OPTICS NON-LINEAR COMPONENTS MEASUREMENT USING BPM SIGNALS

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Abstract

The knowledge of linear and non-linear errors in circular accelerator optics is very crucial for controlling and compensating resonances and their consequent beam losses. This is indispensable, especially for high intensity machines. Fortunately, the relationship between the recorded beam offset signals at the BPMs is a manifestation of the accelerator optics, and can therefore be exploited in the determination of the optics linear and non-linear components. We propose a novel method for estimating lattice non-linear components located in-between the positions of two BPMs by analyzing the beam offset signals of a BPMs triple containing these two BPMs. Depending on the non-linear components in-between the locations of the BPMs triple, the relationship between the beam offsets follows a multivariate polynomial. After calculating the covariance matrix of the polynomial terms, the Generalized Total Least Squares method is used to find the model parameters, and thus the non-linear components. Finally, a bootstrap technique is used to determine confidence intervals of the estimated values. Results for synthetic data are shown.

INTRODUCTION

In high energy particle accelerators, magnetic fields are usually employed for beam focusing and deflection, and electric fields are used for beam acceleration. Synchrotrons use cavities to generate accelerating electric fields synchronized with the beam, and electric magnets to generate focusing and deflecting magnetic fields with a strength depending on beam energy.

Constant magnetic fields generated by dipole magnets are usually used for beam deflection, and constant gradient fields generated by quadrupole magnets are usually used for beam focusing. Periodic sequences of focusing/defocusing quadrupole magnets called FODO cells are usually used for focusing in horizontal and vertical directions. This is called strong focusing. Furthermore, other non-linear magnetic fields generated by sextupole or octupole magnets can be applied on purpose, e.g., for chromaticity compensation.

In addtion to the wanted magnetic fields, the magnets can generate unwanted spurious linear and non-linear fields [1] due to fabrication errors or aging. These error fields in the magnets excite undesired resonances leading together with the space charge tune spread to long term beam losses and reducing dynamic aperture [2, 3]. Therefore, these magnets errors and their impact on the beam must be studied and evaluated in order to control and compensate them for better machine operation, such that the demand for higher beam intensity can be fulfilled. Thus, the measurement of the linear and non-linear error components in circular accelerator optics is indispensable, especially for high intensity machines.

The utilization of non-linear chromaticity measurement in determining the non-linear optics model has been presented in [4, 5]. In [2, 3, 6], The Non-Linear Tune Response Matrix (NTRM) technique has been proposed to be used to diagnose non-linear field components. These methods are however very costly and require long measurement campaigns apart from having difficulties in estimating non-linear components with mixed orders.

In this work, we address a new Lightweight approach for determining optics linear and non-linear components in a circular particle accelerator without requiring heavy measurement campaigns. The relationship between the recorded beam offset signals at the Beam Position Monitors (BPMs) is a manifestation of the accelerator optics, and can be therefore exploited in the determination of the optics linear and non-linear components. A pencil like beam is preferred here in order to get rid of finite beam size effects on the signals. Such a beam can be reached by an optimized one turn injection [6]. We estimate the lattice non-linear components located in-between the positions of two BPMs by analyzing the beam offset signals of a BPMs triple containing these two BPMs. The Generalized Total Least Squares method is used for parameter estimation, and a bootstrap technique is used to determine confidence intervals of the estimated values.

SYSTEM MODEL

Three coordinate axes are defined for each position along the synchrotron ring, which determine the different beam offsets from the closed orbit. Fig. 1 shows the transversal coordinates: x for horizontal and y for vertical offset. The longitudinal coordinate is marked by s.

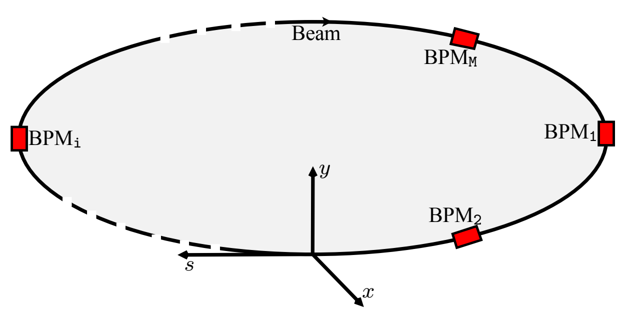

Figure 1: Local coordinates and BPMs.

