CHOICE OF L*: IR OPTICS AND DYNAMIC APERTURE(*)

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Abstract

A design of interaction region (IR) optics from the viewpoint of nonlinear motion and dynamic aperture limitation is discussed for the FCCee Crab Waist collision scheme. We use the first order tune-amplitude shift as a figure of merit to characterize the strength of nonlinear perturbation caused by different sources including the final focus kinematic terms, quadrupole fringe field, octupole field error in QD0 and chromatic sextupoles. Theoretical prediction is compared with the tracking results. Dynamic aperture limited by different nonlinearities in the IR is presented and analyzed.

INTRODUCTION

A drift L* from the interaction point (IP) to the first quadrupole (QD0) in the collider final focus (FF) is an important parameter not only from the viewpoint of a machine detector interface or detector background condition. This drift length also influences the beam optics and dynamics and hence determines the design of the whole IR and beyond. It is essential especially for the Crab Waist (CW) beam-beam collision [1] because this approach assumes that the bulk of luminosity increase comes from an extremely low vertical beta at the IP ($\beta_y^* \le 1$mm), resulting in large chromaticity (for both vertical betatron tune and function) and ~1÷10-km beta in the FF quadrupoles.

Large chromaticity must be corrected by strong sextupole magnets which usually are arranged in pairs and separated by the –I optical transformation [2]. For the ideal kick-like sextupoles such a scheme cancels all geometrical aberrations. For the realistic length sextupoles, the second order aberrations are cancelled exactly while the higher order terms remain and spoil the DA [3].

Very large beta in QD0 amplifies influence of nonlinear imperfections in quadrupole fields (including the fringe fields and the field errors inside) on nonlinear beam dynamics. These effects can also provide the DA reduction.

One more source of the DA shrink is kinematic terms which originated from the fact that due to a large transverse momentum in the first drift, usual paraxial approximation is not still valid and the next momentum terms should be taken into account.

All the above-mentioned effects are discussed and estimated below. The problem is that there is no general criterion to evaluate relative contribution of a particular nonlinear perturbation to the DA size. Fortunately all important effects are of the forth power (octupole-like) in Hamiltonian canonical variables. Basing on this fact we suggest using vertical nonlinear detuning coefficient as a figure of merit to compare different effects depending on L*. Numerical simulation of the DA shows strong and weak points of such approach.

IR ARRANGEMENT

Typical CW IR consists of several optical modules as it is shown in Figure 1. Strong FF quadrupole doublet squeezes the beam at the IP. The final focus telescope (FFT) matches the IP lattice functions to the rest of the IR. The chromaticity correction section YXCCS consists of the sextupoles Y1-Y2 and X1-X2 combined in two pairs with –I transformation inside of each pair.

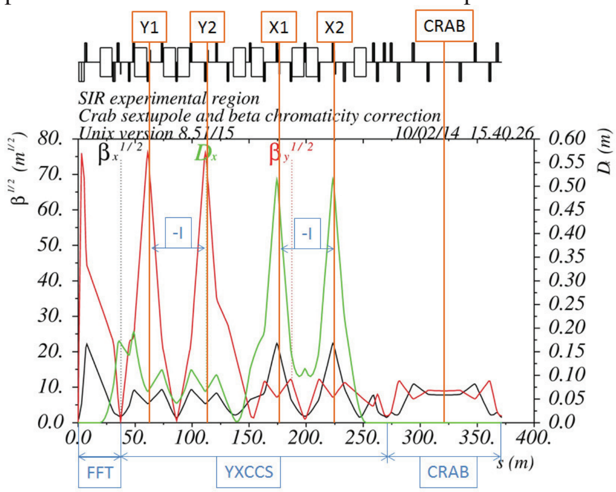

Figure 1: Optical arrangement of the FCCee IR (one half) in the CW mode for L* = 0.7 m.

Dispersion function in the chromaticity correction section is excited by a dipole magnet (BM in Figure 2) and the vertical beta in the Y sextupole pair is as large as in the QD0. Finally the crab sextupole is placed at the end of IR at the proper phase advance with respect to the IP.

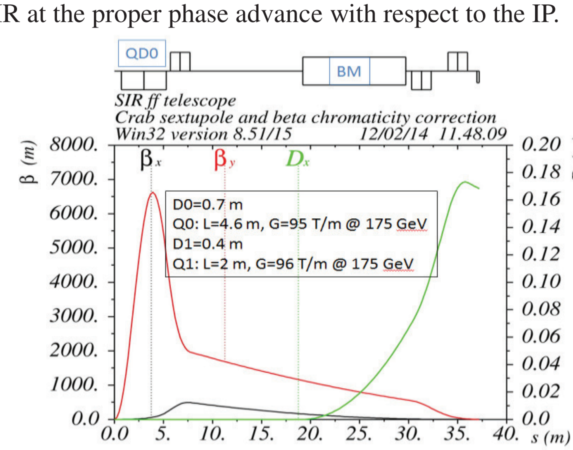

Figure 2: Final focus arrangement.

