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Beam Position Monitors for Circular Accelerators

4 Oct. 2012 IBIC2012, Tsukuba S. Hiramatsu, KEK

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Beam trajectory



Measurement of closed orbit and/or betatron orbit with BPMs around accelerators gives beam optics parameters ($\beta(s), \phi(s), \eta(s)$ etc).

Wideband BPM system measures betatron orbit:

$$x(s) = \sqrt{\varepsilon\beta(s)}\cos\nu\{\phi(s) - \phi_0\}$$

Narrow band BPM system measures closed orbit:

$$x(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu} \int_{s}^{s+C} \sqrt{\beta(s')} \cos\left\{\pi\nu - \phi(s) + \phi(s')\right\} \frac{\Delta B(s)}{B\rho} ds'$$

Electrostatic pickup of BPM - Beam induced charge on pickup electrode -

 The most popular BPM pickup consists of electrostatic induction pickup electrodes. hadron machine (long bunch)

 ---- a pair of parallel plates
 triangle cut pickup electrodes
 diagonal cut cylindrical electrodes
 etc.

> electron machine (short bunch) ---- button electrodes (2 buttons, 4 buttons)

Beam induced charge on the cylindrical pipe

Charge distribution on the pipe surface can be calculated by using mirror charge.

$$\sigma(R,\phi) = -\frac{\lambda}{2\pi R} \cdot \frac{R^2 - r^2}{R^2 + r^2 - 2rR\cos(\phi - \theta)}$$
$$= -\frac{\lambda}{2\pi R} \left\{ 1 + 2\sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \cos n\left(\phi - \theta\right) \right\}$$

 λ : line charge density of the pencil beam at (*r*, θ)



Beam induced charge on BPM electrodes with opening angle of ϕ_0

Induced charge on parallel-plate electrodes:

$$q_{R} = \int_{-\phi_{0}/2}^{\phi_{0}/2} \sigma(R,\phi)Rd\phi$$

$$= -\frac{\lambda}{2\pi} \left\{ \phi_{0} + 4\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{R}\right)^{n} \sin(\frac{n\phi_{0}}{2}) \cos n\theta \right\}$$

$$q_{L} = \int_{\pi-\phi_{0}/2}^{\pi+\phi_{0}/2} \sigma(R,\phi)Rd\phi$$

$$= -\frac{\lambda}{2\pi} \left\{ \phi_{0} + 4\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{R}\right)^{n} \sin\{n(\pi + \frac{\phi_{0}}{2})\} \cos n\theta \right\}$$
Beam charge beam charge in the second seco

Induced signal voltages are expected to be :

$$V_R \propto q_R, \quad V_L \propto q_L$$



Δ/Σ ratio of signals of parallel pickup electrodes



Calculation of beam induced charge with finite boundary element method

- For the case of the beam chamber with arbitrary shape cross section, the charge distribution induced on the chamber surface by the charged beam can be calculated using the finite boundary element method.
- The two dimensional potential $\phi(\mathbf{r})$ in the beam chamber is given by

$$\phi(\mathbf{r}) = \oint_{\text{surface}} \ln\left(\frac{1}{|\mathbf{r} - \mathbf{r}'(s)|}\right) \sigma(s) ds + \lambda \ln\left(\frac{1}{|\mathbf{r} - \mathbf{r}_0|}\right)$$

 λ_{-} : line density of the beam charge (pencil beam) at ${\bf r}_0$ $\sigma(s)$: surface density of beam induced charge on the beam chamber

 $\sigma(s)$ is given by solving the equation for $\phi(r) = 0$ on the chamber surface.

Finite boundary element method (1) for calculation of induced charge distribution

$$\oint_{\substack{\text{chmaber}\\\text{surface}}} \ln\left(\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'(s)|}\right) \sigma(s) ds = -\lambda \ln\left(\frac{1}{|\boldsymbol{r}-\boldsymbol{r}_0|}\right)$$

The line integral is performed on the closed line along the cross section of the inner surface of the beam pipe.



- λ : line charge density of pencil beam
- σ_i : surface charge density on j-th element of the wall

Beam induced charge on the rectangular beam chamber (finite boundary element method)



Beam Induced charge on triangle cut BPM electrodes



Finite boundary element method (2)

Induced harge distribution $\sigma(s)$ and pickup electrode potential V_k can be obtained simultaneously with finaite boundary element method.