Multiple BPMs for acquiring beam horizontal and verti-

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cal offsets are placed at different positions along the accelerator ring. In Fig. 1, an accelerator ring with M BPMs is depicted. The BPM signals must be delayed accordingly such that they correspond to the same beam segment or bunch at every sample.

Let $x_i(t)$ and $y_i(t)$ be the signals at the BPM$_i$ at time t, which is located at the position s$_i$ along the accelerator ring, where $i \in \{1, 2, \cdots, M\}$. These signals correspond to the actual beam horizontal and vertical offsets $\tilde{x}_i(t)$ and $\tilde{y}_i(t)$ at s$_i$ perturbed by noise terms $z_{xi}(t)$ and $z_{yi}(t)$, respectively. This means

$$\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} = \begin{pmatrix} \tilde{x}_i(t) \\ \tilde{y}_i(t) \end{pmatrix} + \begin{pmatrix} z_{xi}(t) \\ z_{yi}(t) \end{pmatrix} \quad (1)$$

The basic focusing optics is composed of FODO cells. Furthermore, other non-linear components are existing along the accelerator ring. These non-linear components could be put on purpose, like chromaticity compensating sextupoles, or a dipole magnet non-linear error with an integrated strength located at some position. In Fig. 2, the optics model is depicted with three BPMs. Focusing and defocusing magnets as well as a non-linear component are shown as $\mathtt{N_i}$. Multiple non-linear components could exist as well.

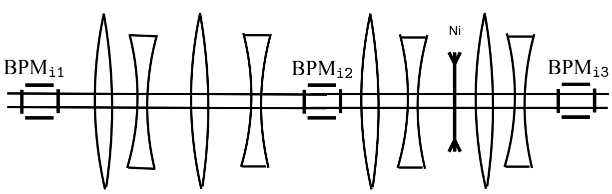

Figure 2: Optics model.

For the sake of simplicity, we assume that non-linear components exist between BPM$_{\mathtt{i2}}$ and BPM$_{\mathtt{i3}}$ and not between BPM$_{\mathtt{i1}}$ and BPM$_{\mathtt{i2}}$. This assumption would not affect the applicability of our approach, if there are no non-linear components between at least two BPMs. In this case, one could take a BPM triple containing these two BPMs. Such BPMs could be for instance bounding a section without magnets errors, or without magnets at all (a drift tube).

Under this assumption, beam offset and angle just before the non-linear components can be written in linear dependence on the beam offsets at BPM$_{\mathtt{i1}}$ and BPM$_{\mathtt{i2}}$, i.e.,

$$\begin{pmatrix} x_{Ni}(t) \\ x_{Ni}(t)' \\ y_{Ni}(t) \\ y_{Ni}(t)' \end{pmatrix} = \mathbf{M}_i \begin{pmatrix} \tilde{x}_{i1}(t) \\ \tilde{x}_{i2}(t) \\ \tilde{y}_{i1}(t) \\ \tilde{y}_{i2}(t) \end{pmatrix}. \quad (2)$$

Since the beam offsets at BPM$_{\mathtt{i3}}$ can be written as a polynomial function of the beam status (horizontal and vertical offsets and angles) before the non-linear components, one can write it as a polynomial of the beam offsets at BPM$_{\mathtt{i1}}$ and BPM$_{\mathtt{i2}}$ depending on the orders and number of the non-linear components. Therefore,

$$\begin{pmatrix} \tilde{x}_{i3}(t) \\ \tilde{y}_{i3}(t) \end{pmatrix} = f(\tilde{x}_{i1}(t), \tilde{x}_{i2}(t), \tilde{y}_{i1}(t), \tilde{y}_{i2}(t)) \quad (3)$$

where $f(.)$ is a multivariate polynomial of order depending on the non-linear components. Hence, one can write for an order N