For the FF parameters shown in Figure 2, $\beta_y^* = 1$ mm yields almost 7 km beta in the middle of QD0.

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QD0 CHROMATICITY

Assuming the vertical beta derivative changes its sign in the middle of QD0 $\alpha_{yOUT} = -\alpha_{yIN}$, one can find the relevant integrated quadrupole strength as

$$(-k_1 L)_{QD0} = 2/L^*. \quad (1)$$

Then, taking into account the beta behaviour over the quadrupole length, the chromaticity produced by QD0 is expressed as (one arm of the IR)

$$\mu'_y \approx -L^*/\beta^*_y. \quad (2)$$

The tune chromaticity associates with the Montague chromatic function a_y [4] excited by QD0 and determining the optical functions behaviour with non-zero $\delta = \Delta p / p_0$ as

$$2\mu'_y = a_y = \frac{d\alpha_y}{d\delta} - \frac{\alpha_y}{\beta_y}\frac{d\beta_y}{d\delta}.$$

To compensate tune and beta chromaticities, we place a pair of vertical sextupoles with $\Delta\mu_y = n\pi$ from QD0 and determine the integrated sextupole strength from the following equation (sextupole pair)

$$\mu'_y + \frac{1}{2}(k_2 2L_s)\beta_{ys}\eta_s = 0. \quad (3)$$

More details on the FCC-ee IR chromatic functions calculation and control can be found in [5].

NONLINEAR TUNE SHIFT

Due to the extremely low vertical beta at the IP and, consequently, extremely large vertical beta in QD0 and chromatic sextupoles, nonlinear motion in the vertical plane determines the IR DA in both directions (in the horizontal one through the coupling terms). To compare relative "power" of different nonlinearities in the IR as a function of L*, we suggest to use a nonlinear detuning coefficient α_{yy} providing the tune shift according to

$$\Delta\nu_y = \alpha_{yy} J_y, \; J_y = A_y^2 / 2\beta_y.$$

where J_y is the vertical action. Fortunately all the nonlinearities we consider here are of the 4th power in canonical variables

$$\Delta H_2 \sim y^n p_y^m, \qquad n + m = 4.$$

and for such (octupole-like) perturbation α_{yy} can be found easily in the first order of perturbative calculation. Moreover, α_{yy} is additive for different sources over the lattice section

$$\alpha_{yy} = \alpha_{yy1} + \alpha_{yy2} + ... = \int_{s_1}^{s_2} (F_1 + F_2 + ...) ds.$$

so one can estimate a combined contribution of several nonlinearities or find a way how to compensate them by introducing nonlinear correctors.

Kinematics Nonlinearity

For large transverse momentum the first order correction of non-paraxiality is given by

$$\Delta H_2 = \frac{1}{8}(p_x^2 + p_y^2)^2, \alpha^k_{xx} = \frac{3}{16\pi}\int \gamma_x^2(s) ds.$$
$$\alpha^k_{xy} = \frac{1}{8\pi}\int \gamma_x(s)\gamma_y(s) ds, \; \alpha^k_{yy} = \frac{3}{16\pi}\int \gamma_y^2(s) ds.$$

where $\gamma_{x,y}(s)$ is the Twiss parameter. The main contribution comes from the first drift [6]

$$\alpha^k_{yy} = \frac{3}{16\pi}\frac{2L^*}{\beta_y^{*2}} \approx -\frac{3}{16\pi}\frac{2\mu'_y}{\beta^*_y}. \quad (4)$$

where 2L* is the distance between two 2QD0s, hence this expression includes both arms of the IR.