For RC >> τ_{b} (bunch length in time),



electrode-k



 $\sigma(s), V_k$ are determined by finite boundary element method

When we have gap between electrode and chamber,

 κ becomes smaller than $2\sin(\phi/2)/(R\phi/2)$



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Beam induced charge on diagonal cut cylinder

Surface density of the beam induced charge on the inner surface of the cylinder

$$\sigma(a,\phi) = -\frac{\lambda}{2\pi a} \cdot \frac{a^2 - r^2}{a^2 + r^2 - 2ar\cos(\phi - \theta)}$$

Induced charge on each diagonal cut electrode has linear dependence on the beam position x (or y).

$$q_{\pm}(x,y) = \int_{0}^{2\pi} a \tan \theta (1 \pm \cos \phi) \sigma(a,\phi;x,y) d\phi$$
$$= -\lambda \frac{L}{2} \left(1 \pm \frac{x}{a} \right)$$





Charge distribution in the KEKB BPM pickup (BPM with 4 button electrodes)



17

Calculated charge distribution by finite boundary element method



Mapping of KEKB BPM calculated by finite boundary method



BPM calibration stand (wire method) (J-Parc MR) T. Toyama





BPM electrode signal at wire calibration (diagonal cut BPM)

T. Toyama

Stretched wire current is not equivalent to the beam.



BPM mapping at calibration stand

BPM calibration stand (KEKB BPM)



Correspondence between beam position and button signal : Mapping



a_i, b_i are determined by least squares to fit the measured mapping data. IBIC2012, Tsukuba

Signal processing

Pickup response

BPM pickup



Equivalent circuit



Q : induced charge on pickup electrode

$$V(\omega) = \frac{j\omega CR}{1 + j\omega CR} \frac{Q(\omega)}{C}$$

Same response as HPF

Button electrode signal of KEKB BPM ($R=Z_0$: cable impedance) (observed through 100m coax cable)



Capacitance of button electrode (KEKB)

M. Tejima



Resonance problem of signal cable



To reproduce bunch shape in the observed signal V, R_L is required to be $R_LC>>$ (bunch length in time) for electrostatic pickup. High R_L causes signal cable resonance.

R connecting between the pickup electrode and the cable is effective to damp the resonance (R=50-100 Ω for Z₀=50 Ω).



$\Delta \Sigma$ processing at PEPII

C. Roberto et al.



Signal bandwidth is designed to be 20 MHz for turn-by-turn measurement with 20 nsec sampling. Measurement bandwidth is programable by DSP.

Δ/Σ processing at KEKB (narrow bandwidth)

The signal frequency is down converted to the IF by a super-heterodyne. In order to avoid errors caused by analog processing to detect signal amplitude, IF signal is directly digitized and the spectrum amplitude of the beam signal is detected by FFT in the DSP.



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Δ / Σ processing at Jparc (wide bandwidth: 10MHz)



Each electrode signal is directly digitized.

The spectrum amplitude of the beam signal is detected by FFT in the DSP.

Log-ratio BPM

F. D. Wells et al.

$$V_{out} = K \ln \frac{A}{B} = 2K \tanh^{-1} \frac{A-B}{A+B}$$







- Output signal is more linear than Δ/Σ for parallel electrode pickup.
- Normalized beam position signal is obtained in real time.
 - ---- Available for real time feedback
- Gain ripple of the log-amplifier against the input signal level causes the beam position error of about ~1%.

Log-ratio BPM at KEKB

M. Tejima



 $X = \log(A/C) - \log(B/D), \quad Y = \log(A/C) + \log(B/D)$



Improved nonlinearity by log-ratio processing.

AM-PM conversion processing



Big technical difficulty to make the system with high precision. Large dynamic range and high real-time bandwidth can be expected. (R. E. Shafer)

Beam based characterization of BPMs

Beam based alignment (BBA) of BPM



Beam orbit x_n at BPM_n changes for $x_m \neq 0$ by changing QM_m strength ΔK_m . Find out the beam orbit x_n unchanged by ΔK_m with steering magnets. The beam orbit unchanged by ΔK_m is the center orbit of the QM_m. The measured beam position by BPM_m is the BPM_m offset δx_m .