$$\tilde{x}_{i3}(t) = \sum_{i+j+k+l \le N} \alpha_{ijkl} \tilde{x}_{i1}(t)^i \tilde{x}_{i2}(t)^j \tilde{y}_{i1}(t)^k \tilde{y}_{i2}(t)^l \quad (4a)$$

$$\tilde{y}_{i3}(t) = \sum_{i+j+k+l \le N} \beta_{ijkl} \tilde{x}_{i1}(t)^i \tilde{x}_{i2}(t)^j \tilde{y}_{i1}(t)^k \tilde{y}_{i2}(t)^l, \quad (4b)$$

where the model parameters α_{ijkl} and β_{ijkl} are manifestation of the linear and non-linear optics between BPM$_{\mathtt{i1}}$ and BPM$_{\mathtt{i3}}$.

PARAMETER ESTIMATION

The measurement and variable dependency model described in Eq. (1) and Eq. (4) constitute an errors-in-variables regression model. Such a model can be solved using a Total Least Squares (TLS) approach.

After collecting K measurements from each BPM, the TLS problem can be stated as

$$\mathbf{Y} \approx \mathbf{X}\beta, \quad (5)$$

where

$$\mathbf{Y} = \begin{pmatrix} x_{i3}(1) & y_{i3}(1) \\ \vdots & \vdots \\ x_{i3}(K) & y_{i3}(K) \end{pmatrix} \in \mathcal{R}^{K\times 2}, \quad (6)$$

and

$$\mathbf{X} = \begin{pmatrix} X_1(1) & \cdots & X_T(1) \\ \vdots & \vdots & \vdots \\ X_1(K) & \cdots & X_T(K) \end{pmatrix} \in \mathcal{R}^{K\times T}, \quad (7)$$

where $X_1(1), \cdots, X_T(i)$ are the polynomial terms $x_{i1}(t)^i x_{i2}(t)^j y_{i1}(t)^k y_{i2}(t)^l$, $i+j+k+l \le N$. β denotes the model parameters α_{ijkl} and β_{ijkl} stacked accordingly. For different plynomial terms in the horizontal and vertical direction, the equation must be splitted and solved separately for each direction.

The matrices $\mathbf{X}$ and $\mathbf{Y}$ contain values from the actual beam offsets and noise perturbation terms, i.e.,

$$\mathbf{X} = \tilde{\mathbf{X}} + \mathbf{Z_X}, \quad (8)$$

$$\mathbf{Y} = \tilde{\mathbf{Y}} + \mathbf{Z_Y}. \quad (9)$$

For the TLS estimator of the true parameters in β to be consistent, i.e., $\beta_{\mathtt{TLS}} \to \beta_{\mathtt{true}}$ in probability as $K \to \infty$, $vec([\mathbf{Z_X}\ \mathbf{Z_Y}])$ must be a zero mean random vector with a multiple of the identity covariance matrix [7, 8].

This condition is unfortunately not fulf lled according to the given structure of $\mathbf{X}$. Therefore, preprocessing (prewhitening) must be undertaken on the data before the TLS estimator can be applied as follows:

1. Estimate the covariance matrix $\mathbf{R_{ZZ}}$ for the rows of the perturbation matrix $\mathbf{Z_X}$

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2. calculate $\mathbf{X_p} = \mathbf{X}\,\mathbf{R}_{\mathbf{ZZ}}^{-\frac{1}{2}}$
3. apply the TLS estimator on

$$\mathbf{C_p} = [\mathbf{X_p}\ \mathbf{Y}] \tag{10}$$

to calculate the parameters $\beta_{\mathtt{TLS,p}}$.