QD0 Fringe Fields

Quadrupole fringe field nonlinearity in the hard-edge approximation is described by the following Hamiltonian

$$\Delta H_2 = -k'_1(s) x^2 y p_y / 2 + k''_1(s)(y^4 - 6x^2y^2)/24.$$

and the vertical detuning coefficient is given by [6]

$$\alpha^f_{yy} = \frac{1}{16\pi} k_{10}(\beta_{y1}\beta'_{y1} - \beta_{y2}\beta'_{y2}).$$

where k_{10} is the central gradient of QD0 and lower digital indices correspond to the entrance and the exit of the quadrupole. Calculating the betas and their derivatives at the QD0 edges

$$\beta_{yeQD0} \approx \frac{L^{*2}}{\beta^*_y}, \qquad \beta'_{yeQD0} \approx \pm\frac{2L^*}{\beta^*_y}.$$

one can obtain the following simple estimation (2×QD0)

$$\alpha^f_{yy} \approx -\frac{1}{2\pi} k_{10}\frac{L^{*3}}{\beta_y^{*2}} \approx -\frac{1}{2\pi} k_{10} L^* \mu'^2_y. \quad (5)$$

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Octupole Error in QD0

An octupole field error (or corrector) in QD0 is described by

$$\Delta H_2 = k_3(s)\left(x^4 - 6x^2y^2 + y^4\right)/24.$$

$$\alpha^o_{yy} = \frac{1}{16\pi}\int k_3(s)\beta_y^2(s)ds = \frac{1}{16\pi}k_3\bar{\beta}_y^2 L_{QD0}. \quad (6)$$

Introducing the relative octupole field error at the aperture radius r_a

$$q = \frac{\Delta B_o(r_a)}{B_Q(r_a)} = \frac{k_3}{6k_1}r_a^2.$$

we can rewrite (2×QD0)

$$\alpha^o_{yy} = \frac{3}{4\pi}\frac{q}{r_a^2}k_{10}L_{QD0}\bar{\beta}_y^2 \approx \frac{3}{2\pi}\frac{q}{r_a^2}\frac{L^{*3}}{\beta_y^{*2}} \approx \frac{3}{2\pi}\frac{q}{r_a^2}L^*\mu_y'^2.$$

Chromatic Vertical Sextupoles

Two defocusing sextupoles (Y1-Y2 in Figure 1) located at $\Delta\mu_y = n\pi$ with respect to QD0 correct locally the tune μ_y' and the beta a_y chromaticity. Their strength can be found from (3). To find the vertical detuning coefficient we can use the first order map through the sextupoles separated by –I transform [3]

Sextupole pair $\quad p_y = -p_{y0} - \frac{(k_2L_s)^2 L_s}{6}\left(y_0^3 + x_0^2 y_0\right).$

Octupole $\quad p_y = p_{y0} - \frac{k_3 L_o}{6}\left(y_0^3 - 3x_0^2 y_0\right).$

By analogy of the sextupole pair map with the octupole one, we can directly use (6) with replacing of $(k_2L_s)^2 \to k_3$:

$$\alpha^{sp}_{yy} = \frac{1}{16\pi}(k_2L_s)^2\bar{\beta}_y^2 L_s \quad .$$

for one IR arm. Substituting μ_y' from (3) and taking into account two vertical chromatic sections, one can find

$$\alpha^{sp}_{yy} = \frac{1}{8\pi}L_s\cdot\mu_y'^2/\eta_s^2. \quad (7)$$

Discussion

Kinematics nonlinearity scales as $\alpha^k_{yy} \sim L^*/\beta_y^{*2} \sim \mu_y'/\beta_y^*$. QD0 fringe field effect grows fast with L^* increase and vertical beta decrease as $\alpha^f_{yy} \sim L^{*3}/\beta_y^{*2} \sim \mu_y'^2 L^*$. Octupole error field inside the QD0 demonstrates the same dependence as the fringe field $\alpha^o_{yy} \sim L^{*3}/\beta_y^{*2} \sim \mu_y'^2 L^*$ and with the L^* increase, tolerance for QD0 field quality becomes tougher. The vertical sextupoles for local chromaticity correction introduce the amplitude dependent tune shift which scales with the first drift length as $\alpha^{sp}_{yy} \sim \mu_y'^2/\eta_s^2$, and there is a possibility to mitigate effectively this effect by increase of the dispersion function in the sextupoles.

ESTIMATION VS SIMULATION

The given above expressions were applied to the Crab Waist FCCee IR lattice provided by A.Bogomyagkov [7]. For a range of L^* from 0.7 m to 3 m an appropriate optical solution was found, the chromaticity was corrected and the vertical detuning coefficient was calculated for various perturbation sources. The vertical beta at the IP is $\beta_y^* = 1$ mm. The QD0 strength $k_1 = -0.16\,\text{m}^{-2}$ is the constant for different L^* but its length changes to fit (1). The estimation results are listed in Table 1.

