order polynomial calculation routine was implemented in this May [7].

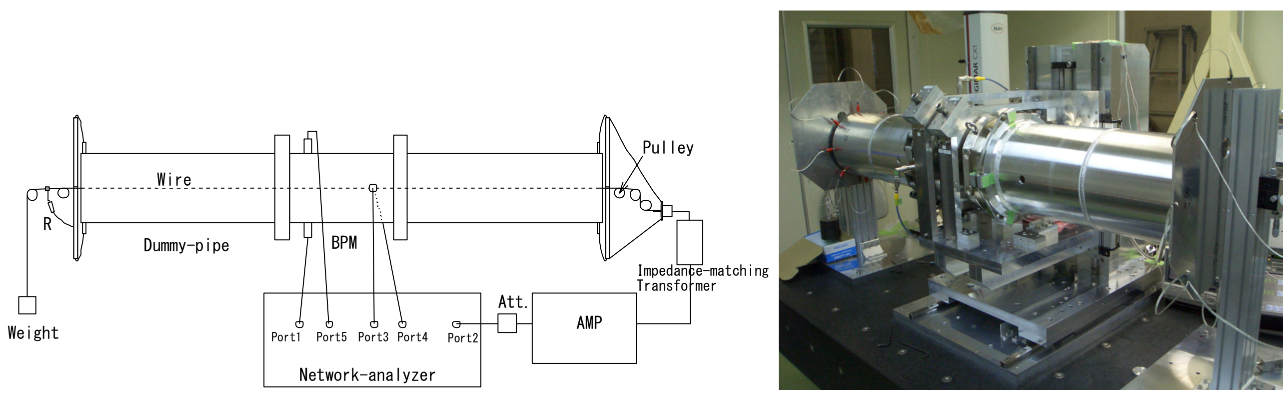

Figure 5: Calibration with stretched-wire method.

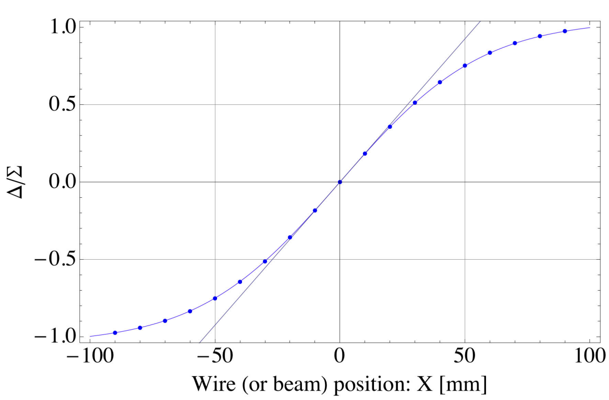

Figure 6: BPM response on the x-axis. Blue curved line: the calculated Δ/Σ vs. the beam position, blue dots: the measured Δ/Σ vs. the wire position, and blue straight line: a linear approximation at the origin.

BPM at the 3-50BT Collimator

We are preparing three additional BPMs at the 3-50 BT collimator section [8][9]. The goal is to measure the beam positions and to calibrate the collimator jaw position. As large beam loss is expected at this section, the rod is chosen as an electrode due to its small surface area that may minimize the charge-up on the electrodes.

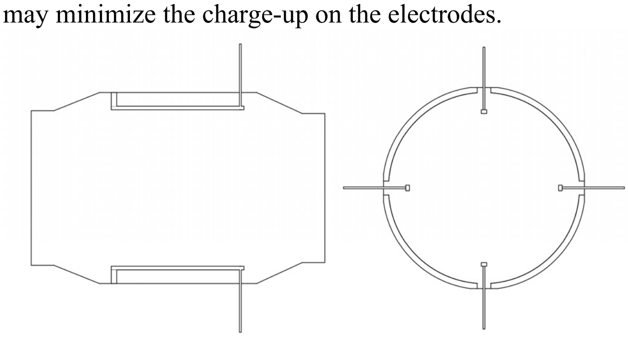

Figure 7: Geometry of the BPM in preparation for the 3-50BT collimator.

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CONCLUSION

We have developed an accurate and efficient analysis method for multi-conductor pickups with the 2D electrostatic analysis including beams in the transverse plane and the transmission line analysis in the longitudinal direction. The method is successfully applied to the BPM with four electrodes at the 3-50BT in J-PARC. The additional BPMs with loop couplers planned for the 3-50BT-collimator section will be analyzed with this method.

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