The estimate of the actual parameters will be then given by

$$\beta_{\mathtt{TLS}} = \mathbf{R}_{\mathbf{ZZ}}^{-\frac{1}{2}}\,\beta_{\mathtt{TLS,p}}. \tag{11}$$

Let $\mathbf{C_p} = \mathbf{U\Sigma V}^T$ be the singular value decomposition of of $\mathbf{C_p}$ def ned in Eq. (10), where $\mathbf{\Sigma} = \mathtt{diag}(\sigma_1, \cdots, \sigma_{T+2})$, and $\sigma_1 \geq \cdots \geq \sigma_{T+2}$ are the singular values of $\mathbf{C}$. The TLS solution for the preprocessed data can be written using Matlab notation as [7, 8]

$$\beta_{\mathtt{TLS,p}} = -\mathbf{V}_{(1:T,T+1:T+2)}\mathbf{V}_{(T+1:T+2,T+1:T+2)}{}^{-1}. \tag{12}$$

The corresponding TLS residuals estimate is given by

$$\mathbf{\Delta C}_{\mathtt{TLS,p}} = -\mathbf{U}diag(0, \cdots, 0, \sigma_{T+1}, \sigma_{T+2})\mathbf{V}^T. \tag{13}$$

Assuming known linear optics along the path of interest, i.e., between $\mathrm{BPM}_{\mathtt{i1}}$ and $\mathrm{BPM}_{\mathtt{i3}}$, the strength and location of the non-linear components can be determined through exhaustive search to meet the estimated parameters.

CONFIDENCE INTERVALS

A very important aspect of the parameter estimates is their reliability. Since the parameter estimation is applied on noised measurements of the beam offsets, the resulting estimates will be noised as well. Therefore, one is interested in knowing how far the estimates are affected by the measurement noise.

Conf dence intervals, which give interval estimates of the parameters, are very good indication of the parameter estimates reliability. They establish some statistical conf dence for the parameters of interest [9].

A conf dence interval of some parameter consists of two bounds, where the true value of the parameter lies between these bounds with a specif ed probability.

The asymptotic distribution of the estimates can ideally be used to determine conf dence intervals. Under some condition, the TLS estimator has a zero mean multivariate normal asymptotic distribution [10]. The covariance matrix of this distribution has a known form, if the moments up to the fourth order of the rows of the errors matrix are of the same form of a normal distribution [10, 11]. The covariance matrix formula in this case depends on the true value of the parameters β, which could be replaced by its consistent estimate.

In our application however, the moments up to the fourth order of the rows of the errors matrix are not of the same form of a normal distribution. The formula for the covariance matrix of the asymptotic distribution gets therefore complicated, and cannot be calculated. Thus, a bootstrap techniques remain as possible ways to calculate conf dence intervals for the estimated parameters.

Bootstrap Conf dence Intervals

The bootstrap is a computer intensive method for statistical inference using the available data without knowing the population distribution.

Let $\mathbf{C} = [\mathbf{X}\ \mathbf{Y}]$. The non-parametric bootstrap procedure to f nd the empirical distribution of a parameter $\hat{\beta}_{ij} = (\beta_{\mathtt{TLS}})_{(ij)}$ is given as in Algorithm 1 [9, 11, 12]:

Calculate $\hat{\beta}_{ij}$ based on $\mathbf{C}$;
for $k = 1$ **to** K **do**
    construct $\mathbf{C}^{*(k)} \in \mathcal{R}^{K\times(T+2)}$ by resampling with replacement from the rows of $\mathbf{C}$;
    recalculate $\hat{\beta}_{ij}^{*(k)}$ based on $\mathbf{C}^{*(k)}$;
end
sort the items $\hat{\beta}_{ij}^{*(k)}$ into an increasing order such that $\hat{\beta}_{ij}^{*(1)} \leq \cdots \leq \hat{\beta}_{ij}^{*(K)}$;

Algorithm 1: Non-parametric bootstrap

The asymptotic consistency of the non-parametric bootstrap procedure for the TLS estimator has been already shown in [11].

The desired bootstrap interval with conf dence probability $1-\alpha$ is $(\hat{\beta}_{ij}^{*(q_1)}, \hat{\beta}_{ij}^{*(q_2)})$, where q_1 is the integer part of $K\frac{\alpha}{2}$ and $q_2 = K - q_1 + 1$.