Beam based alignment (BBA) J-Parc 50 GeV Ring

T. Toyama



Figure 7: Result of the Beam Based Alignment. (a) analysed using BPM89 only, (b) using all BPM's. IBIC2012, Tsukuba

BPM offsets measured by BBA KEKB LER & HER

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OFFSET by BBA @LER







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COD change by BBA (KEKB/LER)



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Beam based pickup gain calibration of 4 button pickups (KEKB)

0.98

0.96

1

Button signal response for the beam position (x,y) :

$$F_{1}(x, y) = 1 + a_{1}x + b_{1}y + a_{2}(x^{2} - y^{2}) + b_{2}(2xy) + a_{3}(x^{3} - 3x^{2}y) + b_{3}(3xy^{2} - y^{3}) + a_{4}(x^{4} - 6x^{2}y^{2} + y^{4}) + b_{4}(x^{3}y - xy^{3}) + \cdots$$

$$\begin{cases} F_2(x, y) = F_1(-x, y), & F_3(x, y) = F_1(-x, -y) \\ F_4(x, y) = F_1(x, -y) \end{cases}$$

i-th button signal at j-th measurement :

$$V_{ij} = g_i q_j F_i(x_j, y_j) \begin{cases} i = 1, \dots, 4 \\ j = 1, \dots, m \end{cases}$$
$$J = \sum_{i=1}^{4} \sum_{j=1}^{m} \{V_{ij} - g_i q_j F_i(x_j, y_j)\}^2$$

Least squares of J give :

$$\begin{cases} g_2 / g_1, \dots, g_4 / g_1 \\ q_1, \dots, q_m, x_1, \dots, x_m, y_1, \dots, y_m \end{cases}$$



²⁰¹ BPM No.

301

101

M. Tejima

V1

401

401

401

V2

V

beam

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BPM offsets after beam based gain calibration (measured by BBA, KEKB LER & HER)

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After gain correction, offset by BBA @ LER



After gain correction, offset by BBA @ HER



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BPM offsets by imbalance of pickup electrode gains(KEKB)

BPM offset (w/o beam based gain calibration)					
LER(horizontal/vertical)		HER(horizontal/vertical)			
average σ	0.110/-0.025 mm 0.49/0.43 mm	-0.097/-0.085 mm 0.34/0.40 mm			
BPIM offset (w/ beam based gain calibration)					
LER(horizontal/vertical) HER(horizontal/vertical)					
average σ	-0.026/-0.034 mm 0.374/0.254 mm	-0.005/-0.052 mm 0.272/0.254 mm			

Beam based gain calibration of diagonal cut BPM (J-Parc)

Vu VR $\begin{cases} V_{L} = \lambda \cdot \left(1 + \frac{x}{a}\right), \quad V_{R} = g_{R} \cdot \lambda \cdot \left(1 - \frac{x}{a}\right) \\ V_{U} = g_{U} \lambda \cdot \left(1 + \frac{y}{a}\right), \quad V_{D} = g_{D} \cdot \lambda \cdot \left(1 - \frac{y}{a}\right) \end{cases}$ $V_{Lj} = -\frac{1}{g_{R}}V_{Rj} + \frac{1}{g_{U}}V_{Uj} + \frac{1}{g_{D}}V_{Dj}, \quad j = 1, \dots, m$ $\begin{pmatrix} -\mathbf{V}_{R1} & \mathbf{V}_{U1} & \mathbf{V}_{D1} \\ \vdots & \vdots \\ -\mathbf{V}_{Rj} & \mathbf{V}_{Uj} & \mathbf{V}_{Dj} \\ \vdots & \vdots \\ -\mathbf{V}_{Rm} & \mathbf{V}_{Um} & \mathbf{V}_{Dm} \end{pmatrix} \begin{pmatrix} \frac{1}{g_R} \\ \frac{1}{g_U} \\ \frac{1}$

The equation is written by

Ax = b

$$\boldsymbol{A} = \begin{pmatrix} -V_{R1} & V_{U1} & V_{D1} \\ -V_{R2} & V_{U2} & V_{D2} \\ \vdots & \vdots & \vdots \\ -V_{Rn} & V_{Un} & V_{Dn} \end{pmatrix}, \quad \boldsymbol{x} = \begin{pmatrix} 1/g_R \\ 1/g_U \\ 1/g_D \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} V_{L1} \\ V_{L2} \\ \vdots \\ V_{L_n} \end{pmatrix}$$

A and *b* have errors \rightarrow Total least squares method Least squares solution : $x_{LS} = (A^T A)^{-1} A^T b$

Total least squares solution

$$\boldsymbol{x}_{TLS} = (\boldsymbol{A}^T \boldsymbol{A} - \boldsymbol{\sigma}_{n+1}^2 \boldsymbol{I})^{-1} \boldsymbol{A}^T \boldsymbol{b}$$

 σ_{n+1} is the smallest singular value of $[A \ b]$

•Good reproduction of gain erros with TLS is confermed by simulation.