Table 1: α_{yy} for the CW FCCee IR Lattice

L^*(m)	0.7	1	2	3
$-2\mu'_y$	1400	2000	4000	6000
$10^{-6}\alpha^k(\text{m}^{-1})$	0.08	0.11	0.24	0.34
$10^{-6}\alpha^f(\text{m}^{-1})$	0.009	0.025	0.21	0.71
$10^{-6}\alpha^{sp}(\text{m}^{-1})$	-8	-16	-64	-144

One can see that the sextupole nonlinearity due to the finite length effect dominates in Table 1. The reason is a rather low dispersion $\bar{\eta}_s \approx 0.05$ m in the Y chromatic section because of the strong requirement to match IR in the tunnel as straight as possible (see details in [5]). A larger dispersion could significantly suppress the Y-sextupoles influence (see comments later to SuperKEKB case).

We checked the estimation by particle tracking with the help of MAD8 and the BINP home-made accelerator design code Acceleraticum [8] for $L^* = 0.7$ m. The IR optics was closed artificially by the linear matrix with the resulting fractional tunes (0.53, 0.57) providing optimum luminosity. The comparison results are given in Table 2.

Table 2: Nonlinear Detuning: Estimation vs Simulation

	Kin	Fringe	Sext.pair
Simulation			
α_{xx} (m^{-1})	60	1100	-2300
$\alpha_{xy} = \alpha_{yx}(\text{m}^{-1})$	380	15300	-0.07×10^6
α_{yy} (m^{-1})	0.075×10^6	0.1×10^6	-14×10^6
Estimation			
α_{yy} (m^{-1})	0.084×10^6	8700	-8×10^6

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Table 2 clearly indicates prevailing of the vertical non-linearity over the horizontal one. The most critical contribution comes from the Y chromatic correction section as it was predicted by the simple estimation.

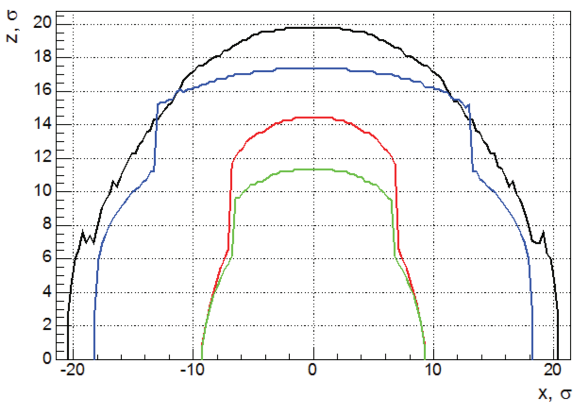

Figure 3: FCCee IR DA for $L^* = 0.7$ m (black), $L^* = 1$ m (red), $L^* = 1.5$ m (green) and $L^* = 2$ m (blue).

A discrepancy in results is explained by the fact that the tracking takes into consideration other sources (like X chromatic section or quadrupole fringes in the Y chromatic module, etc.). Closed ring solution allowed us to find the transverse dynamic aperture for a set of L^*. The relevant plot (in the rms beam sizes) is shown in Figure 3 at the IP azimuth with $\sigma_x = 3.24\times10^{-5}$m, $\sigma_y = 6.52\times10^{-8}$m, $\beta_x^* = 0.5$m, $\beta_y^* = 0.001$m.

With L^* increase the DA in Figure 3 predictably shrinks, but for $L^* = 2$ m it surprisingly grows up. The explanation is a more fortunate design of the optics for the Y chromaticity correction section as it is shown in Figure 4. For $L^* = 2$ m the dispersion function inside the Y sextupoles is slightly larger than that for $L^* = 0.7$ m (≈ 10 cm against ≈ 5 cm) while the vertical beta is almost the same so according to (7) the dispersion function increase compensates enlargement of the first drift length in such a way that the DA is almost equal for both cases.

Anyway, even the largest DA in Figure 3 is not so large, only $\approx 20\sigma_x \times 20\sigma_y$, and the question is if it is possible to open it additionally?