The strength of the bootstrap comes from its ability for statistical inference even in complicated situations, as well as its higher accuracy compared to the normal approximation approach [12]. Higher order accuracy of the bootstrap can be achieved by dealing the studentized distribution of the estimators [9, 12]. This bootstrap technique is called percentile-t bootstrap, and can be performed as in Algorithm 2 [9, 12]:

Calculate $\hat{\beta}_{ij}$ based on $\mathbf{C}$;
calculate $\sigma_{\hat{\beta}_{ij}}$ using bootstrap;
for $k = 1$ **to** K **do**
    construct $\mathbf{C}^{*(k)} \in \mathcal{R}^{K\times(T+2)}$ by resampling with replacement from the rows of $\mathbf{C}$;
    recalculate $\hat{\beta}_{ij}^{*(k)}$ based on $\mathbf{C}^{*(k)}$;
    calculate $\sigma_{\hat{\beta}_{ij}^{*(k)}}$ using nested bootstrap;
    calculate $\beta_{\gamma}^{*(k)} = \frac{\hat{\beta}_{ij}^{*(k)} - \hat{\beta}_{ij}}{\sigma_{\hat{\beta}_{ij}^{*(k)}}}$;
end
sort the items $\beta_{\gamma}^{*(k)}$ into an increasing order such that $\beta_{\gamma}^{*(1)} \leq \cdots \leq \beta_{\gamma}^{*(K)}$;

Algorithm 2: Percentile-t bootstrap

Thus, the percentile-t conf dence interval is $(\hat{\beta}_{ij} - \sigma_{\hat{\beta}_{ij}}\beta_{\gamma}^{*(q_2)}, \hat{\beta}_{ij} - \sigma_{\hat{\beta}_{ij}}\beta_{\gamma}^{*(q_1)})$.

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The bootstrap technique can be performed on the parametric model of the data. In this case, the residuals and the estimated beam offset data for the TLS estimator could be calculated using Eq. (13), similar to [13]. Consequently, one could add the corresponding resamples with replacement from the residuals to the estimated beam offsets, and proceed with constructing new matrices $\mathbf{X}$ and $\mathbf{Y}$ as defined in Eq. (7) and Eq. (6) and solving the the model described in Eq. (5). The confidence interval can be thus calculated by repeating the previous technique many times.

RESULTS

An accelerator section with 3 BPMs containing a sextupolar magnet error with a magnetic field of the form in vertical direction $\overrightarrow{B}_y = k(x^2 - y^2)\overrightarrow{e}_y$ is considered. The beam offset relation polynomial for horizontal beam oscillation of this scenario at BPM_3 is described in Table 1.

Table 1: Optics Model Polynomial

Term	x_1	x_2	x_1^2	x_2^2	x_1x_2	y_1^2	y_2^2	y_1y_2
Coef.	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Value	1	-2	0.6	-0.7	0.7	0.1	-0.1	0.4

We have generated data for a one bunch beam oscillating over 1000 turns considering the model polynomial with horizontal and vertical tunes of 0.29 and 0.19, respectively.

Fig. 3 shows the interval estimates the model parameters using our approach. The measurement noise standard deviation is 20% of the smallest oscillation amplitude.

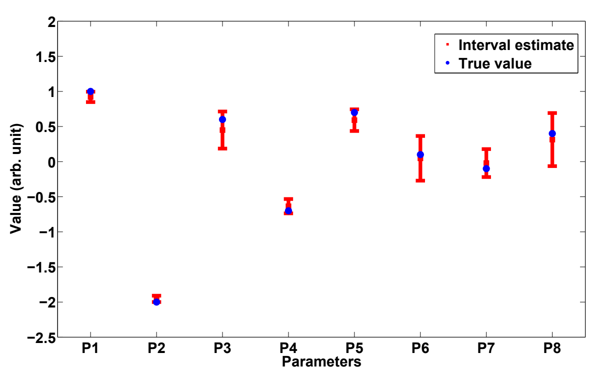

Figure 3: Bootstrap interval estimates.

The results in Fig. 3 can be used to determine the strength and location of the sextupolar magnet error using an exhaustive search. The parameters with a tighter confidence intervals can be given more weight in the search objective function.

In the future, we are going to employ statistical hypothesis testing to detect the order of the model polynomial and non-linear magnet errors.

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