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BPM001	g ₂	g 3	g 4
TLS	1.0062	1.0024	0.9873
LS	1.0103	1.0045	0.9892
BPM002	g_2	g ₃	g_4
BPM002 TLS	g ₂ 0.9568	g ₃ 0.9811	g ₄ 0.9463

Beam based gain calibration with LS and TLS.

Table 2: Test of Beam Based Gain Calibrations



Figure 5: Reconstructed mapping data. Red: (x, y) without correction, Black: (x, y) with TLS.

Resolution of BPM system (3-BPM correlation method) T. Toyama



A, B, C are determind by minimizing

$$F^{2} = \sum_{i} \{x_{3,i} - (Ax_{1,i} + Bx_{2,i} + C)\}^{2}$$

Assuming same resolution of 3 BPMs, the resolution is given by

$$\sigma = \sqrt{\frac{1}{N-1} \frac{\sum_{i=1}^{N} \{x_{3,i} - (Ax_{1,i} + Bx_{2,i} + C)\}^2}{1 + A^2 + B^2}}$$





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Measured BPM resolution of J-Parc MR

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Measured resolution in normal mode (narrow bandwidth)



Resolution in single-pass mode (wide bandwidth)



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Measured BPM resolution of KEKB (3 BPMs mehod)

Coupling impedance of BPM pickups

BPM impedance reduction (KEK 12GeV PS) BPM impedance affected the instability T. Toyama



Resonant impedance (measured/calculation) of WCM-type BPM

	ω /2π (GHz)	Q	$egin{array}{c} R_{shunt} \ (\Omega) \end{array}$	R/Q (Ω)
BPM	0.636/0.667	77/2650	1.5×10 ³ /2.6×10 ⁴	19.4/9.8
	1.498/1.377	230/8222	5.3×10 ³ /3.3×10 ⁴	23/40

IBIC2012, KsuTakayama et al. Phys. Rev. Lett. 78 (1997) p.871



D. Arakawa, in KEK Proceedings 98-1 (Japanese)

Replacement of wall-current-type BPMs to electrostatic-type BPMs improved longitudinal emittance blow-up. (KEK 12GeV PS)



Longitudinal emittance vs. beam intensity

Broadband impedance of diagonal cut BPM (J-PARC, MR)

T. Toyama



No significant resonance, $|Z_L/n| < 0.35 \times 186 \text{ m}\Omega = 65 \text{ m}\Omega << 9 \Omega_{max}$ (Keil-Schnell criterion) Y.Shobuda

Coupling impedance of SLAL PEPII BPM

C. –K. Ng et al.

3D EM field calculation with MAFIA



Acceptable impedance is determined so that the maximum growth rate of coupled-bunch instabilities does not exceed the radiation damping rate.

PEPII BPM with 15mm buttons

Energy loss by beam	126 W	
Power out of one cab	9 W (37 W)*	
Transfer impedance a	0.65 Ω	
Broad-band impedan	0.008 Ω (11 nH)	
Narrow-band	MAFIA	$6.5 \text{ k}\Omega \text{ at} \sim 6.8 \text{ GHz}$
impedance:	accepted	3.4 kΩ



Resonant Impedance of KEKB BPMs K. Shibata



Displacement of BPM pickups by thermal distortion of beam chambers (KEKB)

Displacement sensors for BPM pickups



BPM displacement sensors attached to the sextupole magnet

 BPM chambers are displaced from the setting position by beam pipe deformation due to the heating by strong irradiation of synchrotron radiation.

Displacement of bending chamber



Movement of BPM pickup

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3

2.5

2

1.5

0.5

0

-0.5

-1

0

Displacement [mm]

Tune change by BPM displacement (KEKB)

Displacement of BPM pickups against 6-pole magnets

LER vertical tune change



BPM displacement occurs tune change.

Concluding remarks

- We have many issues to be considered for design and construction of the BPM system.
- It is one of the difficult problems to guarantee the precise measurement of beam positions for different operations of accelerators.
 - For example the operation condition of accelerators at the commissioning stage or the machine study is very different from the routine operation.
- To guarantee the accuracy of the beam position measurements at various operation conditions, it is important to fully understand the beam diagnostic devices and also the accelerator facility.