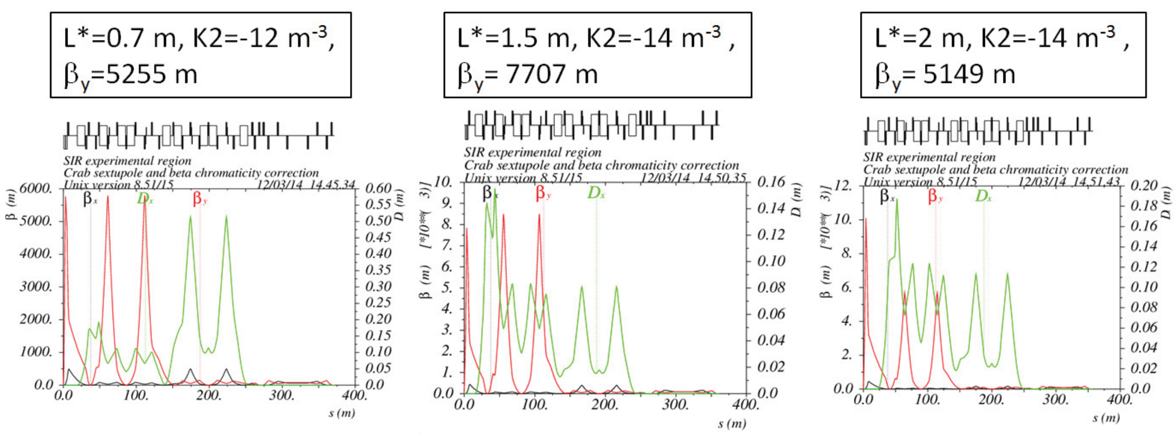

Figure 4: IR optics for three L^* lengths. Vertical betas are indicated in the vertical sextupoles.

The third order aberrations in the sextupole pair with –I transform coming from the finite magnet length can be mitigated by inserting of additional sextupole correctors as it was proposed in [3]. Two low strength (~10% of the main ones) sextupoles also separated by the –I transformation are placed near the main chromatic sextupoles and (if the DA is limited by the sextupoles) can significantly enlarge the DA.

We applied this technique to the FCCee IR with different length of the first drift and indeed obtained sizable improvement of the DA as it is demonstrated in Figure 5. With the sextupole correctors the DA increased up to ~70÷100 sigma in horizontal direction and ~700-800 sigma in vertical direction. This fact confirms that the chromatic sextupoles provide major contribution to the nonlinear beam motion as it was predicted by the α-test.

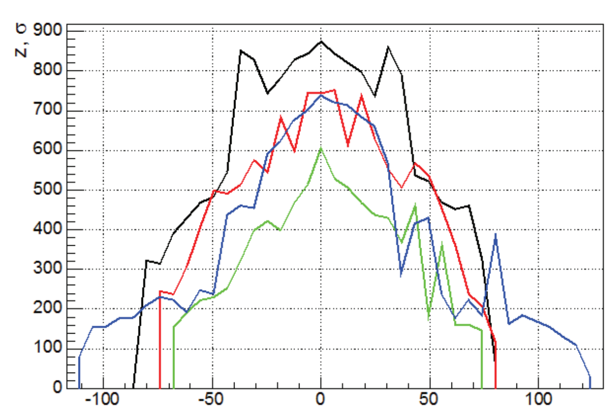

Figure 5: IR DA improved by the sextupole correctors. Colours are the same as in Figure 3.

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The question is which values of α_{yy} correspond to the enlarged DA? The answer is given in Table 3 which compares the vertical nonlinear detuning before and after the sextupole correction for L^*= 0.7 m (uncorrected values are taken from Table 2).

Table 3: α Before and After Correction

	Before	After
α_{xx} (m^{-1})	-2300	51200
$\alpha_{xy} = \alpha_{yx}$($m^{-1}$)	-72000	14000
α_{yy} (m^{-1})	-14×10^6	0.4×10^6

It is clearly seen that the vertical and the coupling alphas are reduced substantially providing the DA enlargement shown in Figure 5. The horizontal alpha has increased but it is still less than the vertical one.

CONCLUSION

To explore nonlinear features of the FCCee collider IR section without knowing in detail the remained lattice we suggested using the vertical amplitude dependent tune shift. One can easily calculate the relevant coefficient by the first order perturbation theory and compare relative contribution of different sources of nonlinear force. We have considered analytically the following nonlinearities: kinematics due to a large transverse momentum at the IP, first quadrupole QD0 fringe field and possible octupole field imperfection, and the sextupoles correcting the vertical FF chromaticity.

It was found that for all considered sources of nonlinearity the vertical detuning is inversely proportional to the IP beta in square $\alpha_{yy} \sim 1/\beta_y^{*2}$, but dependence on L^* differs for them: for kinematic term it is $\alpha_{yy}^* \sim L^*$ while for sextupole pair and for fringe field it is $\alpha_{yy}^{sp} \sim L^{*2}$ and $\alpha_{yy}^{f} \sim L^{*3}$, respectively.

Due to the moderate dispersion function in the sextupole location $\overline{\eta}_s \approx 0.05$m, the sextupole strength is large and this effect determines the size of the IR dynamic aperture. Low strength sextupole correctors can significantly supress the effect of the chromatic sextupoles and open the DA.

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